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## Coherence of Transients

Amos Dotan and William S. Hodgkiss

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<p>This technical report analyzes the performance of various methods of Squared Magnitude Coherency (SMC) estimation when the available data is very short.</p> <p>Three methods of Squared Magnitude Coherency estimation were examined. The first, Coherency estimation using the periodogram and a FFT (the frequency domain). The second, Fitting an AR model to the process, estimating its coefficients, and calculating the SMC from the coefficients (the time domain analysis). The third method is a direct coherency estimation via constrained least squares linear prediction proposed by A. H. Nuttall.</p> <p>For a short data the SMC estimation using the time domain method has a large bias error, and a large confidence interval. The other two methods have reasonable performance for short data.</p> <p>Most of the previous work that calculated the SMC estimation confidence interval, assumed that to the estimated SMC is Gaussian distributed (Rendat &amp; Piersol, Brockwell &amp; Davis). Simulation results show that this assumption holds only for SMC values between 0.3 - 0.8, for long data as well as for short data.</p> <p>Simulation results reveal that as soon as the number of samples is such that the law of large numbers holds (N greater than = 40), all the expressions of the estimated SMC density function developed by Carter et al. and Sakai et al. still hold.</p>					
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## Coherence of Transients

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### ABSTRACT

This technical report analyzes the performance of various methods of Squared Magnitude Coherency (SMC) estimation when the available data is very short.

Three methods of Squared Magnitude Coherency estimation were examined. The first, Coherency estimation using the periodogram and a FFT (the frequency domain). The second, Fitting an AR model to the process, estimating its coefficients, and calculating the SMC from the coefficients (the time domain analysis). The third method is a direct coherency estimation via constrained least squares linear prediction proposed by A. H. Nuttall.

For a short data the SMC estimation using the time domain method has a large bias error, and a large confidence interval. The other two methods have reasonable performance for short data.

Most of the previous work that calculated the SMC estimation confidence interval, assumed that to the estimated SMC is Gaussian distributed ( Bendat & Piersol, Brockwell & Davis ). Simulation results show that this assumption holds only for SMC values between 0.3 - 0.8, for long data as well as for short data.

Simulation results reveal that as soon as the number of samples is such that the law of large numbers holds ( $N$  greater then  $\approx 40$ ), all the expressions of the estimated SMC density function developed by Carter et al. and Sakai et al. still hold.



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## INTRODUCTION

Squared Magnitude Coherence (SMC) and time delay are two quantities needed for passive detection and localization of a radiating source. The source signal, together with environmental noise, is received by sensors at known locations. The coherence function is useful for detection and separation of signals, since it measures the causality and similarity between a pair of signals received by two different sensors.

In the case of two channels, a very common test is to compare the SMC estimate computed using sample sequences from the two channels to a threshold<sup>5</sup>. This is an especially useful test because the statistical behavior of the SMC estimate is well understood in the absence of a common signal, allowing thresholds corresponding to particular false alarm probabilities to be readily established.

When designing an underwater acoustic telemetry system using a chirp DPSK modulation scheme, the coherence between two received pulses is very important in order to evaluate the system performance and gives a great deal of information on the channel characteristics. As only a short data interval is available (each bit is very short), a question arising regarding the quality (in the sense of bias and confidence) of the conventional SMC estimation methods, and whether their performance analysis for short data intervals still holds.

Given two stationary zero mean random processes  $x(t)$  and  $y(t)$ , the squared magnitude coherence (SMC) is defined as :

$$|C(\omega)|^2 = \frac{S_{xy}(\omega) S_{yx}(\omega)}{S_{xx}(\omega) S_{yy}(\omega)} \quad (1.1)$$

Where  $S_{xy}(\omega)$  is the cross power spectrum between the signals  $x(t)$  and  $y(t)$ , and  $S_{xx}(\omega)$  and  $S_{yy}(\omega)$  are the auto power spectra of  $x(t)$  and  $y(t)$  respectively. The most traditional way of SMC estimation uses the periodogram and compute the individual spectra usually via the Fast Fourier Transform<sup>5,10</sup>

All of the methods based on estimation of the auto and cross-spectra have the

advantage of not assuming any signal model, i.e no a priori information about the signal is needed. The disadvantage of these methods is that in cases where there only is limited data length, a tight tradeoff between resolution and estimate stability exists.

A new modeling approach to spectral estimation was proposed in the late sixties by Parzen and Burg<sup>12</sup>. This new method has based on the idea that there is always any a priori knowledge about the processed signals, and uses this information to fit a mathematical model to the process. The drawback of this approach is the need to fit the best model and the need to find the closest model order.

Many deterministic and stochastic discrete time processes encountered in practice are well approximated by a rational transfer function model. In this model an input driving sequence  $\{ \bar{\pi}_n \}$  and the corresponding output sequence  $\{ \bar{x}_n \}$  that is to model the data are related by the linear difference equation,

$$\bar{x}_n = \sum_{i=0}^q [b_i] \bar{\pi}_{n-i} - \sum_{j=1}^p [a_j] \bar{x}_{n-j} \quad (1.2)$$

where  $\bar{x}$  and  $\bar{\pi}$  are  $[d \times 1]$  vectors, and the  $[a]$ 's and  $[b]$ 's are  $[d \times d]$  matrices.

The cross spectral matrix follows from Eq. (1.2):

$$[S(f)] = [A(f)]^{*-1} [B(f)]^* [\Sigma] [B(f)]^T [A(f)]^{T-1} \quad (1.3)$$

where:

$$[A(f)] = [I] + \sum_{k=1}^p [a]_k \exp(-j2\pi k f)$$

$$[B(f)] = [I] + \sum_{k=1}^q [b]_k \exp(-j2\pi k f)$$

and

$[\Sigma]$  is the driving sequence covariance matrix.

Methods of fitting a model and its order using measured data exist Kay<sup>12</sup> (page 1395) Wax<sup>23</sup> and Marple<sup>15</sup>. In the most common models it is assumed that the received time series can be described as an autoregressive (AR) process, moving average (MA) process, or combination of them (autoregressive moving average (ARMA) process)<sup>3, 11, 12, 19</sup>.

Under the assumption of fitting a linear model to the measured process, one method of estimating the SMC is by estimating the cross spectral matrix and substituting the estimated values in Eq. (1.1)<sup>8,15</sup>. Some "semi direct" methods of estimating the SMC based on AR, MA or ARMA process modeling were suggested by Dae Hee Youn et al.<sup>24</sup>, and Nuttall<sup>17</sup>. In these papers, methods of directly estimating the ratio  $\frac{S_{yy}(f)}{S_{xx}(f)}$  and  $\frac{S_{xy}(f)}{S_{xx}(f)}$  were suggested.

The estimator statistical properties are given for only a very few estimation methods. The most detailed statistical analysis is given for the SMC estimation technique using the FFT for the cross spectral matrix estimation. The probability function of the SMC estimator and a numerical method how to calculate it are described in papers by Carter et al.<sup>5-7,13</sup>. The bias of the estimator is calculated in<sup>4,18</sup> and the confidence interval is analyzed by Bendat & Piersol<sup>2</sup> (chap 6), by Fay<sup>9</sup> by Carter<sup>22</sup> and by Brockwell & Davis<sup>3</sup>. The statistical properties of Squared Magnitude Coherence estimation via Multivariate AR modeling is given by Sakai & Tokumaru<sup>20</sup>.

Two important assumption were made in all the statistical analyses of the SMC estimator. The first assumption is that the available data is very long such that the central limit theorem holds, and the second being that the driving signal is white Gaussian noise (WGN)

In many applications such as under water explosion, seismic signals, or other transient events, the received data for analysis is very short by definition and the confidence interval using the traditional FFT method is very large. In this report, we summarize the work done in examining the "performance" of methods of SMC estimation using modern spectral estimation techniques and find their limitation when only short data is available.

## II STATISTICAL METHODS OF SMC ESTIMATION

### II.1 SMC ESTIMATION VIA THE PERIODOGRAM

E. J. Hannan <sup>10</sup> proved that for a time series  $x(n)$  which is generated by a linear process and has a continuous spectrum which is not zero at  $f$ , the asymptotic (the total number of data points is going to infinity) distribution of the SMC estimator has the density:

$$P(c^2/m, c^2(f)) = \frac{2[1-c^2(f)]^m}{B(m-p, p)} c(f)^{2p-1} [1-c^2(f)]^{m-p-1} {}_2F_1(m, m; p; c^2 c^2) \quad (2.1)$$

$$\text{for } f \neq 0, \pi : p = 1, \quad m = \frac{1}{2} N$$

$$\text{for } f = 0, : p = \frac{1}{2}, \quad m = \frac{N-1}{4}$$

$$\text{for } f = \pi : p = \frac{1}{2}, \quad m = \frac{1}{4} N$$

Here,  $B(...)$  is the beta function and  ${}_2F_1$  is the confluent hypergeometric function,  $N$  is the total number of data points.

When using the FFT technique as a SMC estimator, each time series is partitioned into  $n_d$  equal length segments, each having  $n$  data points. The segments may be overlapping or disjoint and each segment is multiplied by a window. The FFT of the weighted  $n$  points is used to estimate the auto and cross power spectral densities which are used to form the SMC estimate.

for  $n_d$  defined as :  $n_d = \frac{N}{n}$ , the density function of the SMC estimator is <sup>5</sup>:

$$P(c^2/n_d, c^2) = (n_d - 1) (1 - c^2)^{n_d} (1 - c^2)^{n_d-2} {}_2F_1(n_d, n_d; 1; c^2 c^2) ; \quad 0 \leq c^2, c^2 < 1 \quad (2.2)$$

It is important to emphasize that Eq. (2.1) and (2.2) hold only in the limit when the number of samples goes to infinity<sup>1</sup>. For all practical purposes these expressions can be used



when the number of points is large enough such that the law of large numbers holds.

Bendat & Piersol<sup>2</sup> give an expression for the confidence interval when the FFT method is used for estimating the SMC. In their analysis they assumed that the estimated SMC is normally distributed<sup>2</sup> [ page 312 in the second ed ], an assumption that holds only for values of SMC approximately between 0.35 and 0.95 and for  $n_d \geq 40$ <sup>2</sup>. [ page 193 in the 1st ed. ].

Scannel & Carter<sup>22</sup> gave a more accurate confidence interval to the SMC estimator as a function of  $n_d$  and the confidence level  $\alpha$ . If the upper and the lower confidence interval are defined as  $A_L(C)$  and  $A_u(C)$  respectively, then

$$\text{prob}[A_L(C) < C < A_u(C) | C] = \alpha$$

where  $\alpha$  is the desired confidence level. They found this interval numerically by taking the estimated SMC density function as in Eq. (2.2). The following figures give their results for confidence level of 80% and 95% and for  $n_d = 8, 16, 32, 64$  and 128.

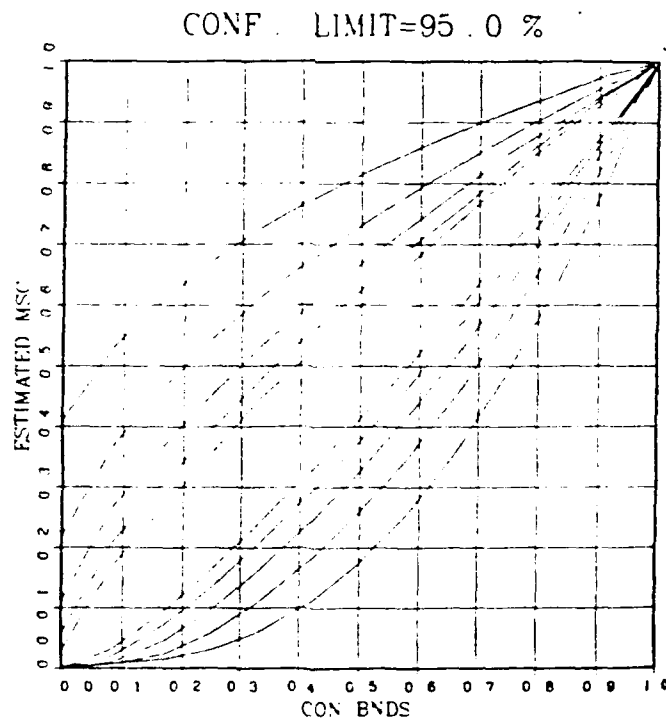
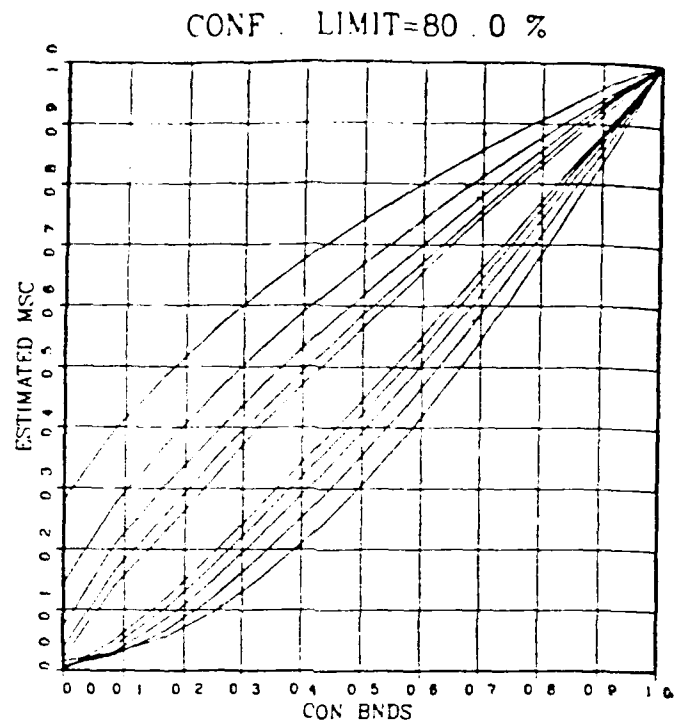


Fig 2.1 80 percent and 95 percent MSC estimate confidence bounds  
for  $N = 8, 16, 32, 64, \text{ and } 128$ .

## II.2 STATISTICAL PROPERTIES OF MULTIVARIATE AUTOREGRESIVE SPECTRAL ANALYSIS

A great deal of effort has been directed toward finding a SMC estimator using modern power estimation method having better performance (in the sense of confidence interval and frequency resolution) for a given length of data. This problem arises especially in cases when only short data is available.

One way of solving this problem is to model the analyzed process. The most general linear model is the autoregressive moving average (ARMA) model.

Using an ARMA model to describe the process is problematic since a set of nonlinear equations is required to be solved in order to estimate the coefficients of the model from the measured data. It is easy to show that any ARMA process can be modeled as an autoregressive (AR) process with an infinite number of coefficients. Taking a sufficient number of coefficients enable us to use an AR model for an ARMA process.

Let us consider a two dimensional p-th order Gaussian AR process:

$$\sum_{j=0}^p [a_j] \bar{x}_{n-j} = [b] \bar{\pi}_n \quad (2.3)$$

where  $[a_0]$  and  $[b]$  are the unit matrices ( $[I]$ ), and  $[a_j]$  are  $[2 \times 2]$  AR coefficients matrices, and  $\{\bar{\pi}_n\}$  is a sequence of Gaussian white noise  $[2 \times 1]$  vectors with mean  $\bar{0}$  and covariance matrix  $[\Sigma]$ .

The estimated coefficient matrices satisfy the following Yule - Walker equation:

$$\begin{bmatrix} [\hat{a}_1], [\hat{a}_2], \dots, [\hat{a}_p] \end{bmatrix} \begin{bmatrix} \hat{R}_0 & \hat{R}_0 & \dots & \hat{R}_0 \\ \hat{R}_{-1} & \hat{R}_{-1} & \dots & \hat{R}_{-1} \\ \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ \hat{R}_{-p+1} & \hat{R}_{-p+1} & \dots & \hat{R}_{-p+1} \end{bmatrix} = \begin{bmatrix} \hat{R}_1, \hat{R}_2, \dots, \hat{R}_p \end{bmatrix} \quad (2.4)$$

for simplicity, let us write Eq. (2.4) as:

$$\begin{bmatrix} [\hat{a}_1], [\hat{a}_2], \dots, [\hat{a}_p] \end{bmatrix} [\hat{\Gamma}] = \vec{\gamma}$$

where

$$[\hat{\Gamma}] = \begin{bmatrix} \hat{R}_0 & \hat{R}_0 & \cdots & \hat{R}_0 \\ \hat{R}_{-1} & \hat{R}_{-1} & \cdots & \hat{R}_{-1} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ \hat{R}_{-p+1} & \hat{R}_{-p+1} & \cdots & \hat{R}_{-p+1} \end{bmatrix}$$

and

$$\vec{\gamma} = [\hat{R}_1, \hat{R}_2, \dots, \hat{R}_p]$$

$\hat{R}_j$  is defined as

$$\hat{R}_j = \frac{1}{N-|j|} \sum_{k=0}^{N-|j|} \bar{x}_{k+j} \bar{x}_k^T$$

The estimated spectral density matrix  $[\hat{S}(f)]$  of Eq. (2.3) is given by :

$$[\hat{S}(f)] = ([\hat{A}(f)]^*)^{-1} [\hat{\Sigma}] ([\hat{A}(f)]^{-1})^T \quad (2.5)$$

where:

$$[\hat{A}(f)] = [I] + \sum_{k=1}^p [\hat{a}]_k \exp(-j2\pi f)$$

and:

$$[\hat{\Sigma}] = \hat{R}_0 + \sum_{k=1}^p \hat{A}_k \hat{R}_{-k}$$

The SMC estimator is:

$$SMC = \frac{s_{21}(f) s_{12}(f)}{s_{11}(f) s_{22}(f)} \quad (2.6)$$

where:  $s_{ij}$  is the  $ij$  element of  $[\hat{S}(f)]$

Following Sakai & Tokumaru<sup>20,21</sup>, the SMC estimator distribution  $F(c'|c)$  is given by :

$$F(c'|c) = \text{Pr}(c' \leq c' | c) = \quad (2.7)$$

$$= \frac{1}{2\pi\sqrt{v}} \int_{\phi=0}^{2\pi} \int_{r=0}^{\sqrt{c'}} r \exp \left[ -\frac{1}{2} \left( r\sqrt{K/m} - \sqrt{\frac{Kn}{m}} \right)^2 \right] dr d\phi$$

changing integration order and integrating with respect to  $r$  leads to:

$$F(c'|c) = \frac{1}{2\pi\sqrt{v}} \int_{\phi=0}^{2\pi} \left\{ \frac{1}{Ka} \exp \left[ \frac{K}{2} \left( \frac{n}{m} - r \right) \right] \left[ \exp\left(-\frac{\gamma^2}{2}\right) - \exp\left(-\frac{\eta^2}{2}\right) \right] \right. \\ \left. + \sqrt{\frac{2\pi nK}{m}} \left[ \operatorname{erf}(\gamma) - \operatorname{erf}(\eta) \right] \right\} d\phi \quad (2.8)$$

where:

$$v = \left[ \frac{p}{N} \right]^2 (1-c)^3, \quad K = \frac{N}{p} \frac{1}{(1-c)^2}$$

$$m = 1 - c \sin^2(\phi), \quad n = c \cos^2(\phi)$$

$$\gamma = \sqrt{\frac{Kn}{m}}, \quad \eta = \sqrt{K n c'}$$

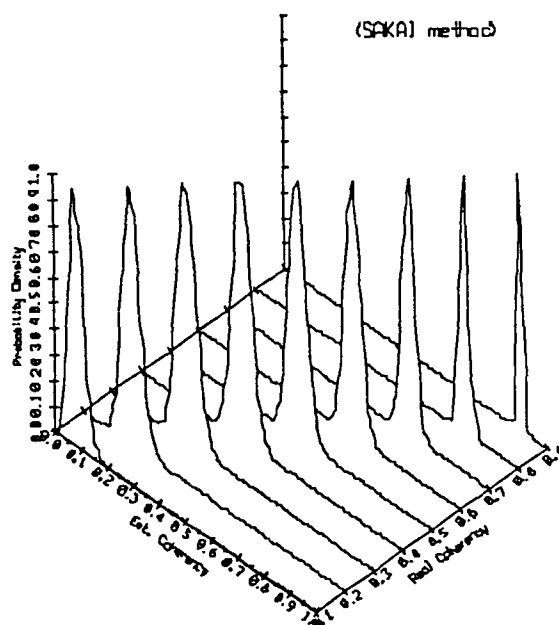
The result above is slightly bit different than that of Sakai & Tokumaru<sup>20</sup>.

Differentiating Eq. (2.7) with respect to  $c'$  leads to the probability density function of the SMC estimator  $P(c'|c)$  :

$$P(c'|c) = \frac{1}{2\pi\sqrt{v}} \int_{\phi=0}^{2\pi} \exp \left[ -\frac{K}{2} \left( \sqrt{m c'} - \sqrt{\frac{n}{m}} \right)^2 \right] d\phi \quad (2.9)$$

Plots of the distribution function and the density function for various values of  $c'$ , and the ratio  $\frac{N}{p}$  are enclosed. Figures 2.2 - 2.7 give the probability density, the cumulative distribution, the mean and the squared standard deviation of the estimated SMC values as a function of the real SMC, for large values of the ratio  $\frac{N}{p}$  (2048, 1024, and 512 respectively).

# Prob. Dens. Func. of Est. Smc



# Com. Prob. Distr. of Est. Smc

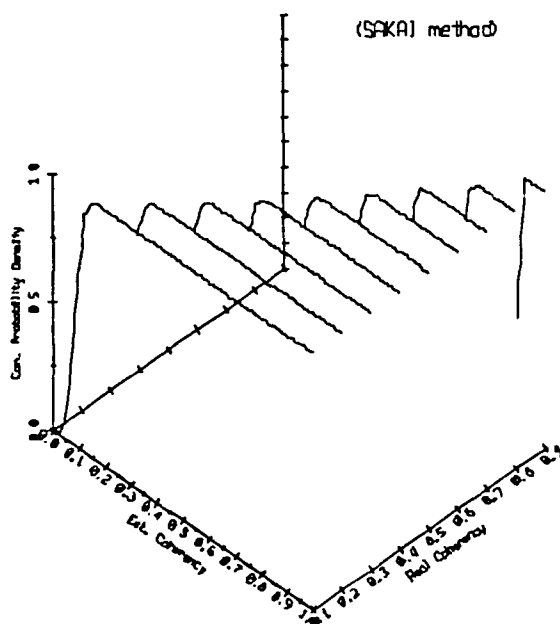
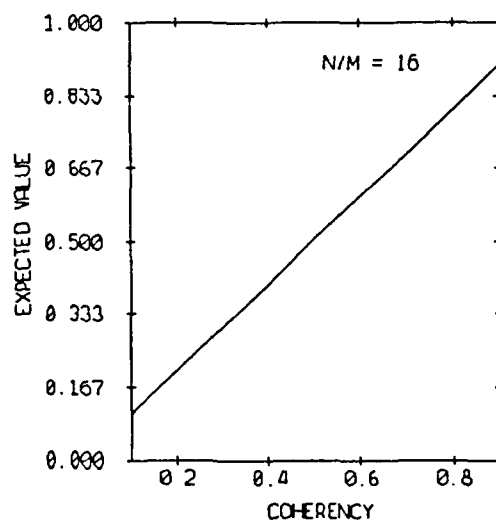
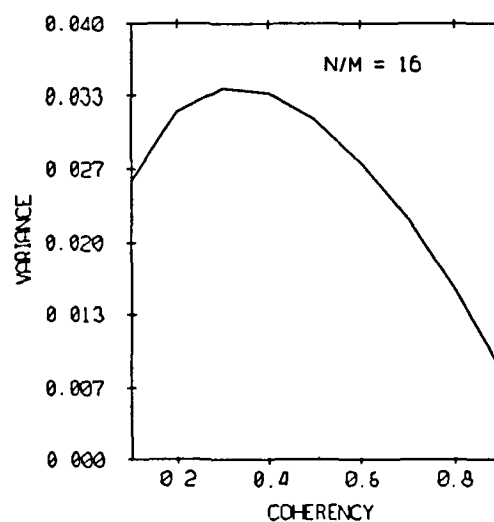


fig 2.2 : probability density and comulative distribution of the estimated SMC as a function of the real SMC for  $\frac{p}{N} = 2048$

## EXP. VALUE of S.M.C ESTIMATION

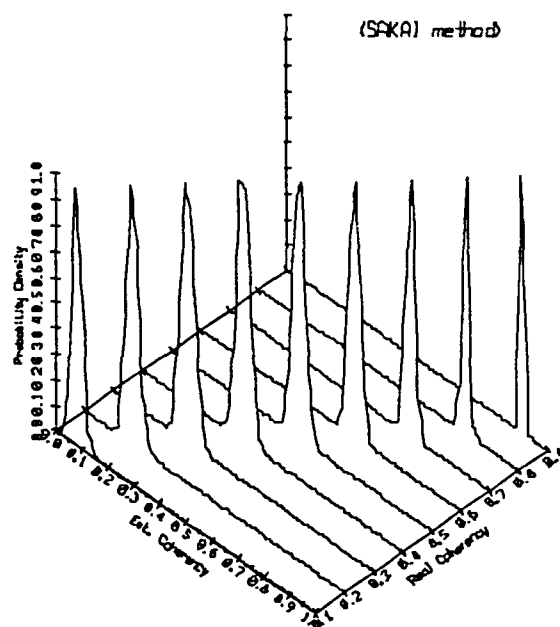


## VARIANCE of S.M.C ESTIMATION



**Figure 2.3 :** Mean and squared standard deviation of the estimated SMC as a function of the real SMC for  $\frac{N}{p} = 2048$ .

# Prob. Dens. Func. of Est. Smc



# Com. Prob. Distr. of Est. Smc

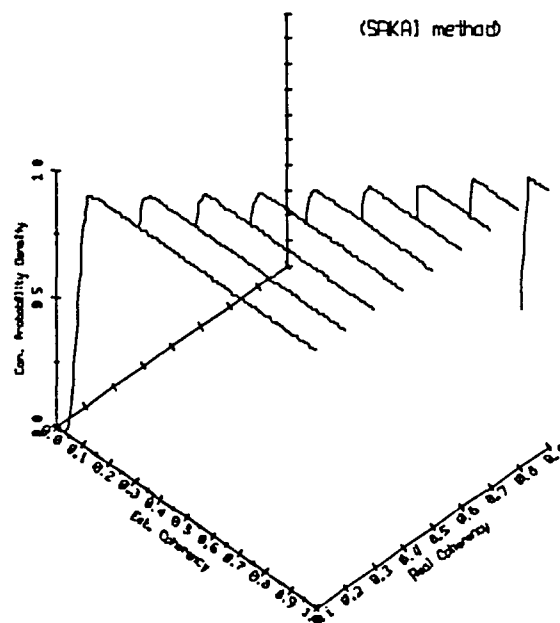
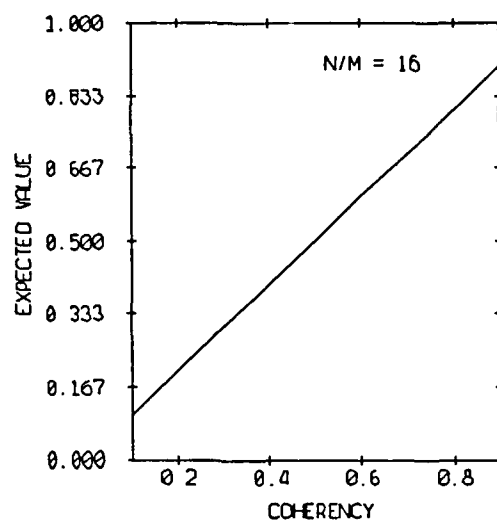


fig 2.4 : probability density and comulative distribution of the estimated SMC as a function of the real SMC for  $\frac{p}{N} = 1024$



## EXP. VALUE of S.M.C ESTIMATION



## VARIANCE of S.M.C ESTIMATION

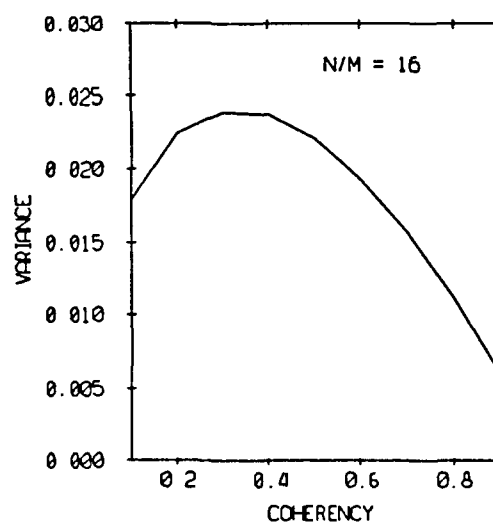
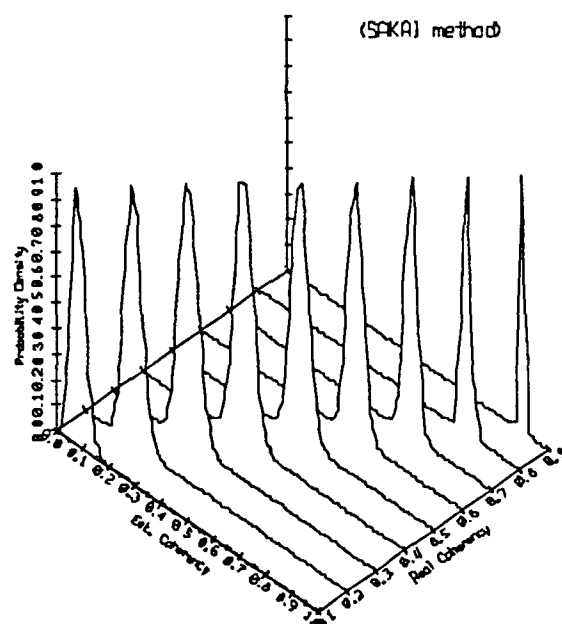


Figure 2.5: Mean and squared standard deviation of the estimated SMC as a function of the real SMC for  $\frac{N}{p} = 1024$ .

# Prob. Dens. Func. of Est. Smc



# Com. Prob. Distr. of Est. Smc

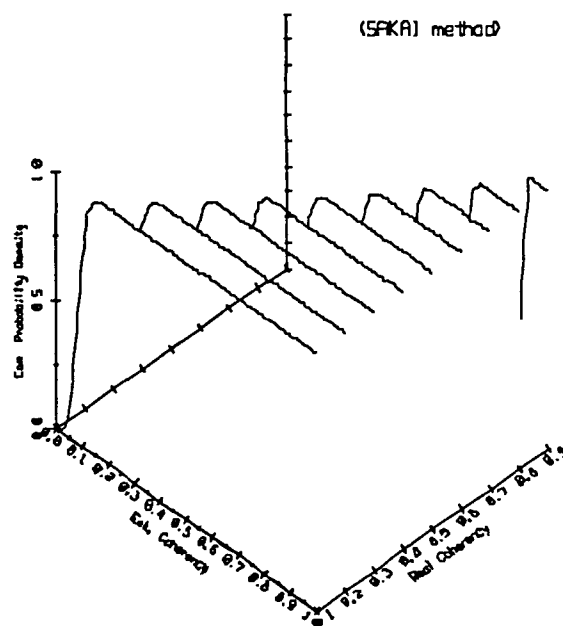
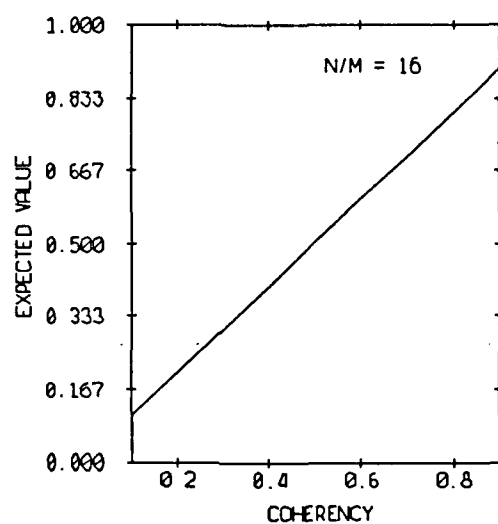
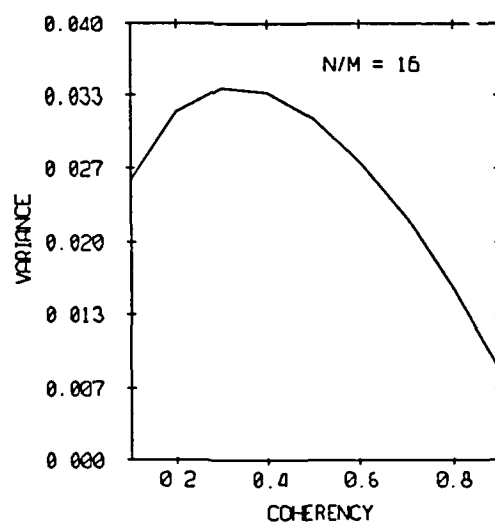


fig 2.6 : probability density and comulative distribution of the estimated SMC as a function of the real SMC for  $\frac{P}{N} = 512$

## EXP. VALUE of S.M.C ESTIMATION



## VARIANCE of S.M.C ESTIMATION



**Figure 2.7:** Mean and squared standard deviation of the estimated SMC as a function of the real SMC for  $\frac{N}{P} = 512$ .

### II.3 OTHER TECHNIQUES OF SMC ESTIMATION

Nuttall<sup>17</sup> suggested a technique which enables estimation of the coherence directly. For two given data sequences  $x(n)$  and  $y(n)$ , two filters are used to linearly estimate  $y$  from  $x$  and also  $x$  from  $y$ . The product of the two filter transfer function is then used as an estimate of the SMC. In this way, no denominator estimates (Eq. (1.1)) are required.

For stationary processes  $x$  and  $y$ , the linear filter with input  $x$  that minimizes the mean square error in predicting the value of  $y$  at the same time instant has a transfer function given by  $\frac{S_{yx}(\omega)}{S_{xx}(\omega)}$ . The linear predictor of  $x$  based upon data recorded  $y$  is accomplished by a linear filter with transfer function  $\frac{S_{xy}(\omega)}{S_{yy}(\omega)}$ . The product of these two transfer function is then precisely the SMC between  $x$  and  $y$ .

The linear estimate of  $y_n$  from the available data record  $\{x_m\}$  and the linear estimate of  $x_m$  from the data record  $\{y_n\}$  are given by

$$\begin{aligned} y_n &= \sum_{k=-q}^p a_k x_{n-k} \\ \hat{x}_m &= \sum_{k=-p}^q b_k y_{m-k} \end{aligned} \tag{2.10}$$

In Chapter 3 and Appendix A we give some simulation results of Nuttall's SMC estimation method.

### II.4 THE EFFECT OF SHORT DATA ON SMC ESTIMATION

As was described in the previous chapters two main methods of SMC estimation have been used. One of them gives the analysis in the frequency domain, where the periodogram plays a fundamental role in the cross spectrum matrix estimation. The other

gives the analysis in the time domain, by which we mean that we postulate some parametric model and the data are fitted to this model by estimating the parameters. The knowledge of these parameters enables us to find directly the SMC function<sup>17</sup>.

Although the analysis described by Carter<sup>5</sup>, Amos and Koopmans<sup>1</sup>, and Hannan<sup>10</sup> holds only in the limit [for  $N \rightarrow \infty$ ], for all practical purposes for  $N$  greater than 40 the law of large numbers holds and it can be assumed that this analysis is valid. It is obvious that the variance of the estimator and the bias becomes larger as the number of points decrease, and hence the confidence level increases. Estimation of the SMC in the frequency domain is very sensitive to the available data length. From Figure 2.1 (Carter<sup>22</sup>), we see that for a relatively small number of time segments, the confidence interval is relatively large.

Brockwell and Davis<sup>3</sup> [Chapter 10.8] proved that for a one dimensional AR process, the AR model parameters are jointly Gaussian in the limit. Following their proof, it is easy to show that their result can also be expanded to the  $d$  dimensional case, i.e:

$$\sqrt{N} ([\hat{a}] - [a]) \Rightarrow Normal [\bar{0}, ([\Sigma][\Gamma]^{-1})] \quad (2.11)$$

where

$$[\Gamma] = \begin{bmatrix} R_0 & R_0 & \cdots & R_0 \\ R_{-1} & R_{-1} & \cdots & R_{-1} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ R_{-p+1} & R_{-p+1} & \cdots & R_{-p+1} \end{bmatrix}$$

and

$$[\Sigma] = R_0 + \sum_{k=1}^p A_k R_{-k}$$

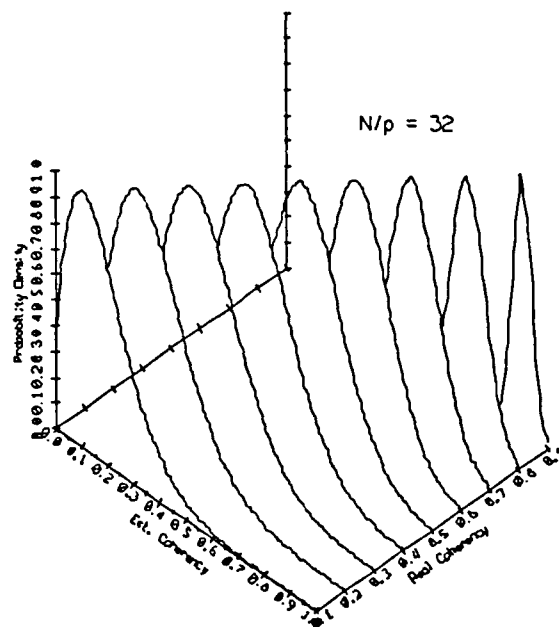
For all practical purposes for  $N$  greater than 40, the law of large numbers holds, and so Eq. (2.11) holds also. Hence, for short data, the statistics of the model estimated coefficients remains jointly Gaussian.

In the time domain analysis, the statistics of the SMC estimator depends only on the statistics of the model estimated coefficients. As these statistics remain jointly Gaussian for short data, the statistics of the estimated SMC does not change also, and the analysis

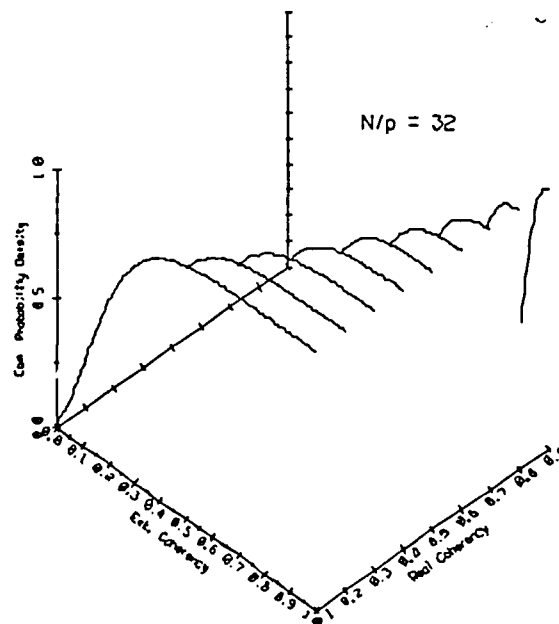
made by Sakai<sup>20</sup> holds.

Figures 2.8 - 2.13 gives the density function, the distribution function, the squared standard deviation (  $\sigma^2$  ) and mean value of the SMC estimation as a function of the real SMC and the ratio  $\frac{N}{p}$  (  $N$  is the data length, and  $p$  is the estimated model order ), for a short data length ( i.e the statistics of the estimated SMC as a function of the real SMC for  $\frac{N}{p} = 32, 16, 8$  )

Prob. Dens. Func. of Est. Smc

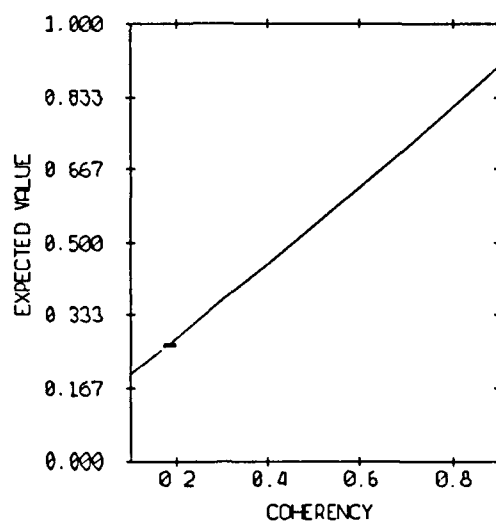


Cum. Prob. Distr. of Est. Smc

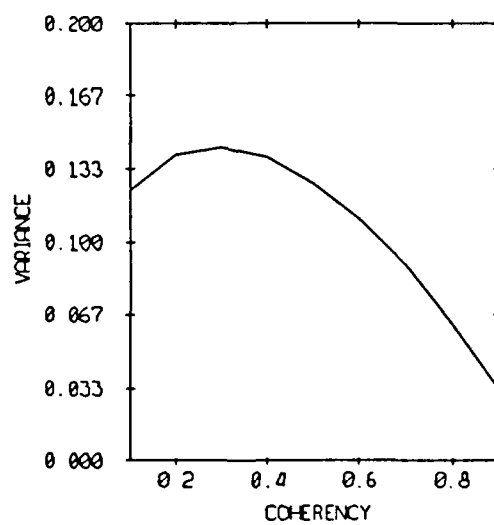


**Figure 2.8.** Probability density and cumulative distribution of the estimated SMC as a function of the real SMC for  $\frac{N}{p} = 32$ .

## EXP. VALUE of S.M.C ESTIMATION



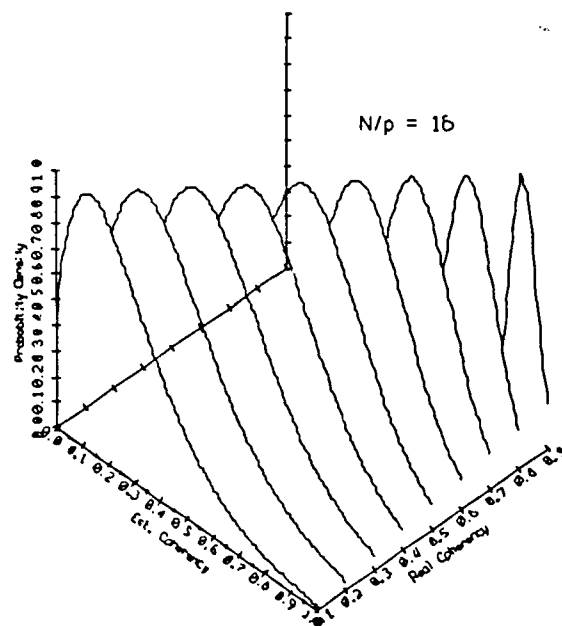
## VARIANCE of S.M.C ESTIMATION



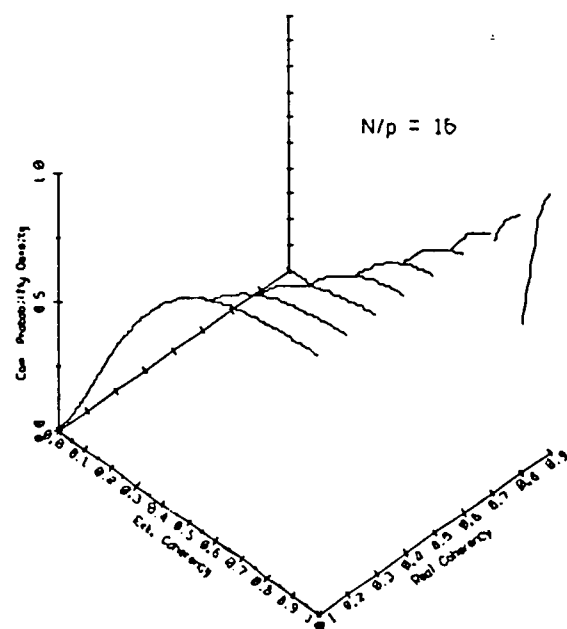
**Figure 2.9.** Mean and squared standard deviation of the estimated SMC as a function of the real SMC for  $\frac{N}{p} = 32$ .



Prob. Dens. Func. of Est. Smc

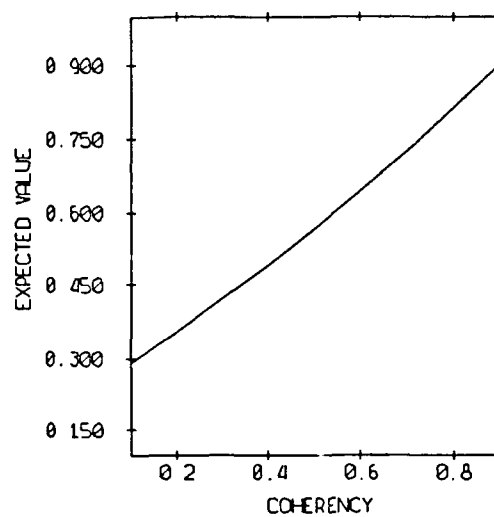


Cum. Prob. Distr. of Est. Smc

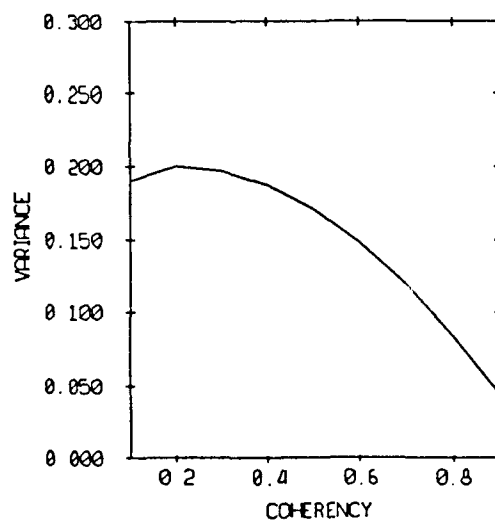


**Figure 2.10.** Probability density and cumulative distribution of the estimated SMC as a function of the real SMC for  $\frac{N}{p} = 16$ .

## EXP. VALUE of S.M.C ESTIMATION

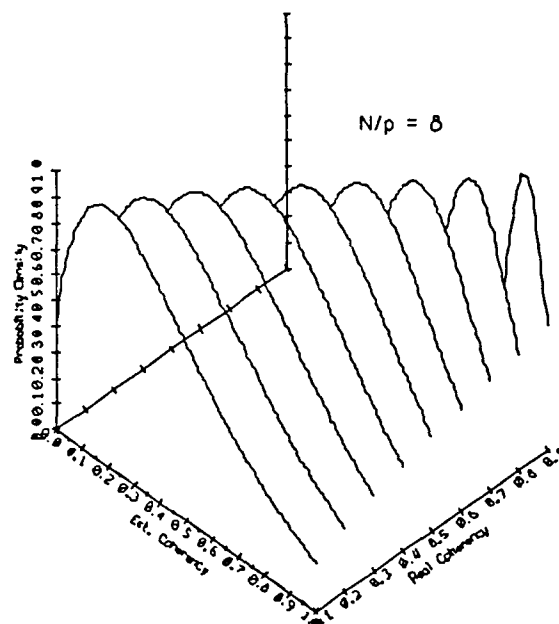


## VARIANCE of S.M.C ESTIMATION



**Figure 2.11.** Mean and squared standard deviation of the estimated SMC as a function of the real SMC for  $\frac{N}{p} = 16$ .

Prob. Dens. Func. of Est. Smc



Com. Prob. Distr. of Est. Smc

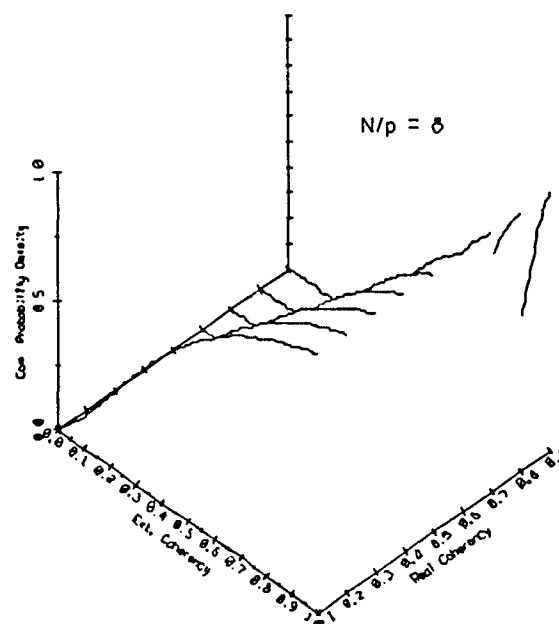
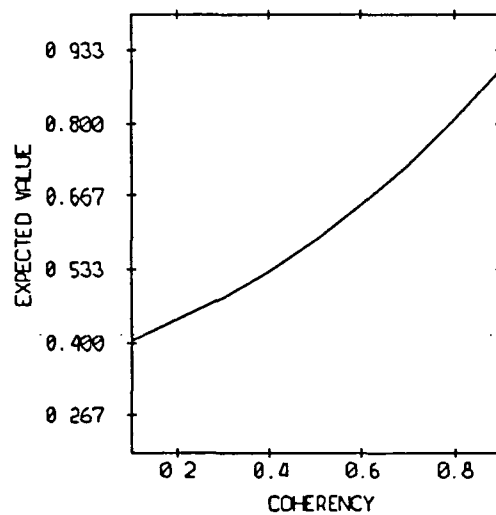
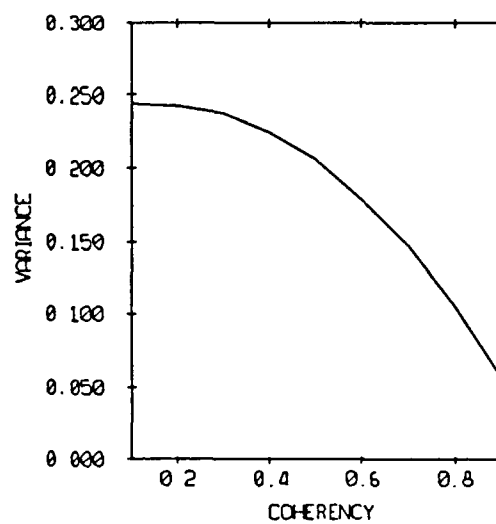


Figure 2.12. Probability density and cumulative distribution of the estimated SMC as a function of the real SMC for  $\frac{N}{p} = 8$ .

## EXP. VALUE of S.M.C ESTIMATION



## VARIANCE of S.M.C ESTIMATION



**Figure 2.13.** Mean and squared standard deviation of the estimated SMC as a function of the real SMC for  $\frac{N}{p} = 8$ .

### III SIMULATION RESULTS

Some simulations were run in order to compare the performance of various methods of SMC estimation . Two models, which are most often used in the literature, were chosen. The first model (denoted as model 1) was a first order ( $p = 1$ ) two channel autoregressive process which was used to generate data ensembles. Data was generated according to the recursion <sup>14-16</sup>

$$\bar{x}_n = [a_1] \bar{x}_{n-1} + \bar{n}_n \quad (3.1)$$

where:

$$[a_1] = \begin{bmatrix} 0.85 & -0.75 \\ 0.65 & 0.55 \end{bmatrix}$$

$$\bar{x}_n = \begin{bmatrix} x_n^1 \\ x_n^2 \end{bmatrix}$$

and  $\bar{n}_n$  is a zero mean white Gaussian noise with the following covariance matrix

$$[\Sigma] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Figure 3.1 gives the power spectra of  $x_n^1$  and  $x_n^2$  for the first model.

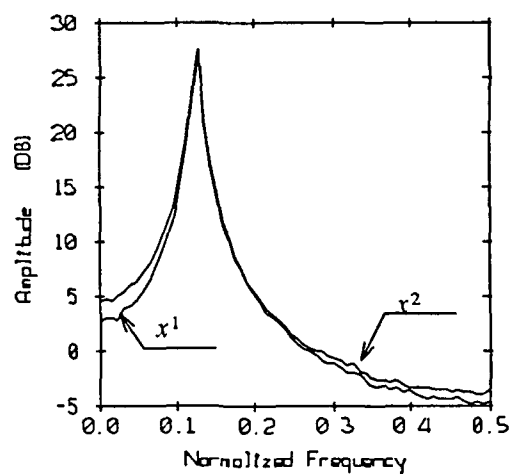


Figure 3.1. The power spectra of the signals of the first model.

The second model (denoted as model 2) is the model used by Sakai and Tokumaru<sup>20,21</sup>

$$\bar{x}_n = \sum_{i=0}^3 [a_i] = \bar{x}_{n-i} + \bar{n}_n \quad (3.2)$$

where

$$[a_1] = \begin{bmatrix} 0.3 & 0 \\ 0 & -0.5 \end{bmatrix}, [a_2] = \begin{bmatrix} 0.2 & 0 \\ 2.0 & 0 \end{bmatrix}, [a_3] = \begin{bmatrix} 0.1 & 0 \\ 0.5 & 0 \end{bmatrix}$$

and  $\bar{n}_n$  is a zero mean white Gaussian noise with the following covariance matrix

$$[\Sigma] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Figure 3.2 gives the power spectra of  $x_n^1$  and  $x_n^2$  for the second model

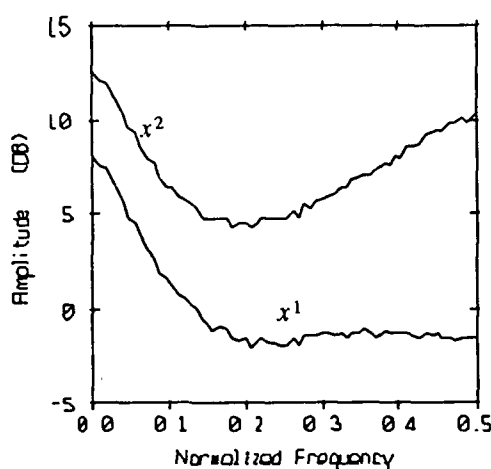


Figure 3.2. The power spectra of the signals of the second model.

In the simulation that we ran, three methods of SMC estimation were examined. The first was the traditional way using the FFT, the second was the method suggested by Nuttall<sup>17</sup>, and the third was via modeling the process, estimating its coefficients and using Eq. (2.5) and (2.6)

The simulation results are given in appendix A. Figure A.1 and Figure A.2 show the true SMC corresponding to the models defined by Eq. (3.1) and (3.2) respectively. Figure A.3 gives the estimated SMC of the process defined by Eq. (3.1)(model No. 1), when the FFT method was used.

For the case when FFT method was used, the given data was divided into 50% overlapped 128 points segments. On each individual data segment, the SMC was calculated (using Hamming window). The estimated SMC is the average calculated SMC over all data segments. For a given data length of  $N$  points, the number of averages ( $n_d$ ) is :

$$n_d = \frac{N}{64} - 1$$

In order to have sufficient data on the statistical behavior of the estimated SMC, 1000 experiments were run on nonoverlapped measured data.

Figure A.4 shows the squared standard deviation and the normalized squared standard deviation of the estimator. The normalized squared standard deviation was defined as

$$\frac{\text{var } \hat{c} - (E(\hat{c}))^2}{c}$$

Figures A.5 - A.20 give the histogram of the estimated SMC at 16 different normalized frequencies  $\bar{\omega}_i$  such that:

$$\bar{\omega}_i = \frac{1}{32}, \frac{2}{32}, \frac{3}{32}, \dots, \frac{15}{32}, \frac{16}{32}$$

Figures A.21 - A.38 and Figures A.39 - A.56 are the same as Figures A.3 - A.20, but for segment length of 256 and 128 points, respectively.

Figures A.57 - A.188 show the results of SMC estimation using Nuttall's method, for data length of 512, 256, and 128 points. Following his simulation we chose  $p = q = 6$  (Eq. 2.10). In order to get the statistics of the estimator, 1000 Monte Carlo experiments were run. Figure A.57 shows the results of all the runs, for data segments of 512 points each. Figure A.57.1 gives the averaged estimated SMC, averaged all over the 1000 experiments. Figure A.58 shows the squared standard deviation and the normalized squared standard deviation of the SMC estimator. Figures A.59 - A.74 give the statistics of the estimated SMC for 16 different frequencies (the same as in Figures A.5 - A.20). Figures A.75 - A.87 and Figures A.88 - A.100 give the statistics of the two filters coefficients (the  $a$ 's and the  $b$ 's in Eq. (2.10)). Figures A.101 - A.144 and Figures A.145 - A.188 are the same as Figures A.57 - A.100 but for data length of 256 and 128 points respectively.

Figures A.189 - A.260 give the simulation result of the SMC estimation following Eq. (2.6) where model No. 1 was taken. 100 run of each experiment were done. Four cases were examined :

- a) - Data length of 1024 points was taken, "estimated" model order was assumed to be 1 (the real model order)



- b) - Data length of 256 points was taken, "estimated" model order was assumed to be 1
- c) - Data length of 128 points was taken, "estimated" model order was assumed to be 1
- d) - Data length of 512 points was taken, "estimated" model order was assumed to be 4

The reason of choosing case 'd' is the desire to compare the estimated SMC statistics with the result of case 'b' and to check if the statistics of the estimated SMC depends only on the ratio  $\frac{N}{p}$  as in Eq. (2.8) and not in the values of  $N$  and  $p$  separately. Both cases 'b' and 'd' have the same ratio  $\frac{N}{p}$ . ( $\frac{n}{p} = \frac{128}{1}$ )

Figures A.189 - A.206 show the average estimated SMC, the squared standard deviation the normalized squared standard deviation and the histograms of the estimated SMC in several frequencies for the case where 1024 points data length was taken and model order of 1 was assumed. Figures A.207 - A.224 shows the case where 256 point data length was chosen, and model order of 1 was assumed. Figures A.225 - A.227 give the results for 128 point data length and model order was assumed to be 1. Figure A.243 - A.258 summarize the case when data length of 512 points was taken, and model order was assumed to be 4.

#### IV DISCUSSION ON RESULTS

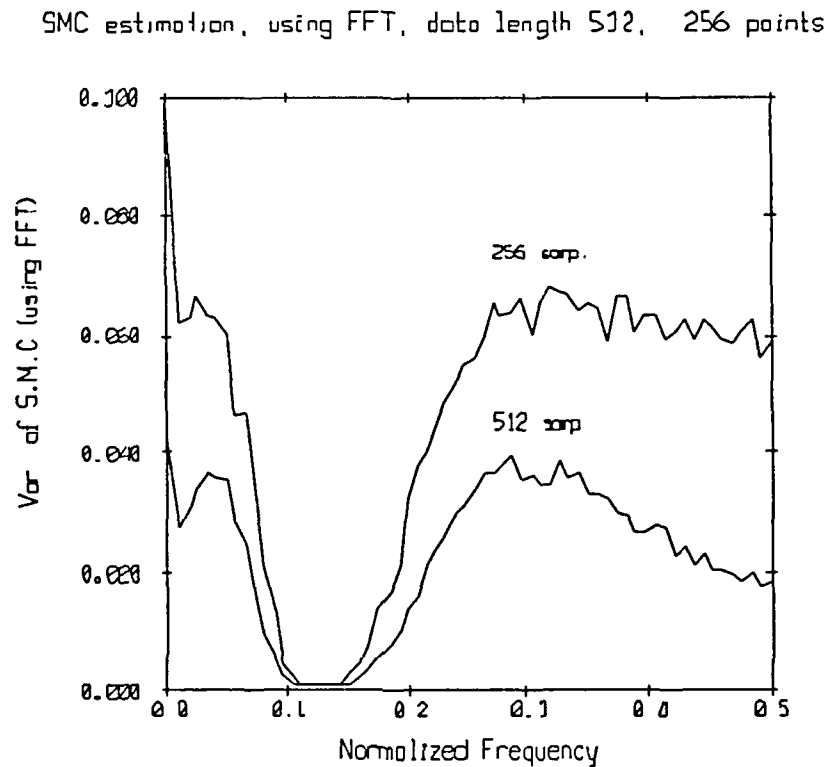
The statistics of the model estimated coefficients were calculated using the simulation results. The coefficients were calculated for the cases when method of estimating the coherence in the time domain was used, or when Nuttall's method is used. The results show that the statistics of the estimated coefficients remains almost Gaussian even for relatively short data (Figures A.163 - A.188), a result that was expected.

Looking at the probability density function of the estimated SMC obtained either from the analytic expression based on Sakai's work<sup>20</sup> [Figures 2.2 -2.7] or from simulation results based on SMC estimation using the frequency domain (the FFT method)(on model No. 1), the time domain (modeling the process as an AR process), or using Nuttall's method gives roughly the same behavior of the estimated SMC probability density function as a function of the real SMC.

For SMC values between 0.3 and 0.8, the probability density function is symmetric and Gaussian-like. Hence the assumption of Brockwell and Davis<sup>3</sup>, and of Bendat and Piersol<sup>2</sup> about the Gaussian behavior of the density function when they calculated the confidence level of the estimated SMC was reasonable. For low values of SMC (less than 0.3) and for high values of SMC (above 0.8) the probability density function is totally different from Gaussian. It is an asymmetric function where the asymmetry becomes larger and larger as the values of the SMC become closer to zero or to one. For these values of SMC, Brockwell's or Bendat's method of calculating the confidence interval gives an inaccurate answer.

The behavior of the estimated SMC density function remains roughly the same even when relatively short data is used, but as expected, the standard deviation of the estimated coherency became larger. Figure 4.1 gives the squared standard deviation ( $\sigma^2$ ) of the estimated SMC of the process defined by model No. 1 (Eq. 3.1) when the method

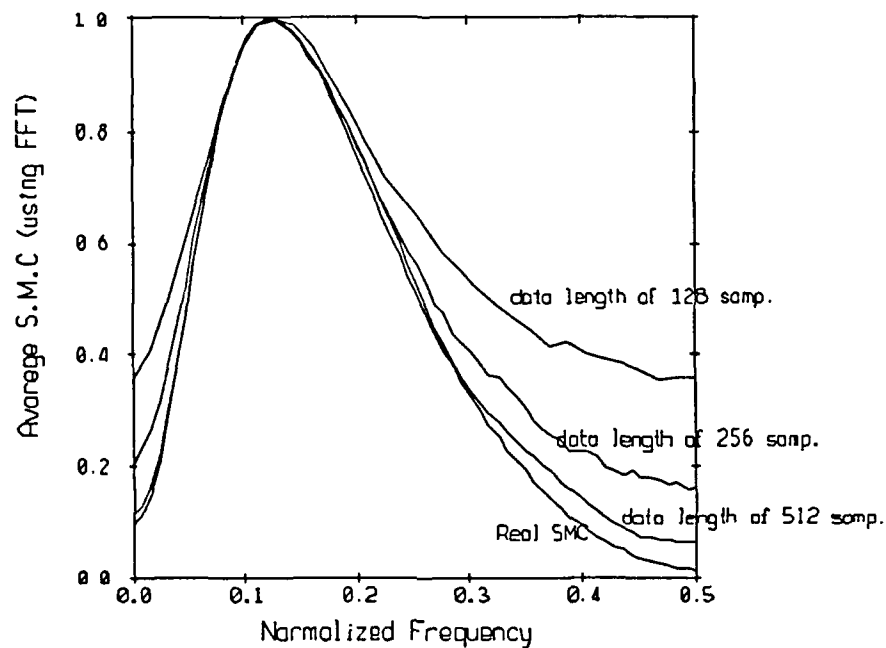
based on the FFT was used to estimate the SMC.



**Figure 4.1.** Variation of squared standard deviation of estimated SMC as a function of data length (model No. 1, using FFT method).

When the FFT method is used to estimate the SMC, it can be seen that the estimator is biased and the bias increases as the data length decreases. This phenomena is strongly dominant for low values of coherence where the bias error is much larger than the real value of the SMC itself. Moreover, careful inspection in Figure 4.2 shows that the bias (as well the standard deviation) does not depend only on the real SMC value but also on the frequency, a result which is not given in the expression by Carter and Sakai.

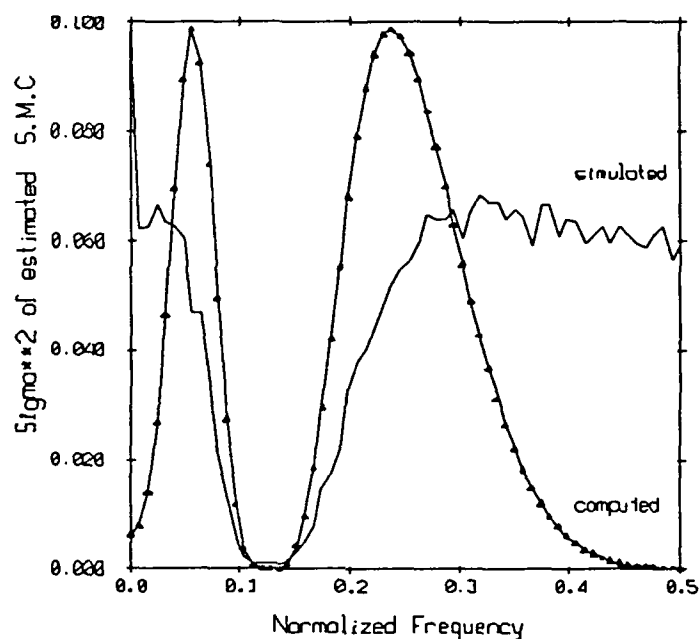
## SMC estimation, using FFT



**Figure 4.2.** Bias variation of the estimated SMC as a function of data length (model No. 1, using FFT method)

Figure 4.3 gives the simulation results and calculated results of the estimated SMC variance for data length of 256 points (i.e.  $n_d = 3$ ). The calculated result was obtained by using the approximation given by Carter et al<sup>5</sup>. As expected, in the region where the statistics of the estimated SMC are Gaussian-like, the calculated and the simulated variance are relatively similar. For low values of SMC, the statistics of the estimated SMC is not Gaussian any longer and the calculated and the simulated variance are different. As a conclusion, we see that the calculated variance is a relatively good upper bound on the estimated SMC variance for SMC values larger than 0.3 even for very short data.

SMC estimation, using FFT, segment length 256 points



**Figure 4.3.** Comparison of the calculated and simulated  $\sigma^2$  of the estimated SMC, for  $n_d=3$ , when FFT method is used.

For model No 1, assuming model order as the real order (1), a comparison of the SMC estimation results between the "FFT" method, Nuttall's method and the "direct" method based on the frequency domain, was made. The comparison shows that the direct method (Eq. 2.4 - Eq. 2.6) is superior in the sense of bias error as well as the variance of the estimation. The worse method is the FFT method which has the largest bias and variance. The difference in the performance between the three methods is emphasized especially at low values of SMC. Figure 4.4 shows the average SMC estimation by the three method for model No 1, model order taken as 1 (the real order), and data length of 128 samples.

SMC estimation, data length 128 points

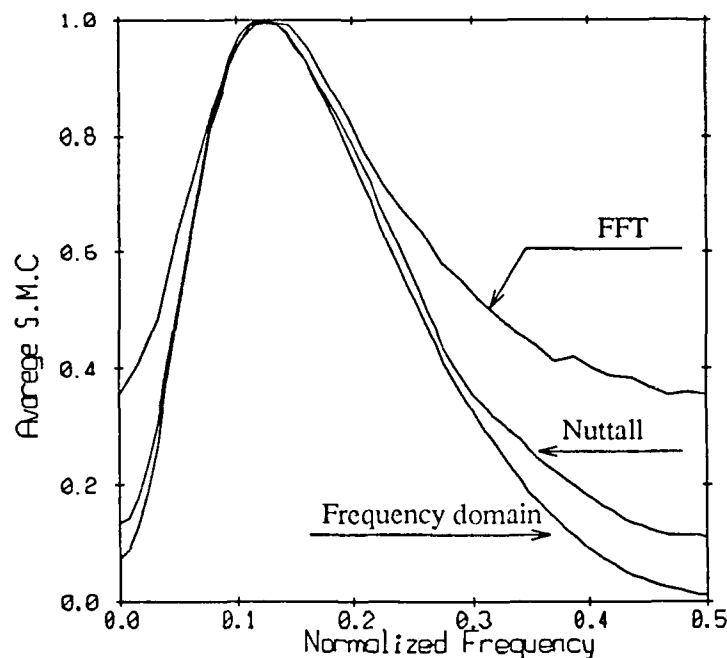


Figure 4.4. Comparison of the average estimated SMC for model No 1, data length 128 samples, and assumed model order is 1.

Examining the results of the SMC estimation when model No 2 is used and comparing it with the results when model No 1 was taken leads to the same conclusion as before: the direct method is the best and the FFT is the worst. An interesting point is that the values of the bias and the variance were slightly different, even though same data length was taken, or different data lengths were taken but such that the ratio  $\frac{N}{p}$  was the same. The comparison shows that the SMC estimation is worst for model No 2.

One of the results we obtained following Sakai and Tokumaru<sup>20</sup> is that the statistics of the estimated SMC depend among other parameters on the ratio between the data length,  $N$ , and the model order,  $p$ . Comparing the simulation results of two cases, 'b' (Figures A.207 - A.225) and 'd' (Figures A.243 - A.270) which have the same ratio  $\frac{N}{p}$

( $\frac{N}{p} = 128$ ), leads to a different conclusion. The statistics of the estimated SMC, the variance and the bias in these two cases, are slightly different (compare Figure A.207.1 with A.243.1 and Figure A.208.1 with A.244.1 ), which leads to the conclusion that the statistics of the estimated SMC depend not only on the ratio  $\frac{N}{p}$  but also on the parameters  $N$  and  $p$  themselves.

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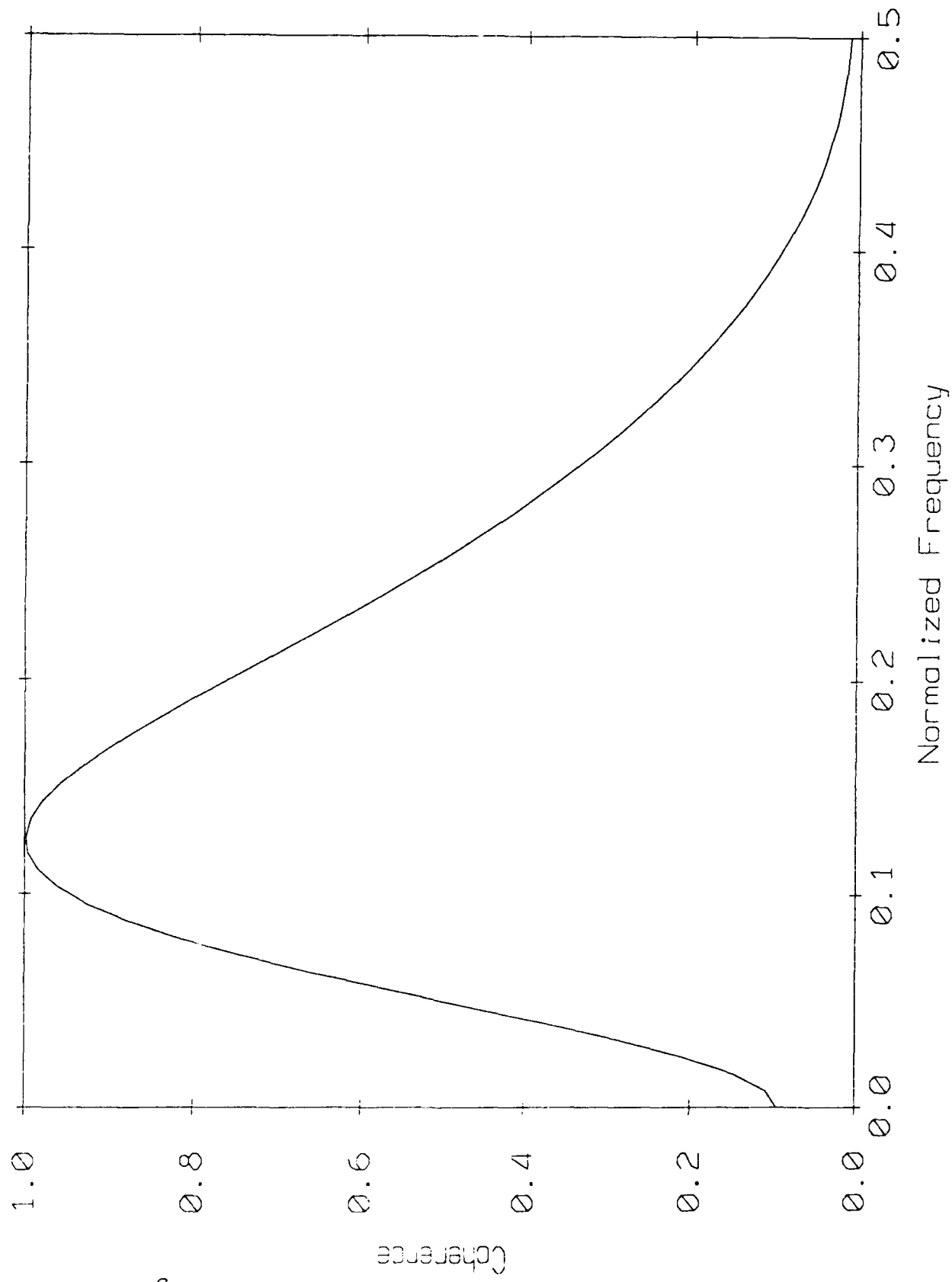
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**APPENDIX A**

# Real Value of Coherency



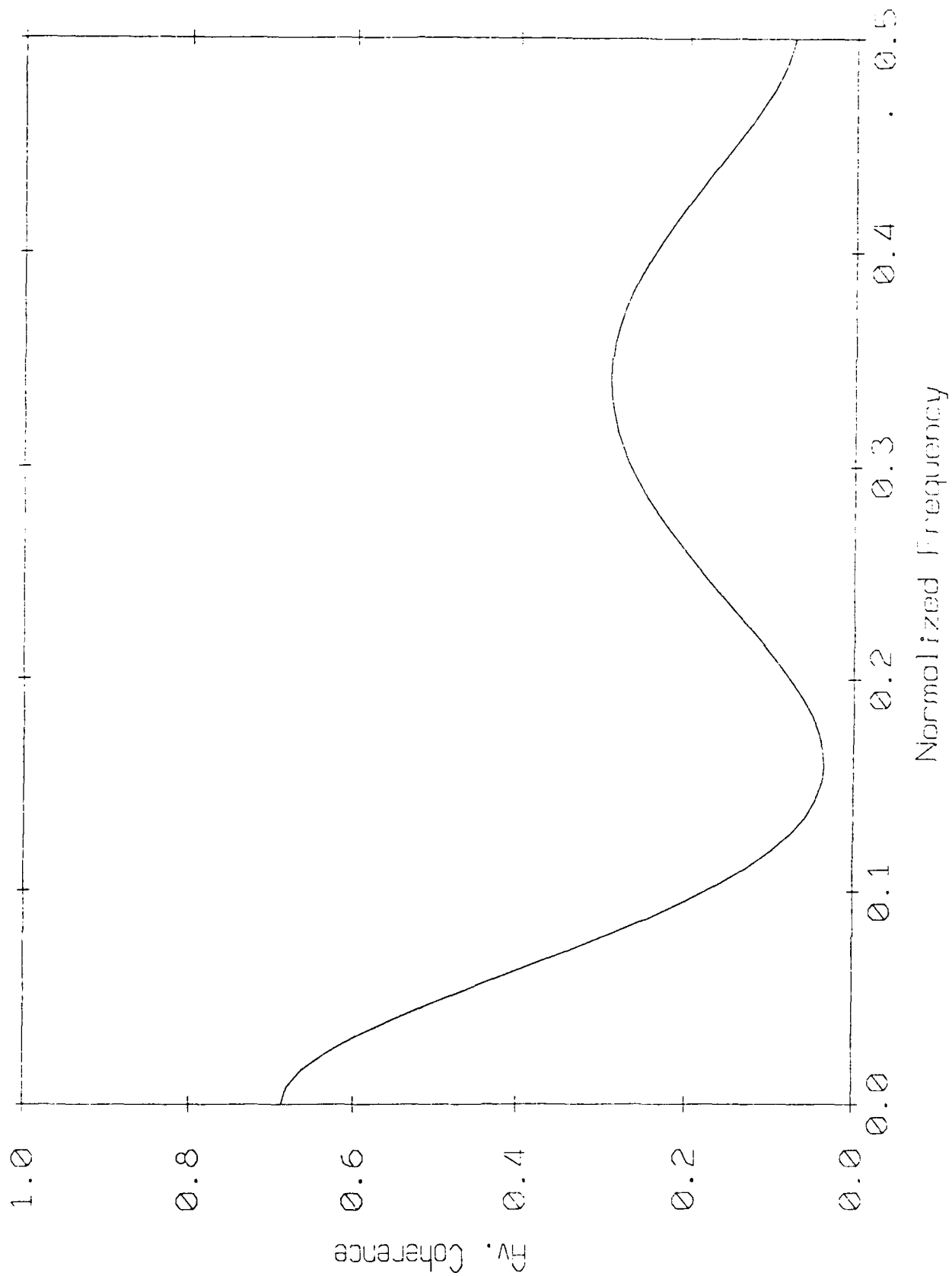


Figure A.2

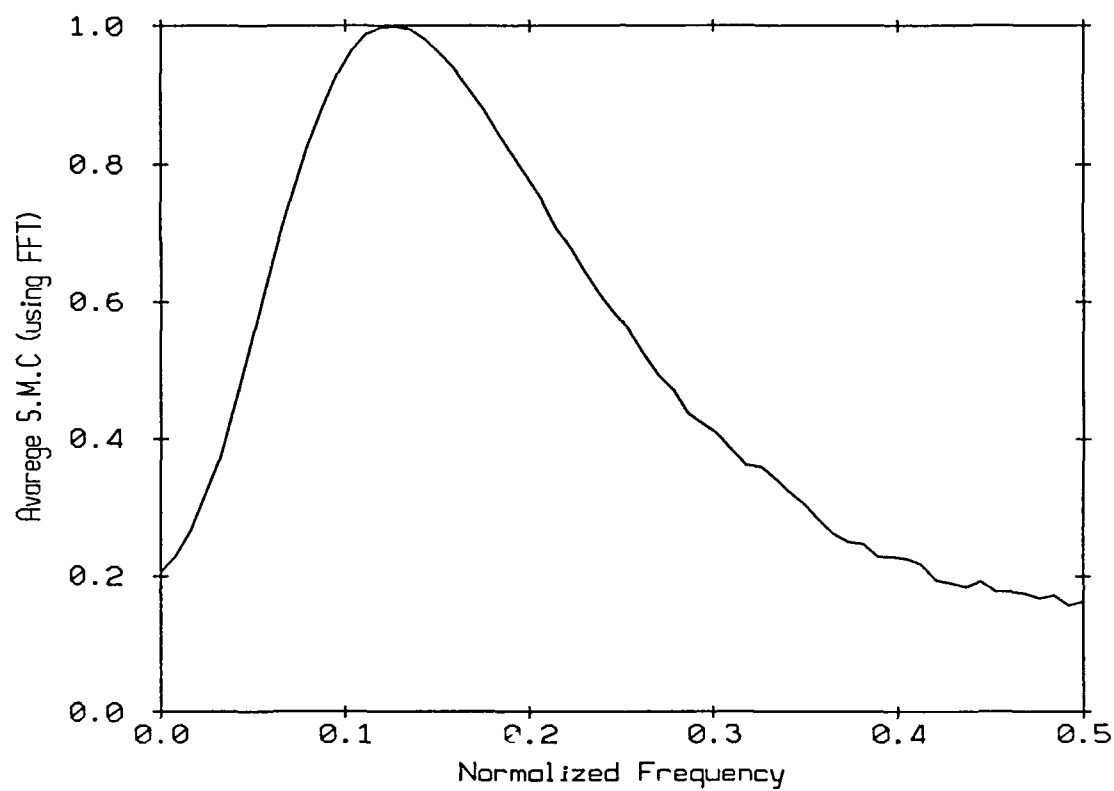


Fig. A.3 : SMC estimation, using FFT method, data length 512 points,  
Average estimated SMC, over 1000 runs

, 1000 runs.

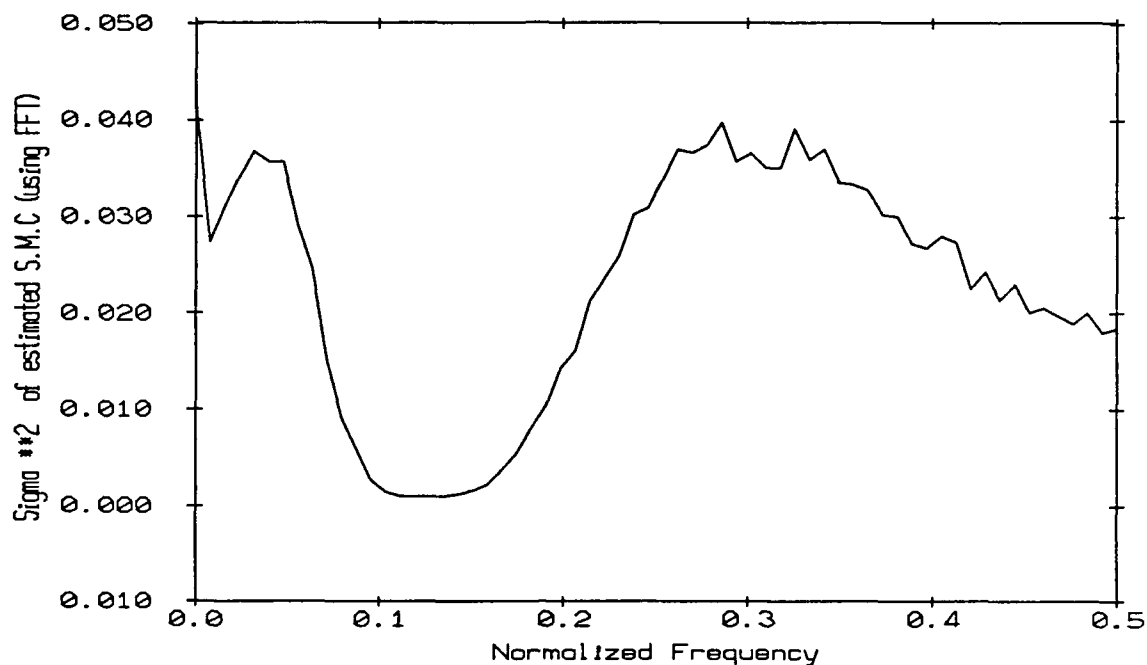


Fig. a.3 : SMC estimation, using FFT method, data length 512 points,  
Squared Standard Deviation of estimated SMC data length 512 points, 1000 runs

, 1000 runs.

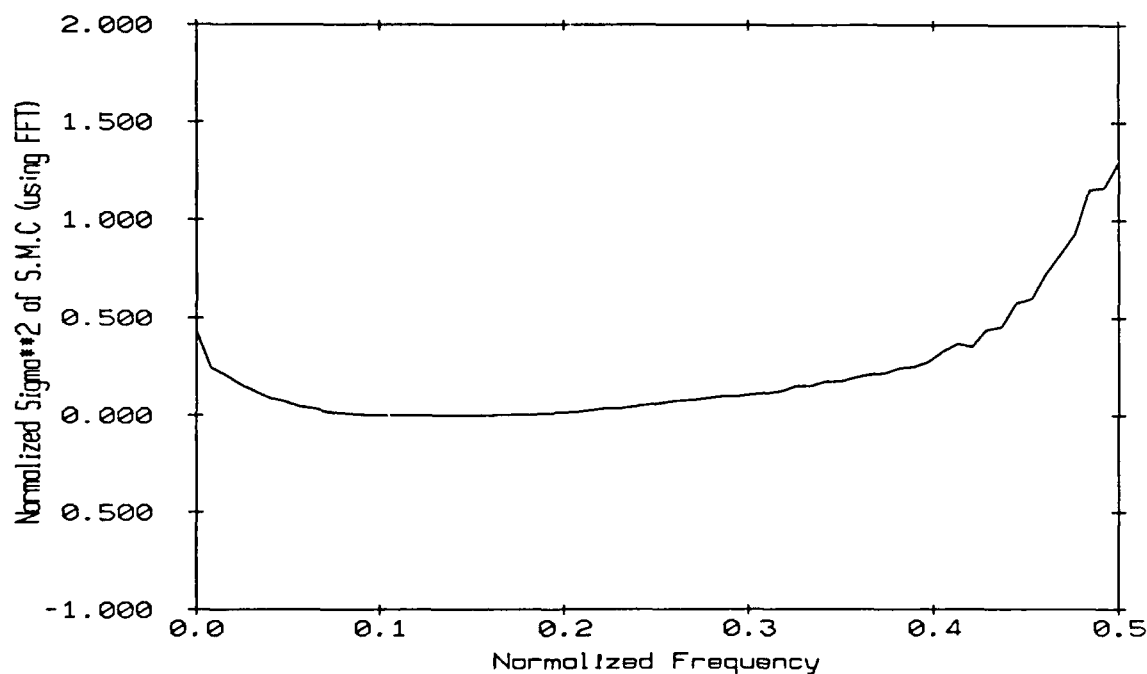


Fig. a.3.1: SMC estimation, using FFT method, data length 512 points,  
Normalized squared standard deviation of estimated SMC over 1000 runs

## Histogram of Coherency (via FFT) at freq. = 1/32

Mean=0.31533, Var=0.33912e-01

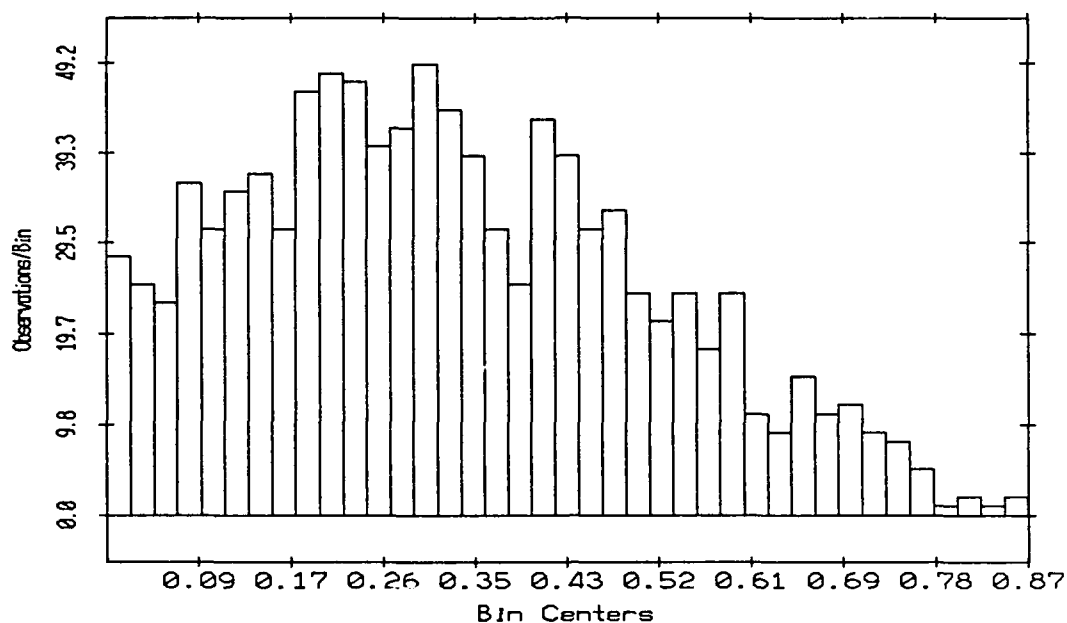


Fig. A.5 : SMC estimation, using FFT method, data length 512 points,  
Histogram of estimated SMC at normalized frequency = 1/32

## Histogram of Coherency (via FFT) at freq. = 2/32

Mean=0.61198, Var=0.28391e-01

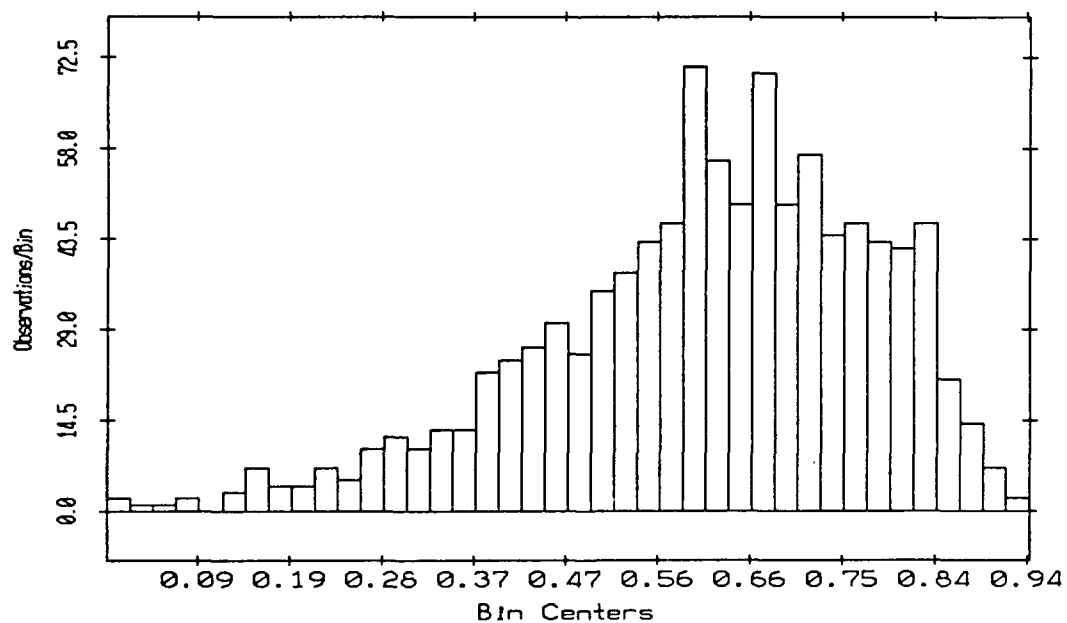


Fig. A.6 : SMC estimation, using FFT method, data length 512 points,  
Histogram of estimated SMC at normalized frequency = 2/32



Histogram of Coherency (via FFT) at freq. =  $3/32$ 

Mean=0.87914, Var=0.71511e-01

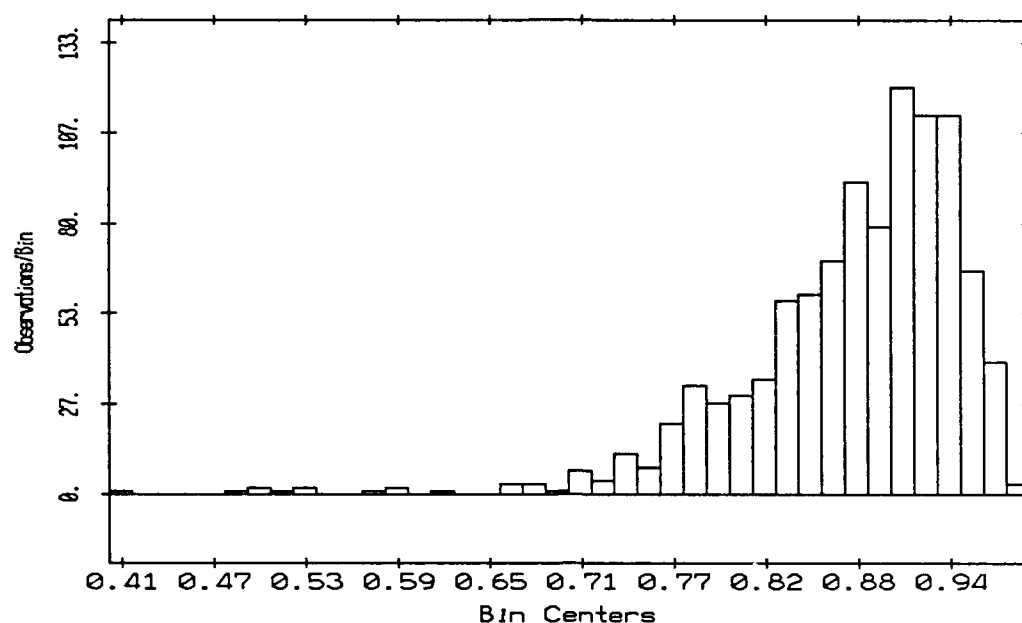


Fig. A.7: SMC estimation, using FFT method, data length 512 points,  
Histogram of estimated SMC at normalized frequency =  $3/32$

Histogram of Coherency (via FFT) at freq. =  $4/32$ 

Mean=0.99733, Var=0.17401e-02

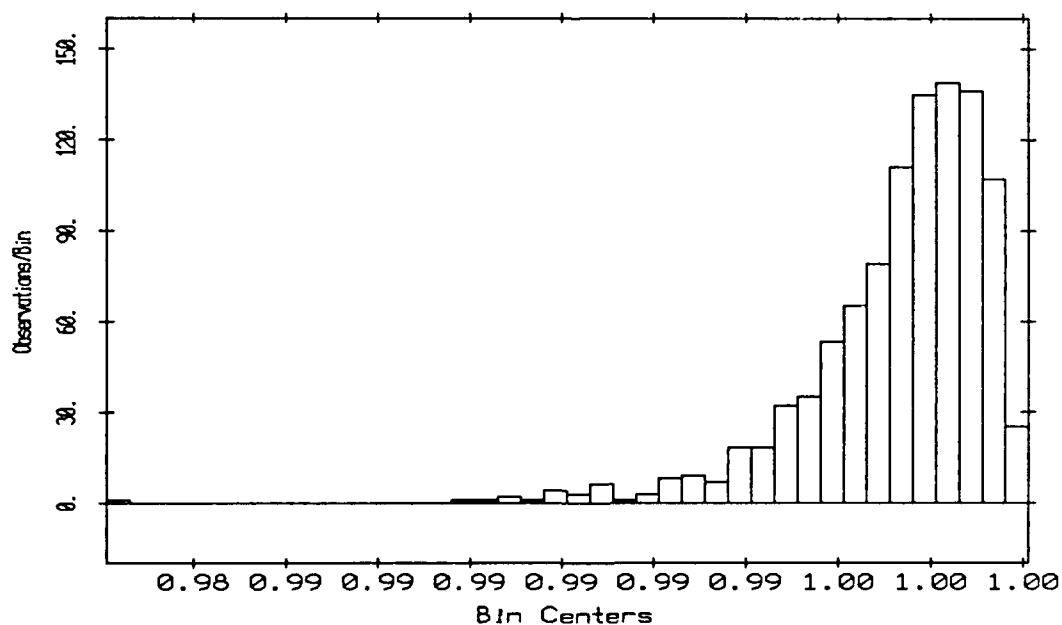


Fig. A.8: SMC estimation, using FFT method, data length 512 points,  
Histogram of estimated SMC at normalized frequency =  $4/32$

# Histogram of Coherency (via FFT) at freq. = 5/32

Mean=0.96151, Var=0.23971e-01

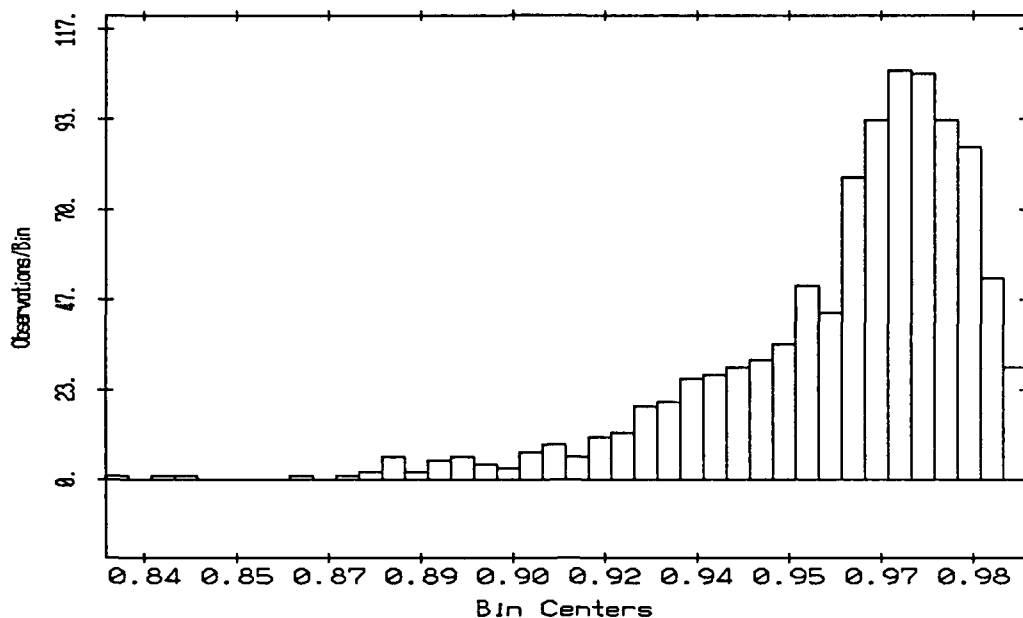


Fig. A.9 : SMC estimation, using FFT method, data length 512 points,  
Histogram of estimated SMC at normalized frequency = 5/32

# Histogram of Coherency (via FFT) at freq. = 6/32

Mean=0.84600, Var=0.86578e-01

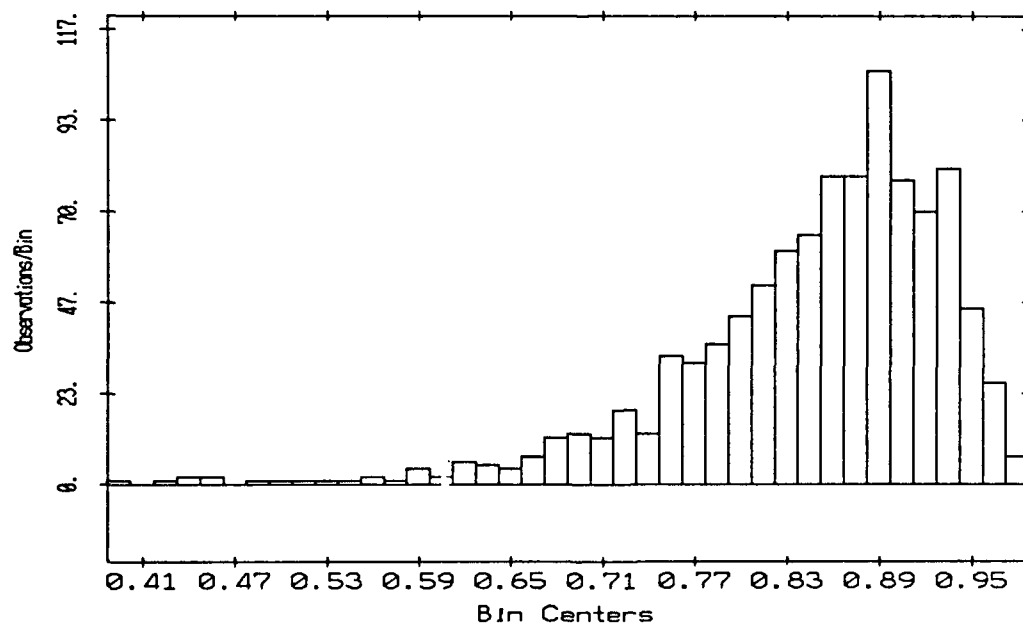


Fig. A.10 : SMC estimation, using FFT method, data length 512 points,  
Histogram of estimated SMC at normalized frequency = 6/32

Histogram of Coherency (via FFT) at freq. =  $7/32$   
 Mean=0.70804, Var=0.14292

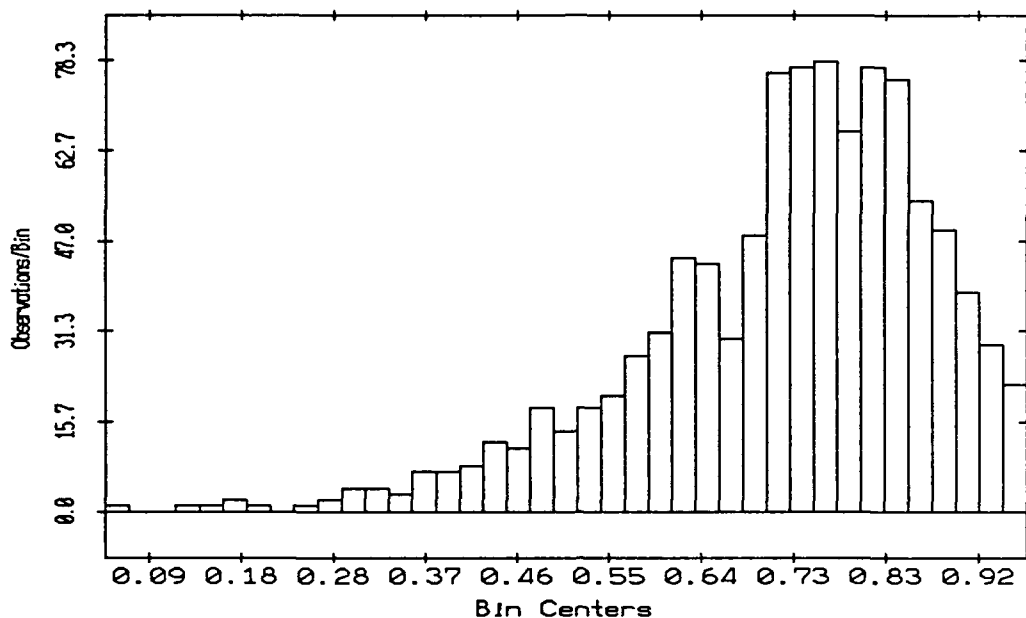


Fig. A.11 : SMC estimation, using FFT method, data length 512 points,  
 Histogram of estimated SMC at normalized frequency =  $7/32$

Histogram of Coherency (via FFT) at freq. =  $8/32$   
 Mean=0.58322, Var=0.17481

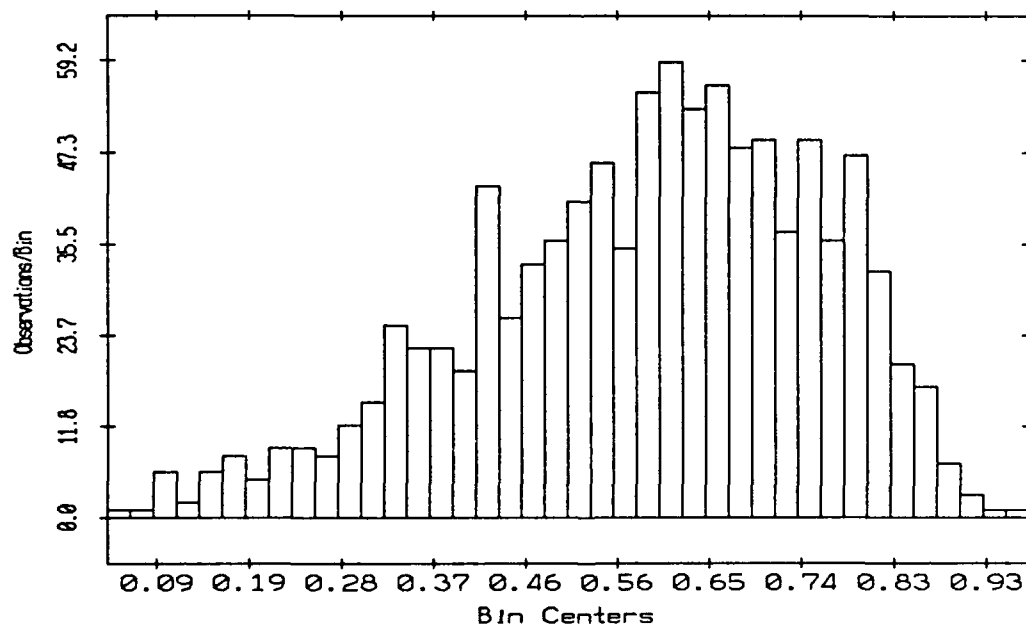


Fig. A.12 : SMC estimation, using FFT method, data length 512 points,  
 Histogram of estimated SMC at normalized frequency =  $8/32$

Histogram of Coherency (via FFT) at freq. =  $9/32$   
 Mean=0.47135, Var=0.19310

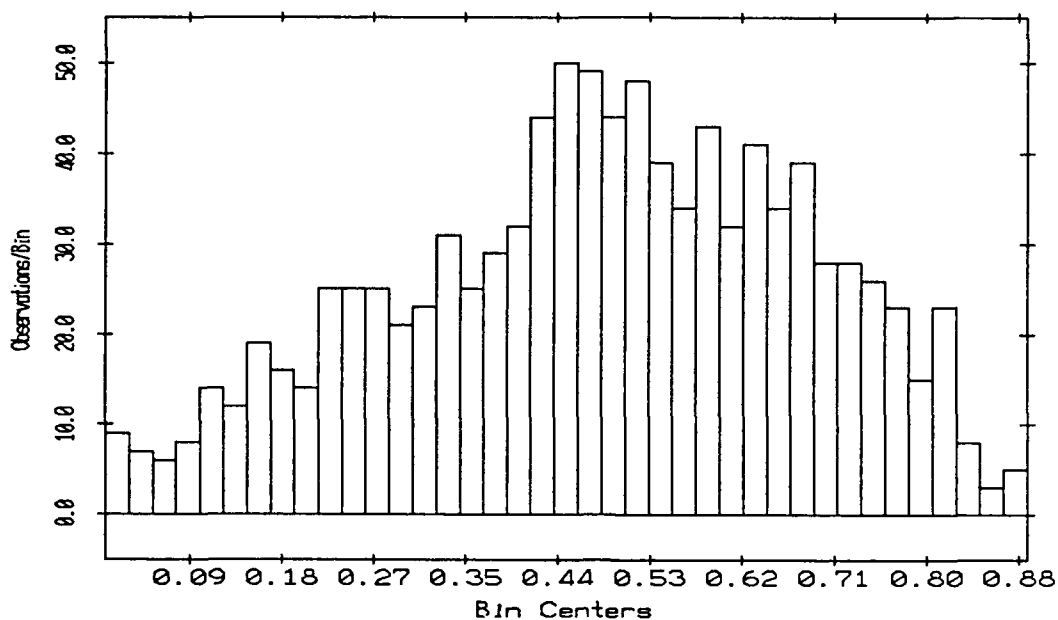


Fig. A.13: SMC estimation, using FFT method, data length 512 points,  
 Histogram of estimated SMC at normalized frequency =  $9/32$

Histogram of Coherency (via FFT) at freq. =  $10/32$   
 Mean=0.38251, Var=0.18719

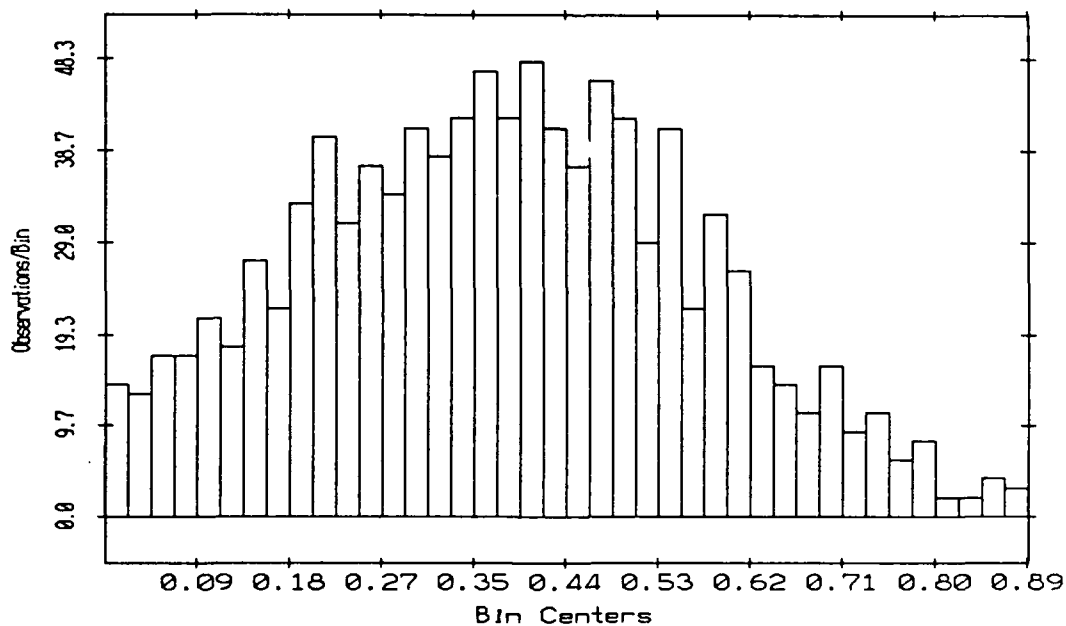


Fig. A.14: SMC estimation, using FFT method, data length 512 points,  
 Histogram of estimated SMC at normalized frequency =  $10/32$

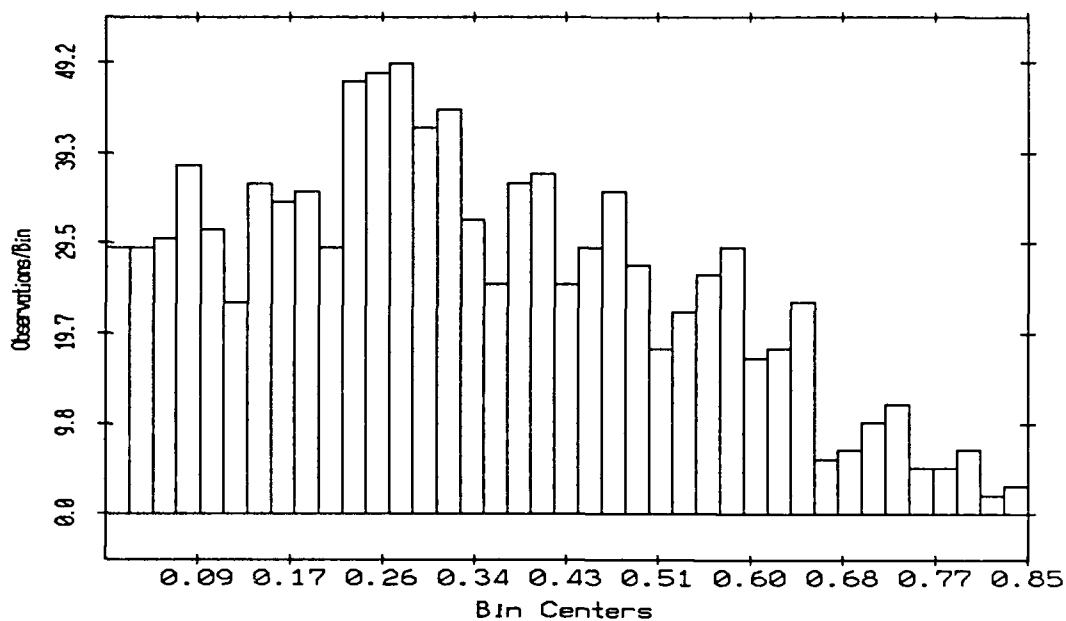
Histogram of Coherency (via FFT) at freq. =  $11/32$ Mean= $0.32144$ , Var= $0.19197$ 

Fig. A.15 : SMC estimation, using FFT method, data length 512 points,  
Histogram of estimated SMC at normalized frequency =  $11/32$

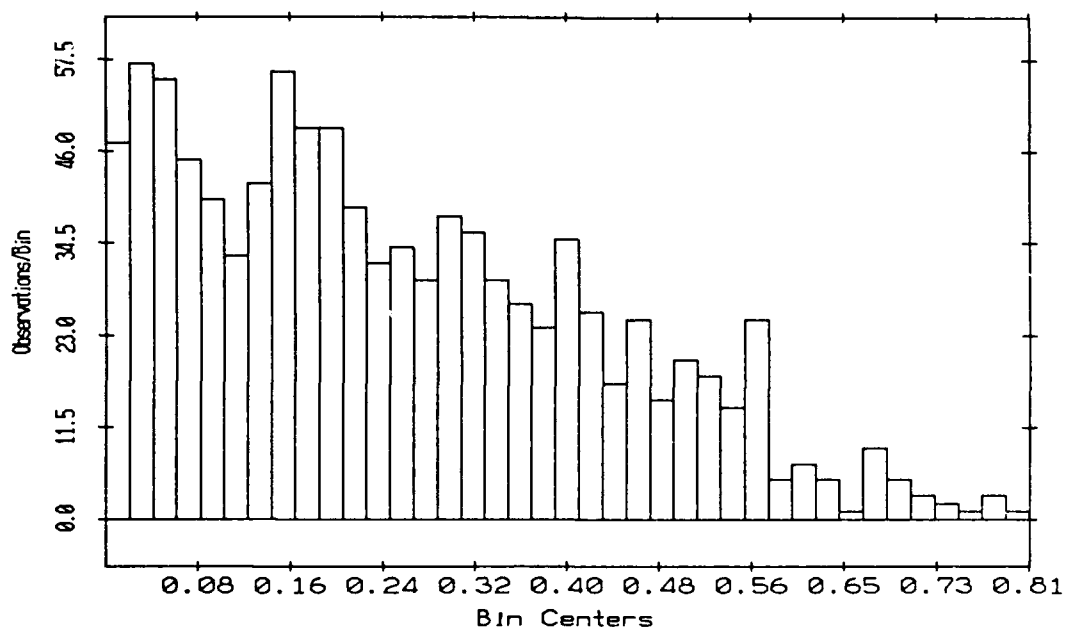
Histogram of Coherency (via FFT) at freq. =  $12/32$ Mean= $0.24881$ , Var= $0.17365$ 

Fig. A.16 : SMC estimation, using FFT method, data length 512 points,  
Histogram of estimated SMC at normalized frequency =  $12/32$

Histogram of Coherency (via FFT) at freq. =  $13/32$   
 Mean=0.22513, Var=0.16700

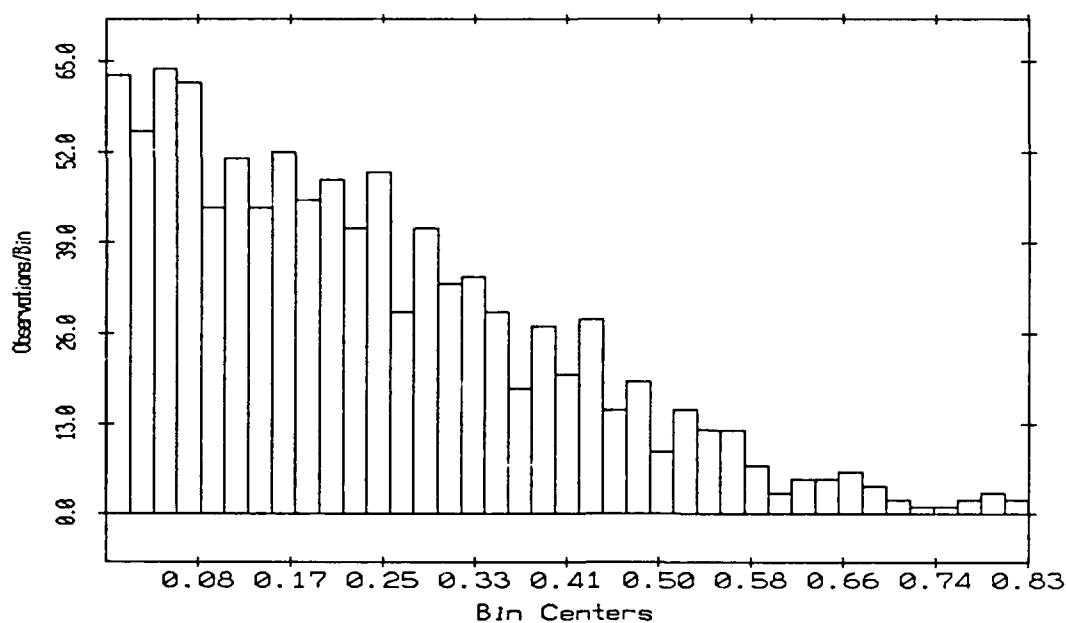


Fig. A.17: SMC estimation, using FFT method, data length 512 points,  
 Histogram of estimated SMC at normalized frequency =  $13/32$

Histogram of Coherency (via FFT) at freq. =  $14/32$   
 Mean=0.18256, Var=0.14605

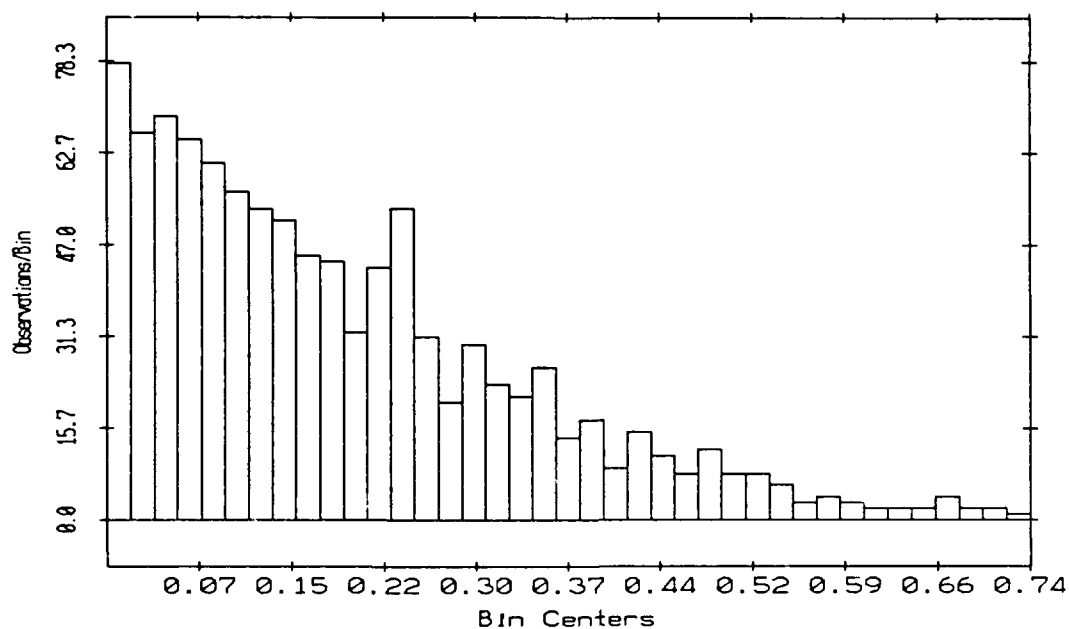


Fig. A.18: SMC estimation, using FFT method, data length 512 points,  
 Histogram of estimated SMC at normalized frequency =  $14/32$

## Histogram of Coherency (via FFT) at freq. = 15/32

Mean=0.17365, Var=0.14045

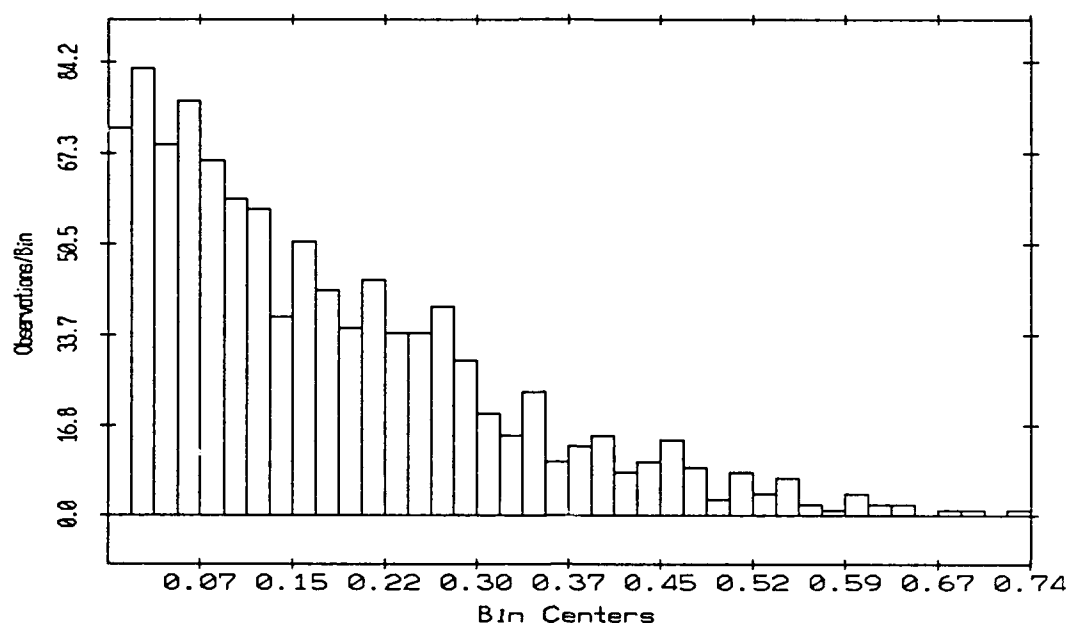


Fig. A.19: SMC estimation, using FFT method, data length 512 points,  
Histogram of estimated SMC at normalized frequency = 15/32

## Histogram of Coherency (via FFT) at freq. = 16/32

Mean=0.16208, Var=0.13555

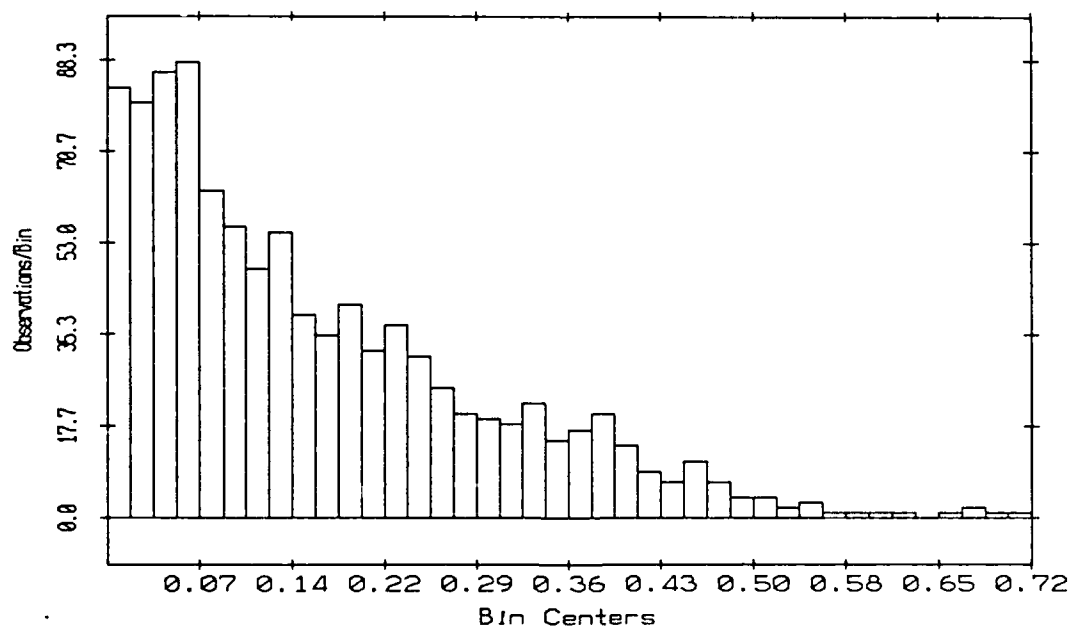


Fig. A.20: SMC estimation, using FFT method, data length 512 points,  
Histogram of estimated SMC at normalized frequency = 16/32

1000 runs.

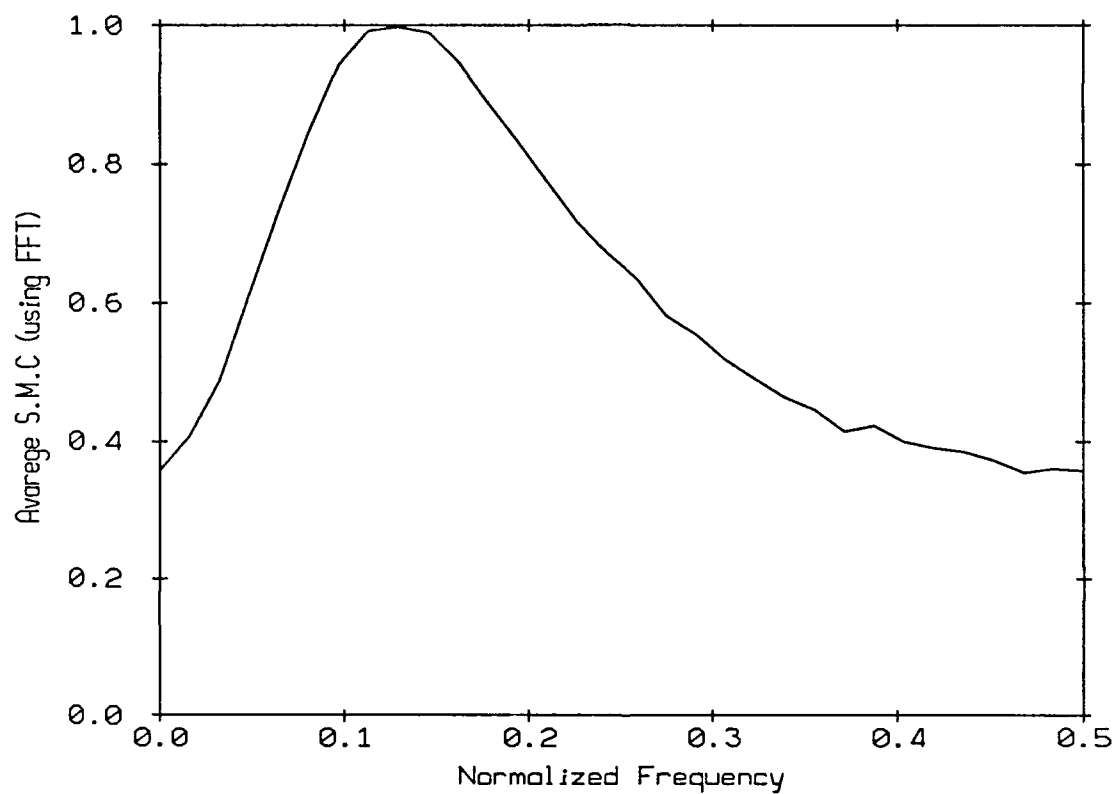


Fig. A.21: SMC estimation, using FFT method, data length 256 points,  
Average estimated SMC over 1000 runs



SMC estimation, using FFT, segment length 256 points

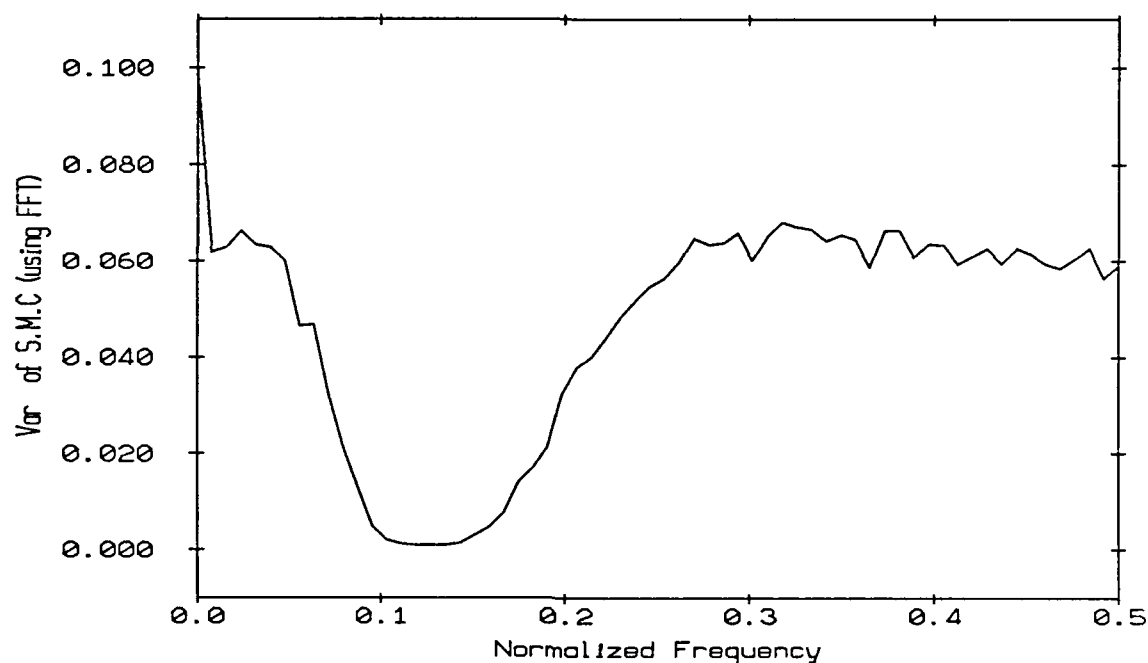


Fig. a.22 : SMC estimation, using FFT method, data length 256 points,  
Squared Standard Deviation of estimated SMC data length 256 points, 1000 runs

SMC estimation, using FFT, segment length 256 points

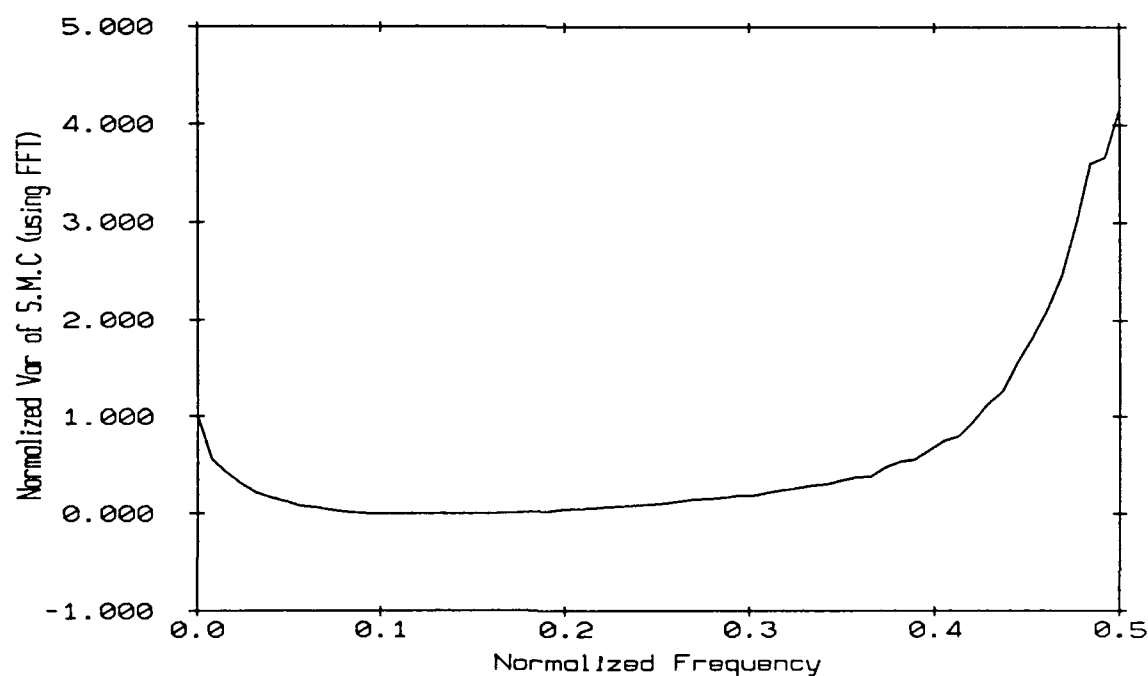


Fig. a.22.1: SMC estimation, using FFT method, data length 256 points,  
Normalized squared standard deviation of estimated SMC over 1000 runs

Histogram of Coherency (via FFT) at freq. =  $1/32$   
 Mean=0.44059, Var=0.65610e-01

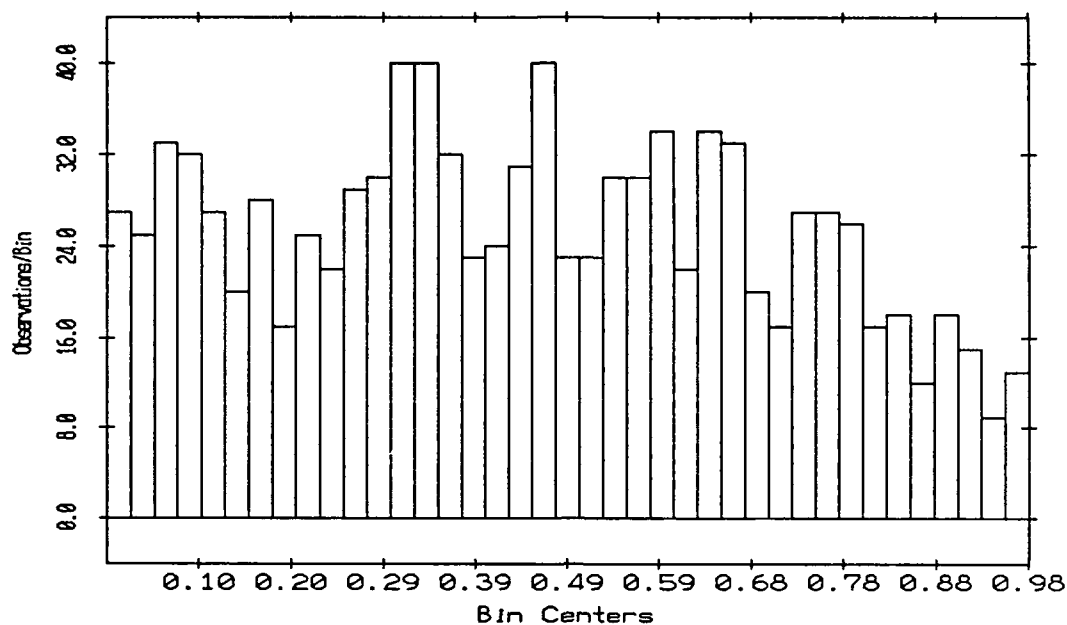


Fig. A.23 : SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $1/32$

Histogram of Coherency (via FFT) at freq. =  $2/32$   
 Mean=0.67301, Var=0.45800e-01

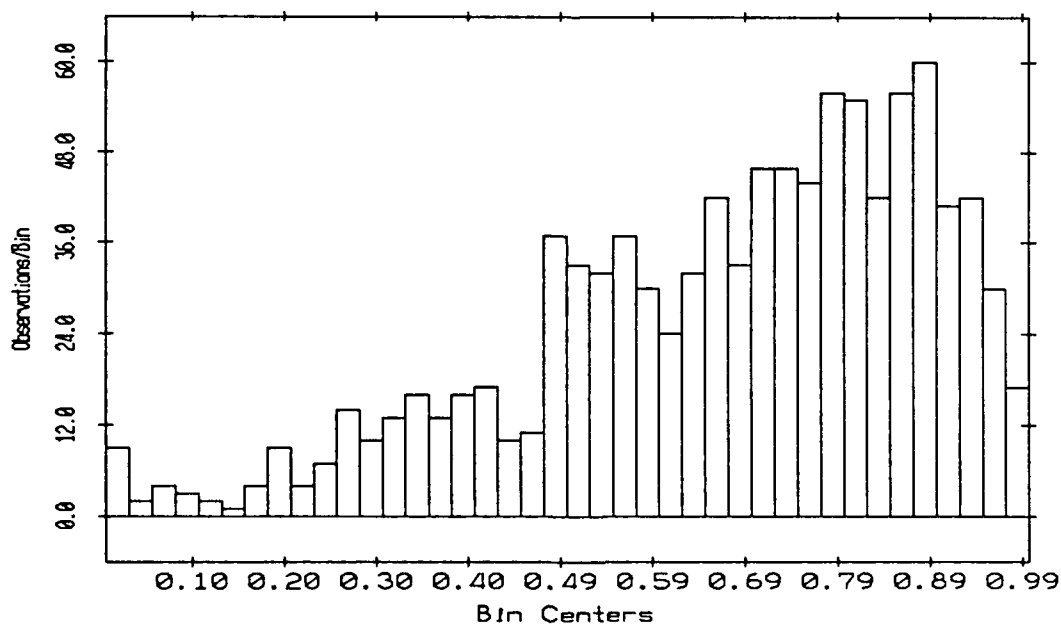


Fig. A.24 : SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $2/32$

Histogram of Coherency (via FFT) at freq. =  $3/32$   
 Mean=0.88903, Var=0.12180e-01

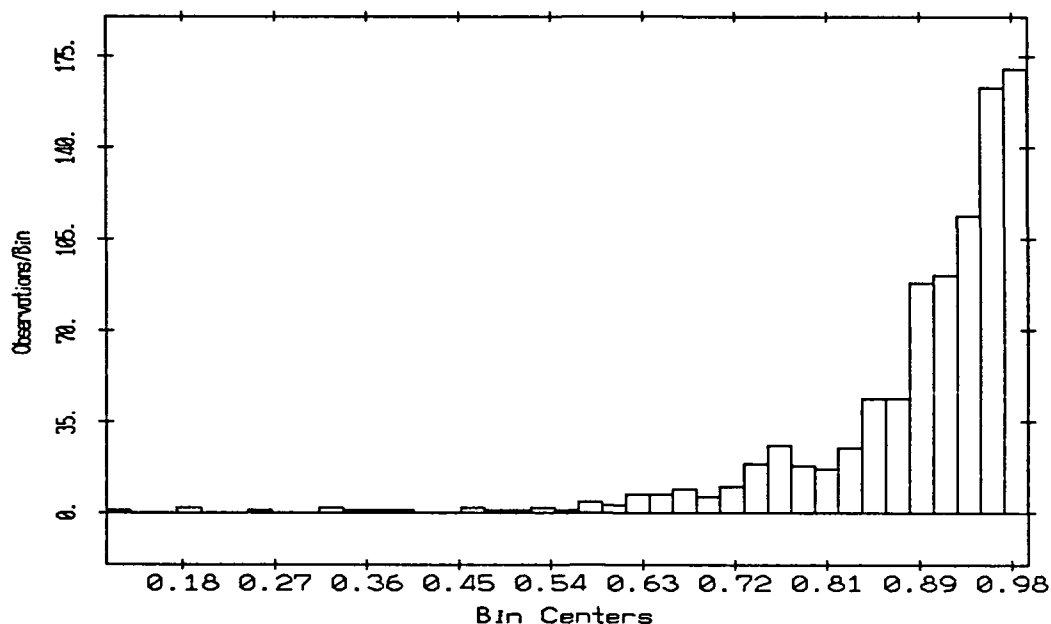


Fig. A.25 : SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $3/32$

Histogram of Coherency (via FFT) at freq. =  $4/32$   
 Mean=0.99725, Var=0.14475e-04

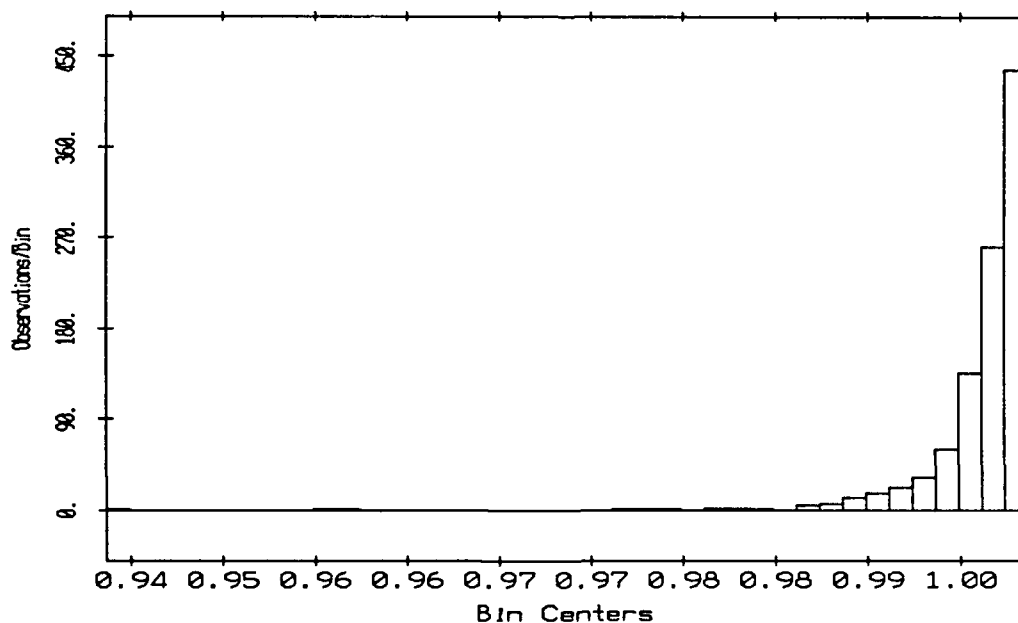


Fig. A.26 : SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $4/32$

Histogram of Coherency (via FFT) at freq. =  $5/32$   
 Mean=0.96087, Var=0.21010e-02

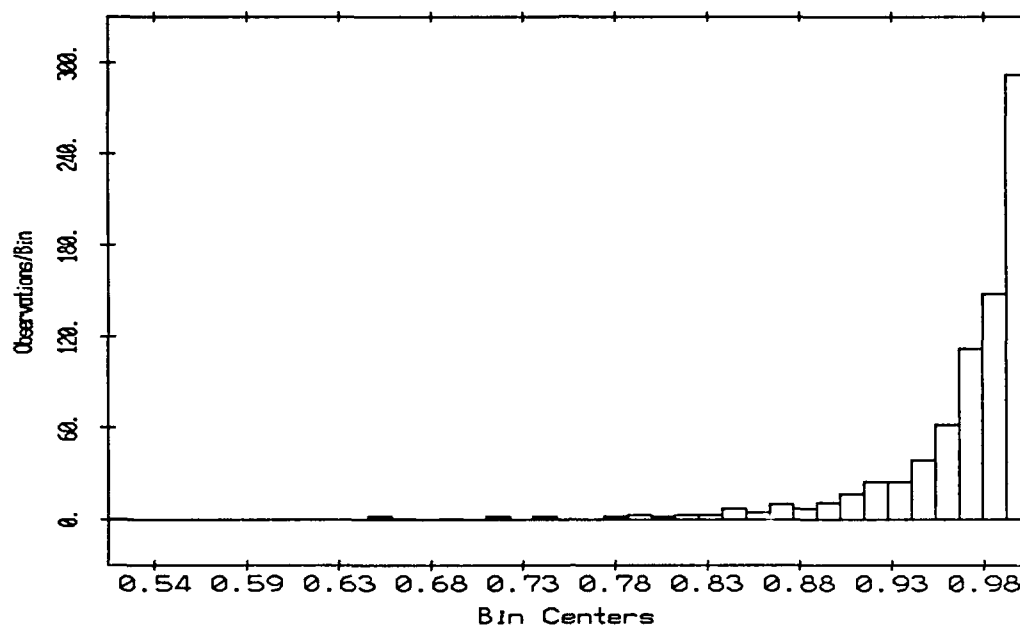


Fig. A.27: SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $5/32$

Histogram of Coherency (via FFT) at freq. =  $6/32$   
 Mean=0.85901, Var=0.16591e-01

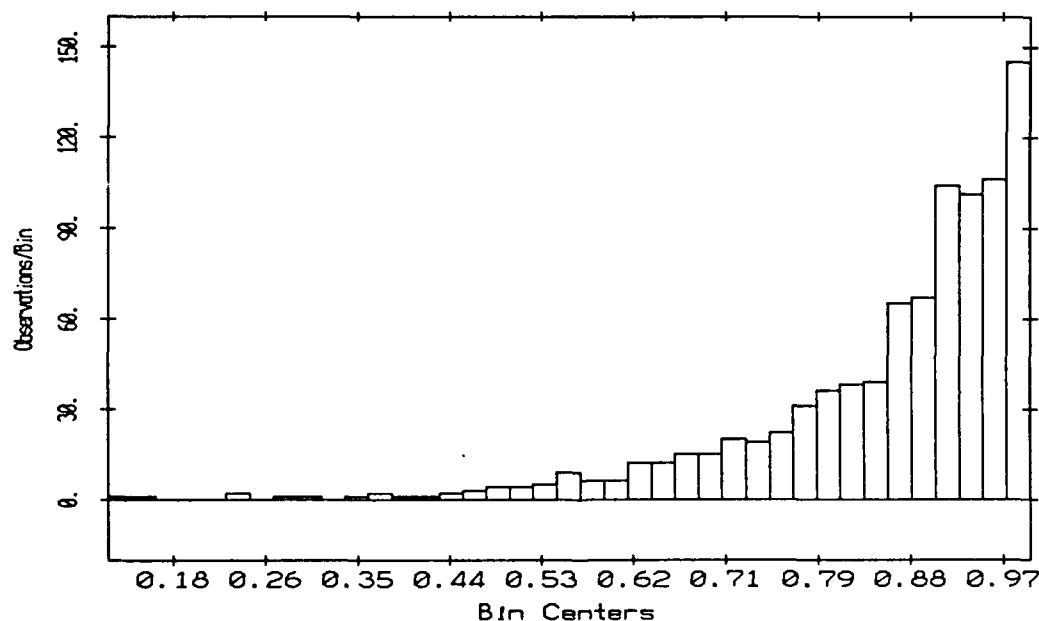


Fig. A.28: SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $6/32$

Histogram of Coherency (via FFT) at freq. =  $7/32$   
 Mean=0.74159, Var=0.39253e-01

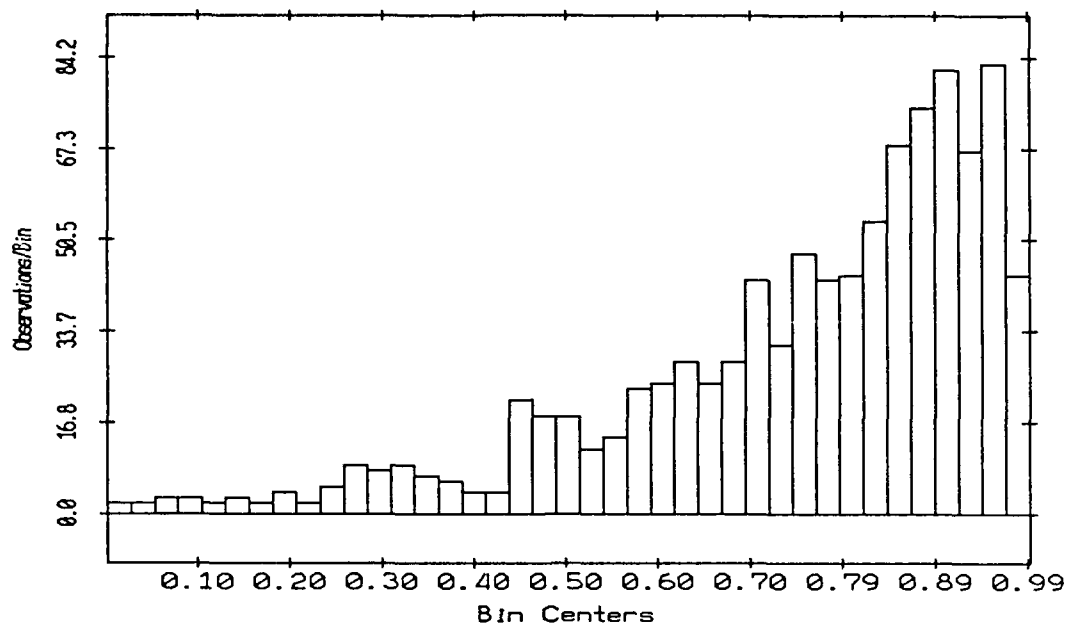


Fig. A.29 : SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $7/32$

Histogram of Coherency (via FFT) at freq. =  $8/32$   
 Mean=0.65741, Var=0.54133e-01

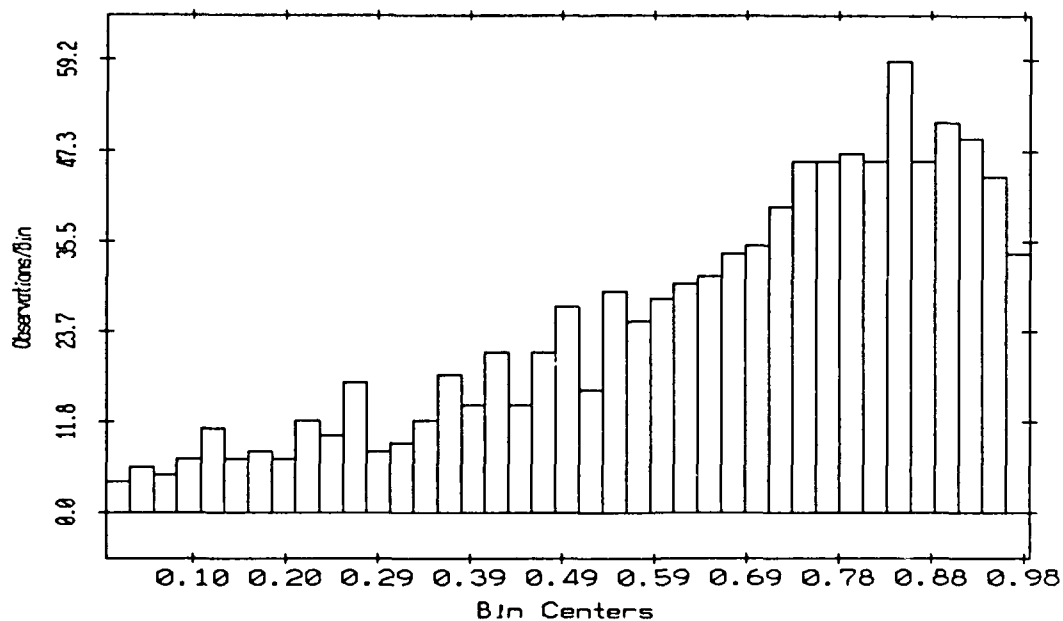


Fig. A.30 : SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $8/32$

Histogram of Coherency (via FFT) at freq. = 9/32  
 Mean=0.57301, Var=0.63562e-01

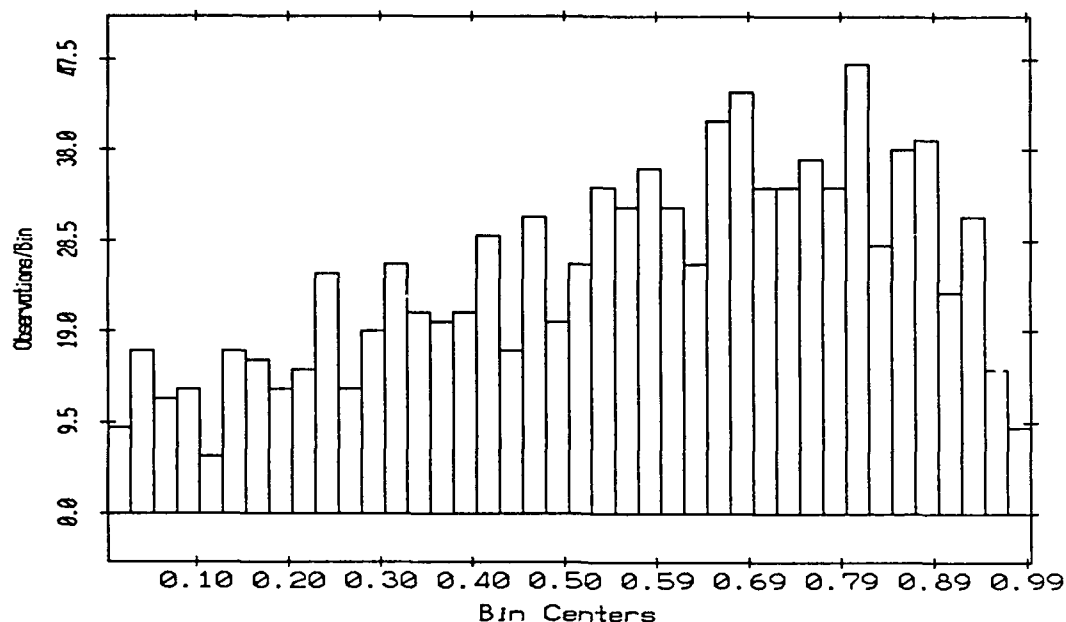


Fig. A.31 : SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency = 9/32

Histogram of Coherency (via FFT) at freq. = 10/32  
 Mean=0.51373, Var=0.64967e-01

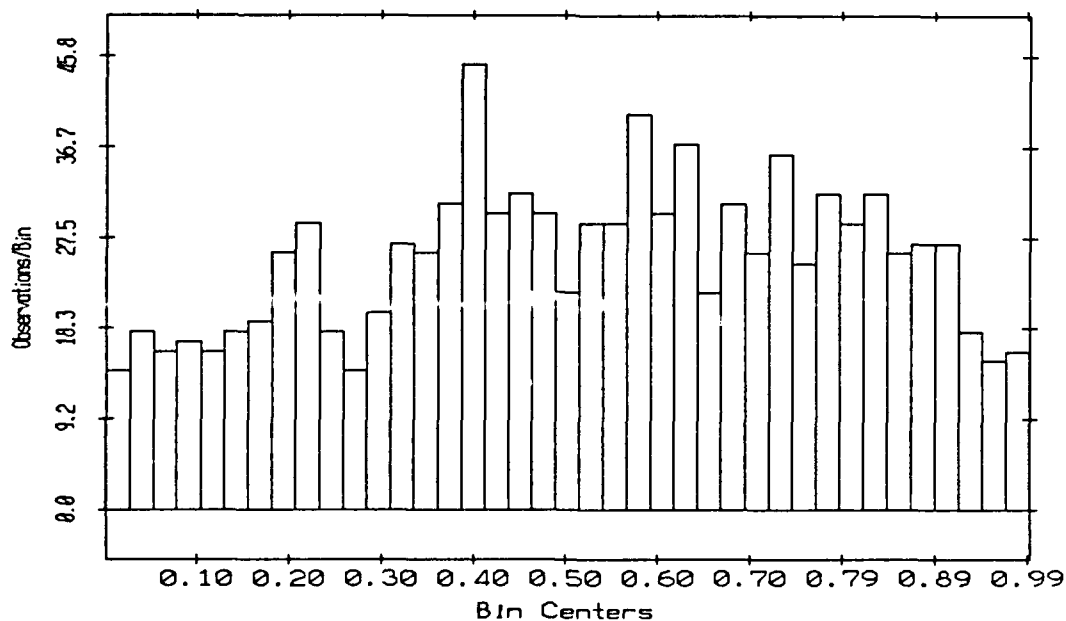


Fig. A.32 : SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency = 10/32

Histogram of Coherency (via FFT) at freq. =  $11/32$   
 Mean=0.47074, Var=0.63934e-01

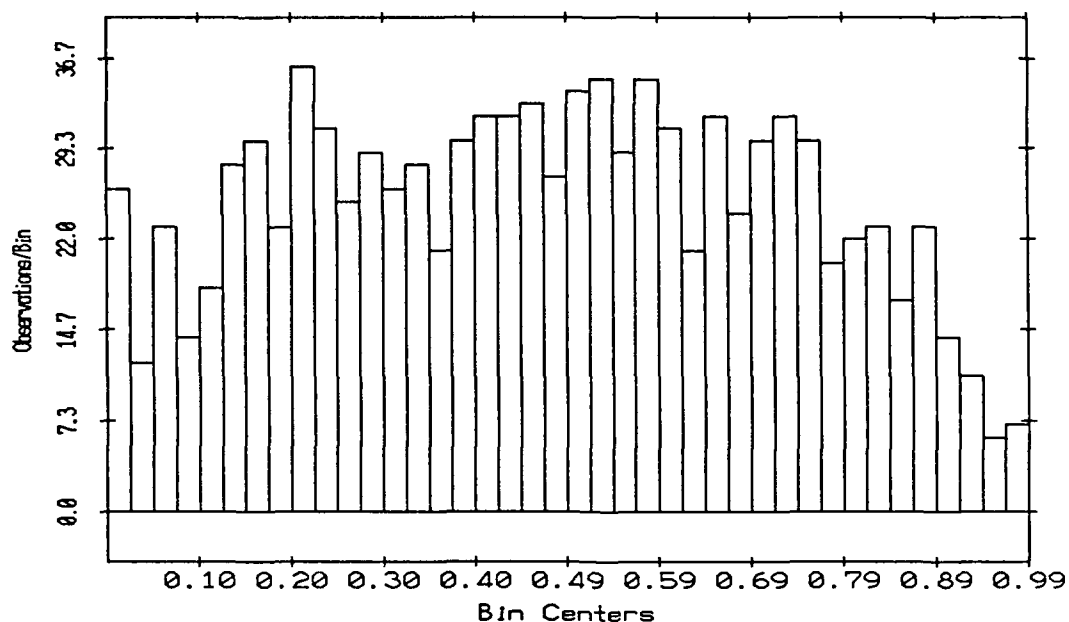


Fig. A.33: SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $11/32$

Histogram of Coherency (via FFT) at freq. =  $12/32$   
 Mean=0.41124, Var=0.66123e-01

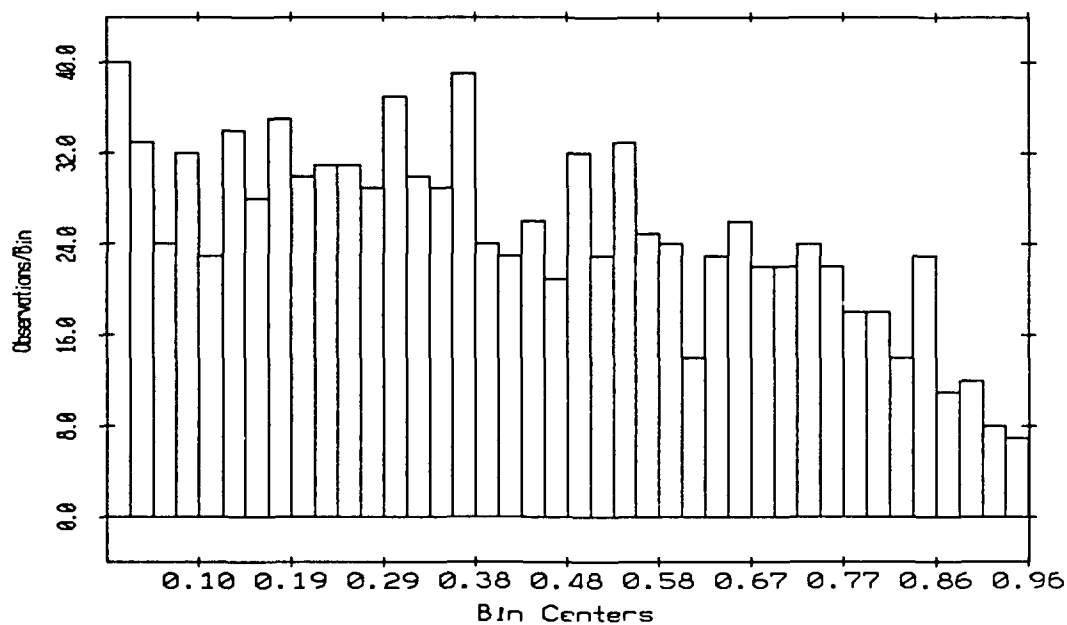


Fig. A.34: SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $12/32$

Histogram of Coherency (via FFT) at freq. = 13/32  
 Mean=0.40009, Var=0.63278e-01

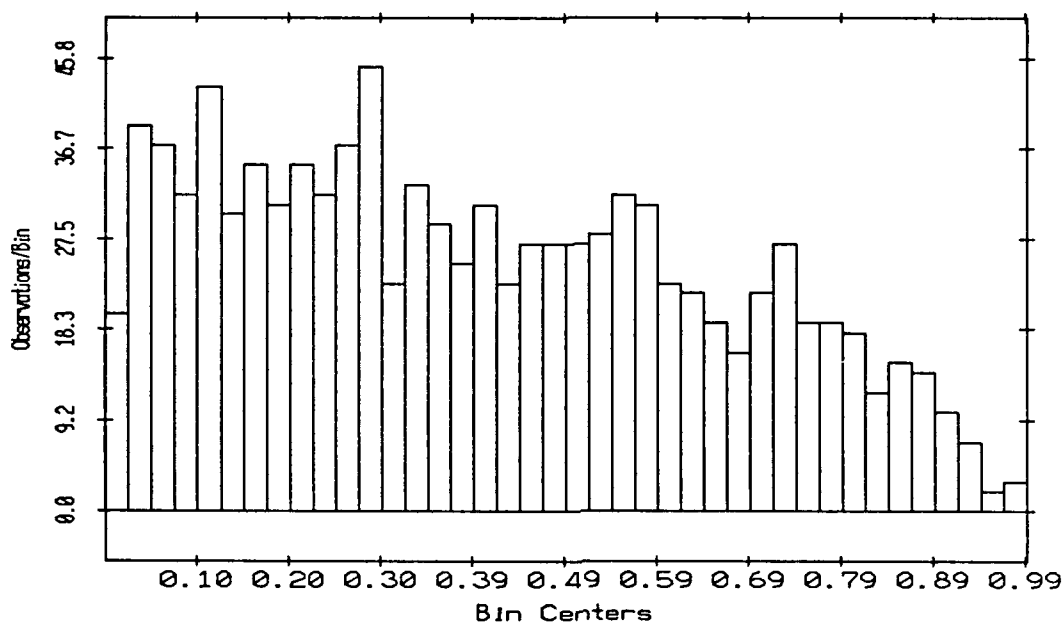


Fig. A.35: SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency = 13/32

Histogram of Coherency (via FFT) at freq. = 14/32  
 Mean=0.37710, Var=0.59276e-01

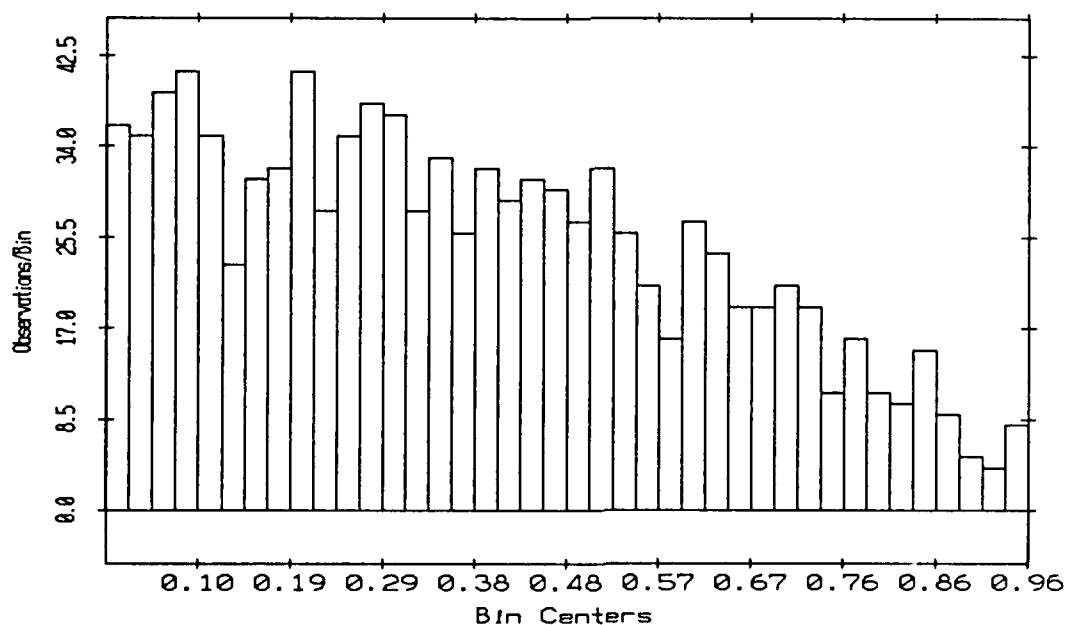


Fig. A.36: SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency = 14/32



Histogram of Coherency (via FFT) at freq. = 15/32  
 Mean=0.35917, Var=0.58582e-01

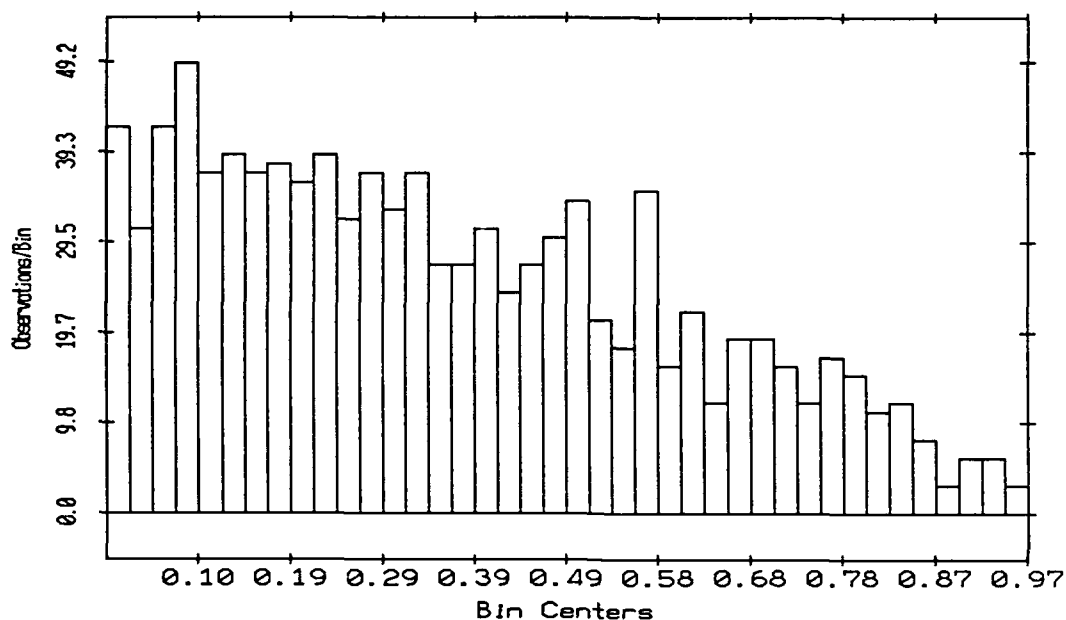


Fig. A.37: SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency = 15/32

Histogram of Coherency (via FFT) at freq. = 16/32  
 Mean=0.35494, Var=0.58997e-01

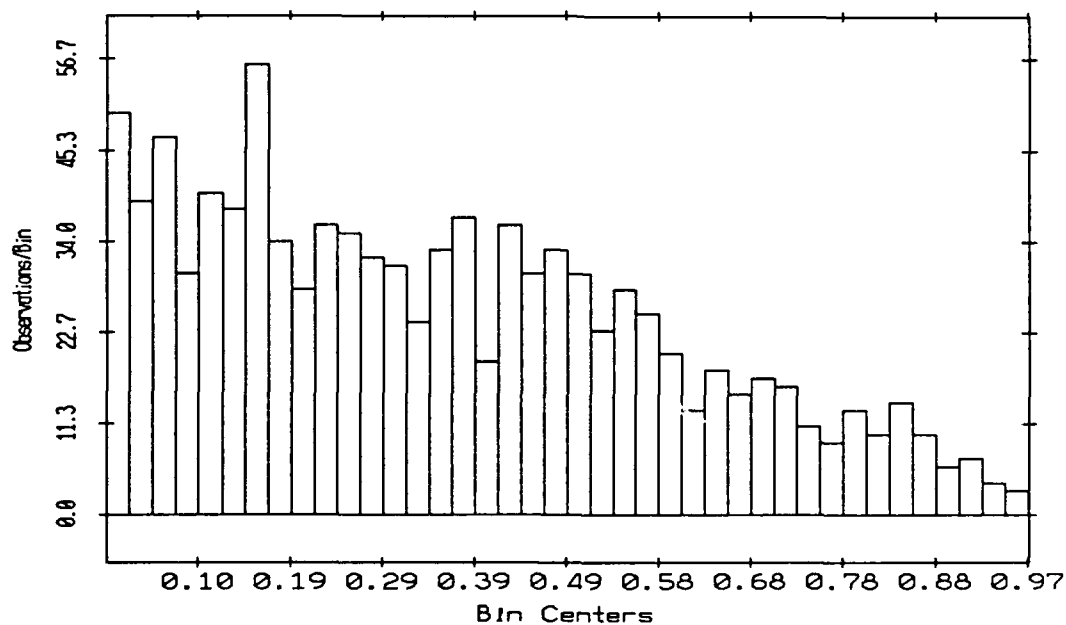


Fig. A.38: SMC estimation, using FFT method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency = 16/32

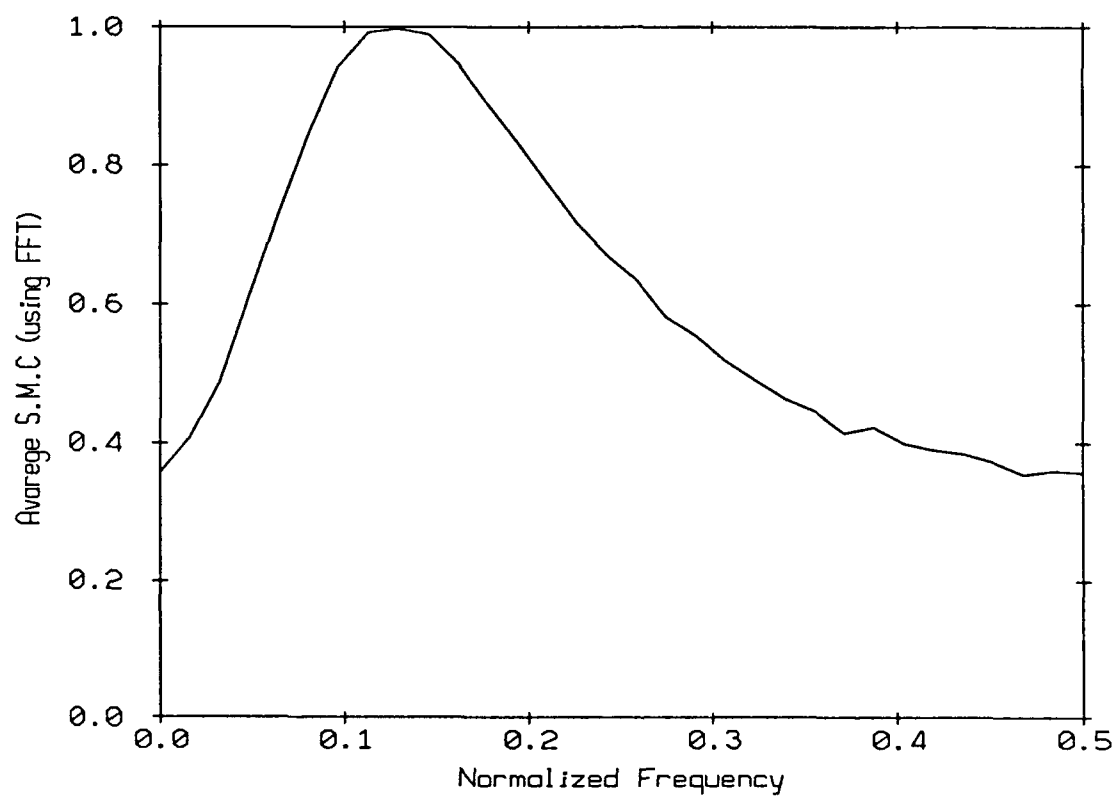


Fig. A.39: SMC estimation, using FFT method, data length 128 points,  
Average estimated SMC over 1000 runs

SMC estimation, using FFT, segment length 128 points

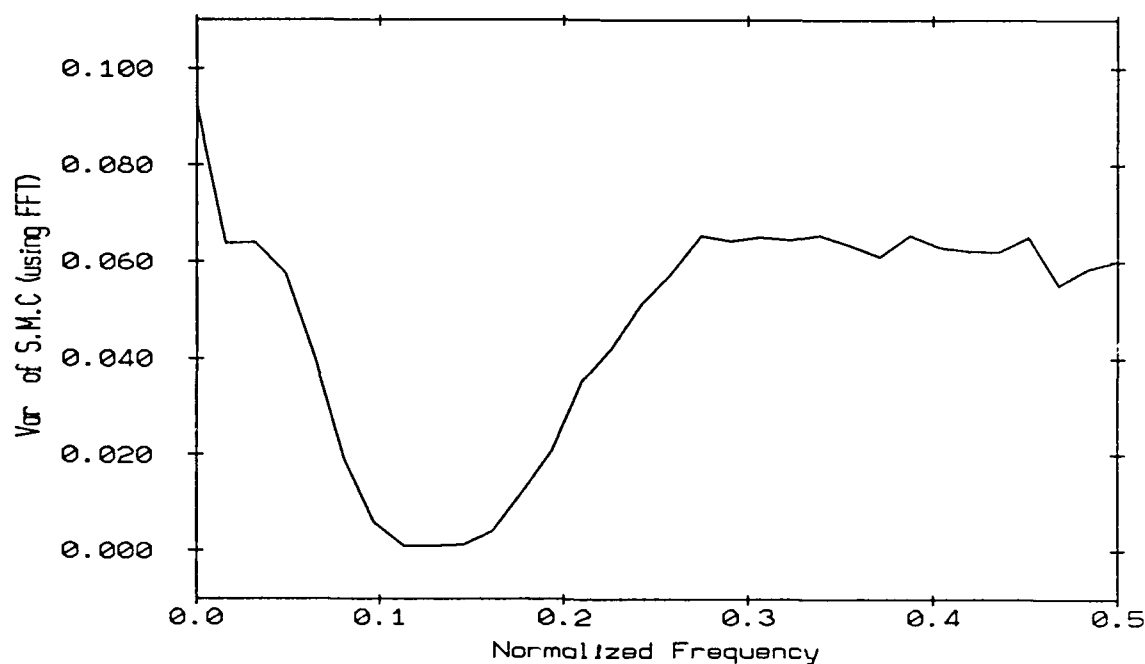


Fig. A.40 : SMC estimation, using FFT method, data length 128 points,  
Squared Standard Deviation of estimated SMC data length 128 points, 1000 runs

SMC estimation, using FFT, segment length 128 points

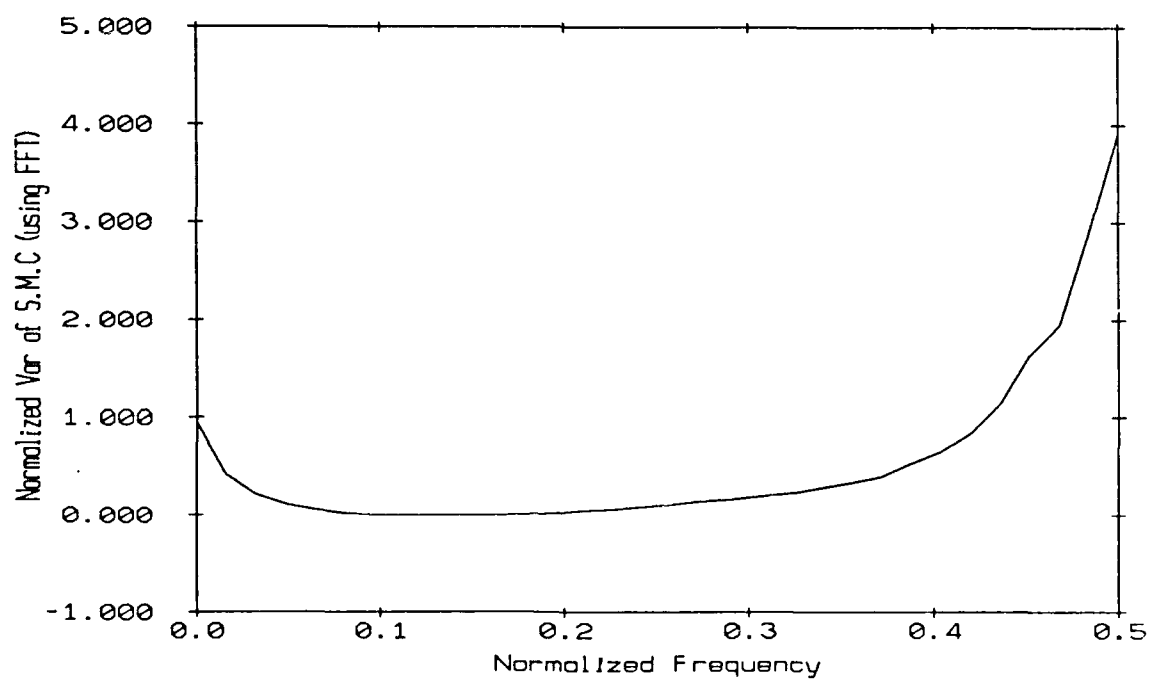


Fig. A.40.1: SMC estimation, using FFT method, data length 128 points,  
Normalized squared standard deviation of estimated SMC over 1000 runs

## Histogram of Coherency (via FFT) at freq. = 1/32

Mean=0.40700, Var=0.63370e-01

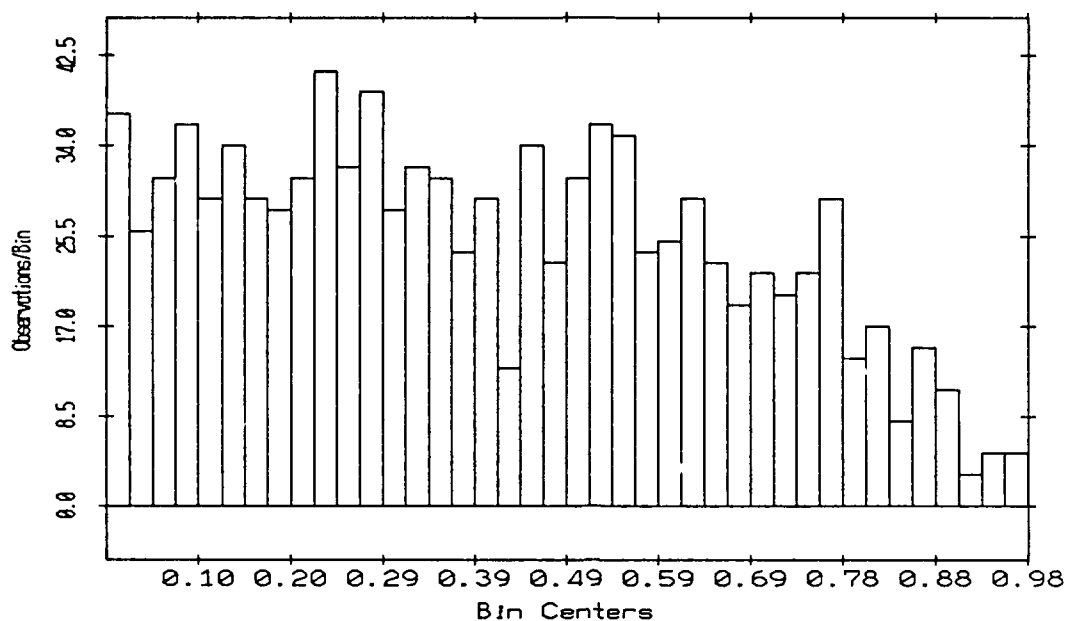


Fig. A.41 : SMC estimation, using FFT method, data length 128 points,  
Histogram of estimated SMC at normalized frequency = 1/32

## Histogram of Coherency (via FFT) at freq. = 2/32

Mean=0.61133, Var=0.56863e-01

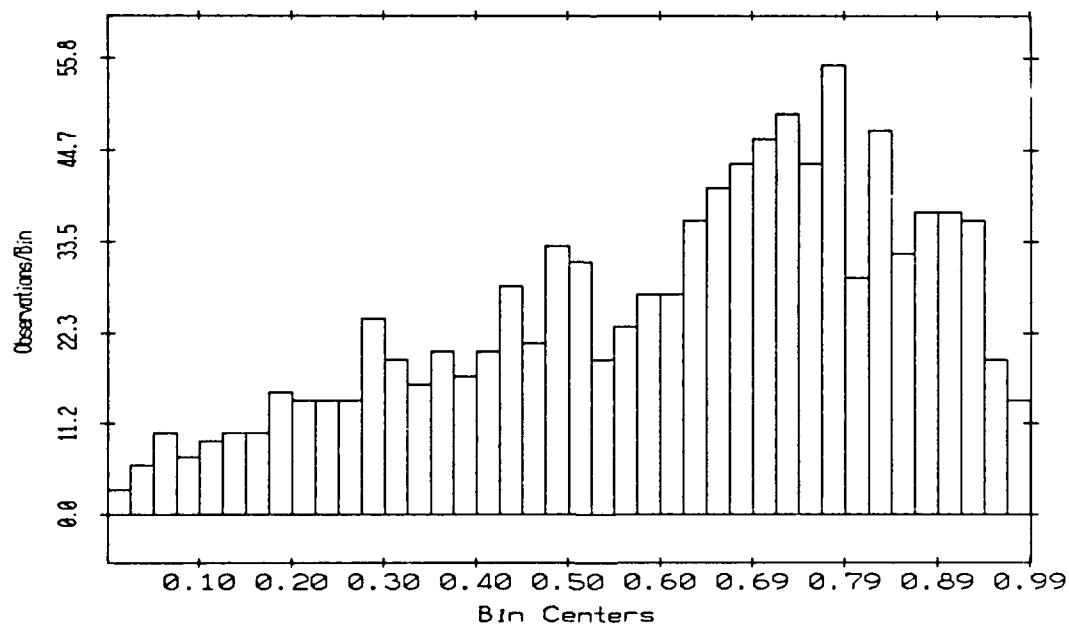


Fig. A.42 : SMC estimation, using FFT method, data length 128 points,  
Histogram of estimated SMC at normalized frequency = 2/32

Histogram of Coherency (via FFT) at freq. =  $3/32$   
Mean=0.84649, Var=0.18714e-01

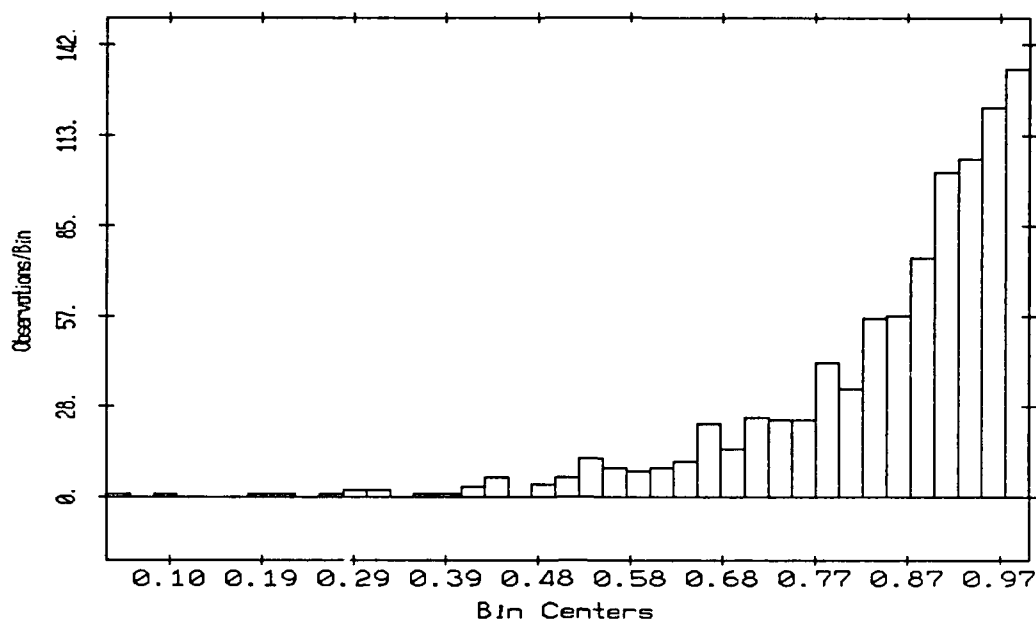


Fig. A.43 : SMC estimation, using FFT method, data length 128 points,  
Histogram of estimated SMC at normalized frequency =  $3/32$

Histogram of Coherency (via FFT) at freq. =  $4/32$   
Mean=0.99246, Var=0.94314e-04

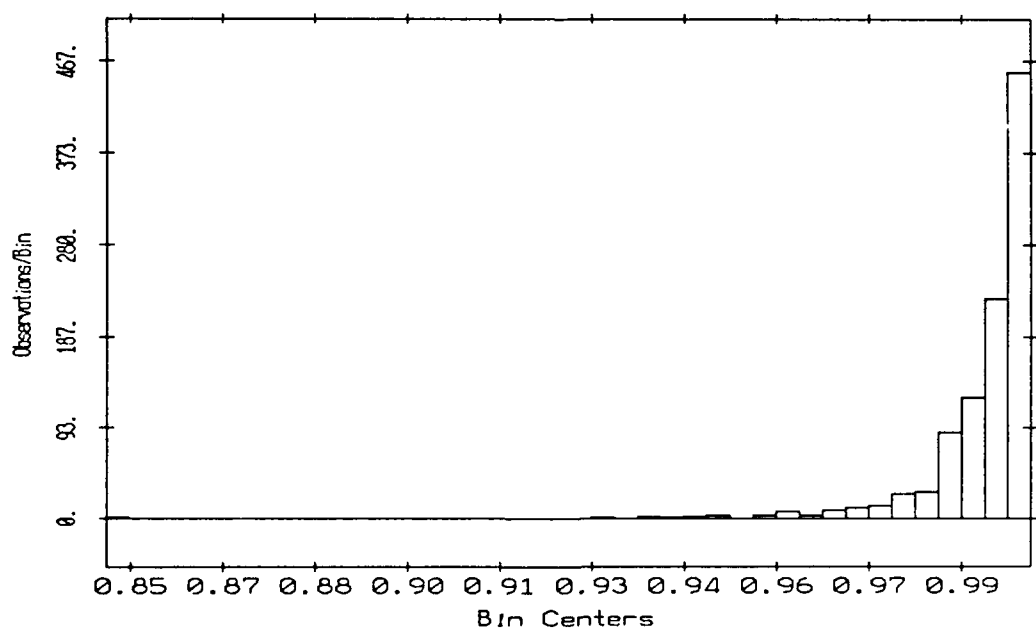


Fig. A.44 : SMC estimation, using FFT method, data length 128 points,  
Histogram of estimated SMC at normalized frequency =  $4/32$

Histogram of Coherency (via FFT) at freq. =  $5/32$   
 Mean=0.98893, Var=0.23667e-03

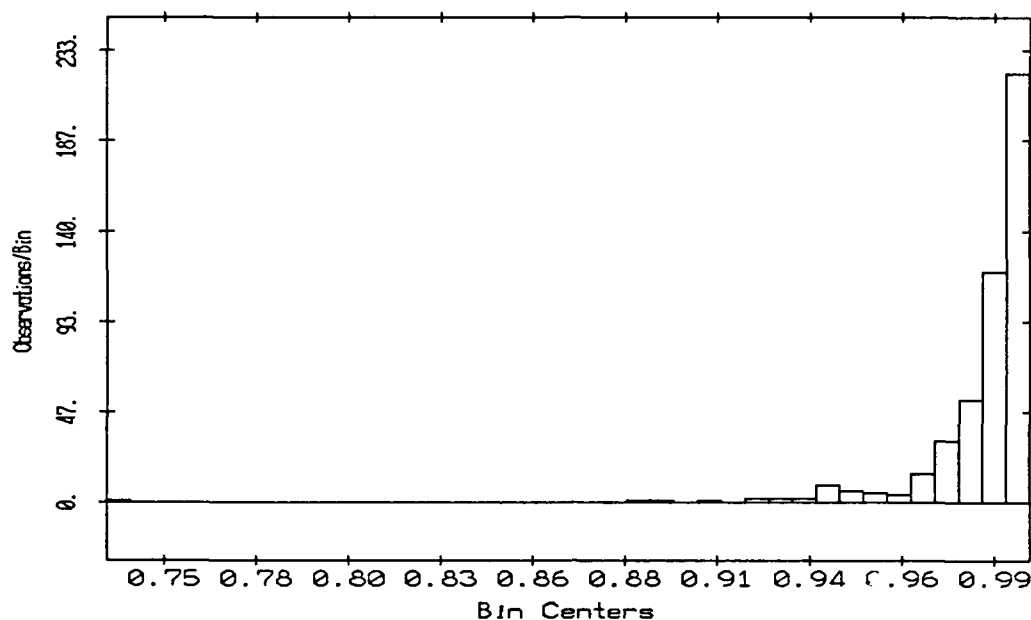


Fig. A.45 : SMC estimation, using FFT method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $5/32$

Histogram of Coherency (via FFT) at freq. =  $6/32$   
 Mean=0.88868, Var=0.12155e-01

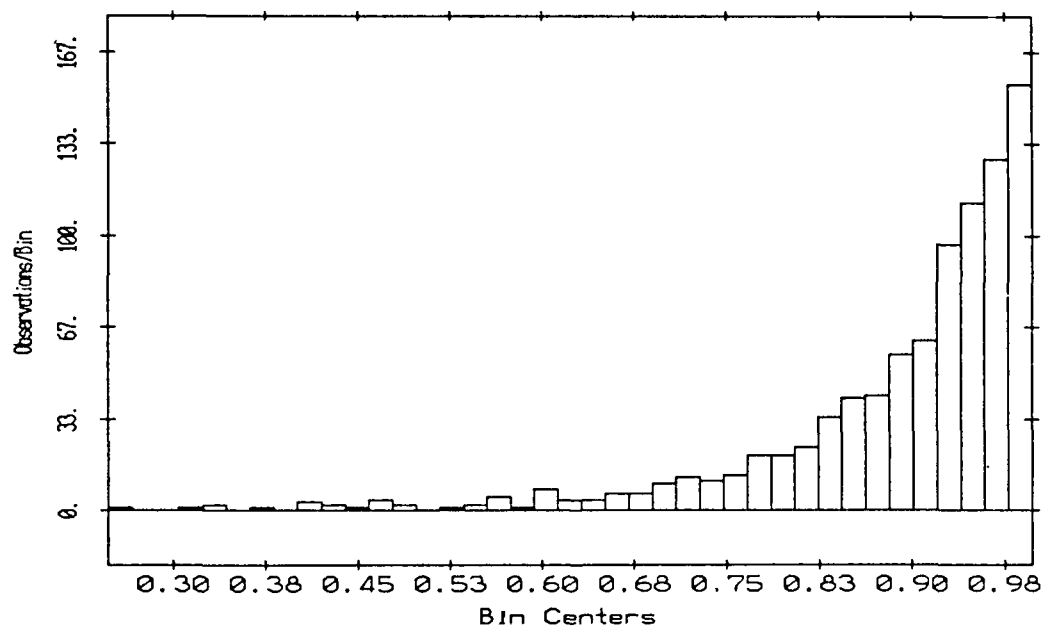


Fig. A.46 : SMC estimation, using FFT method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $6/32$

Histogram of Coherency (via FFT) at freq. =  $7/32$   
 Mean=0.77360, Var=0.34611e-01

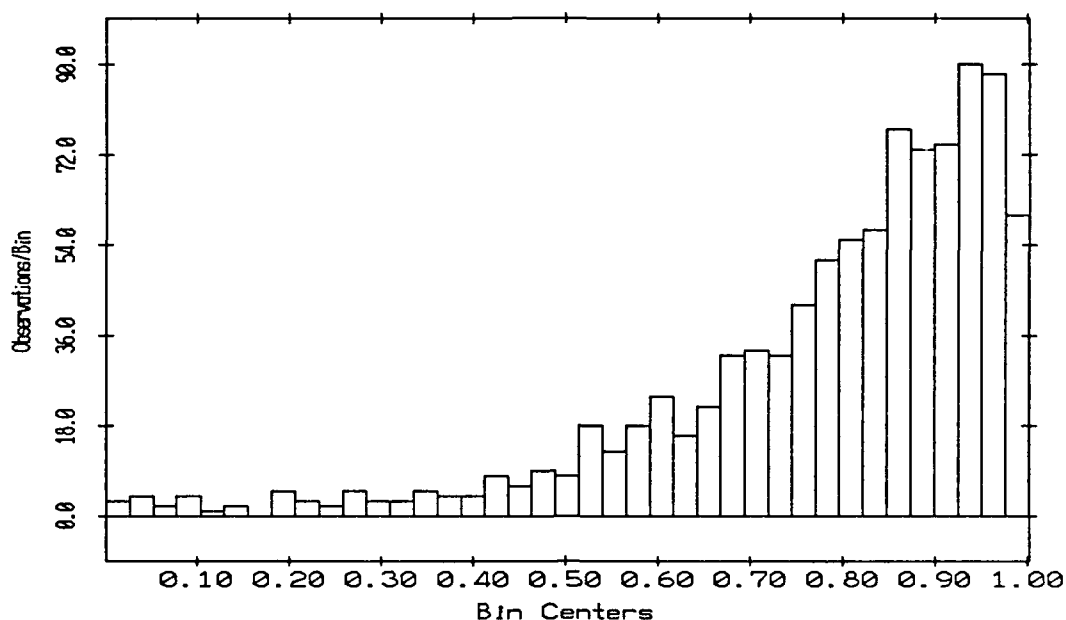


Fig. A.47 : SMC estimation, using FFT method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $7/32$

Histogram of Coherency (via FFT) at freq. =  $8/32$   
 Mean=0.67181, Var=0.50669e-01

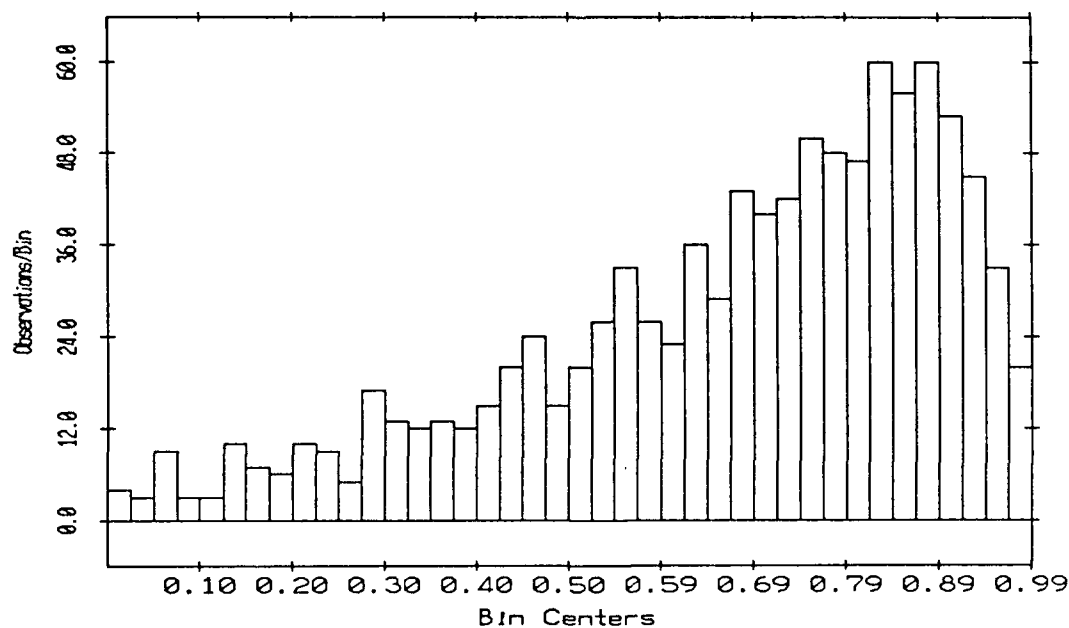


Fig. A.48 : SMC estimation, using FFT method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $8/32$

Histogram of Coherency (via FFT) at freq. = 9/32  
 Mean=0.58249, Var=0.64663e-01

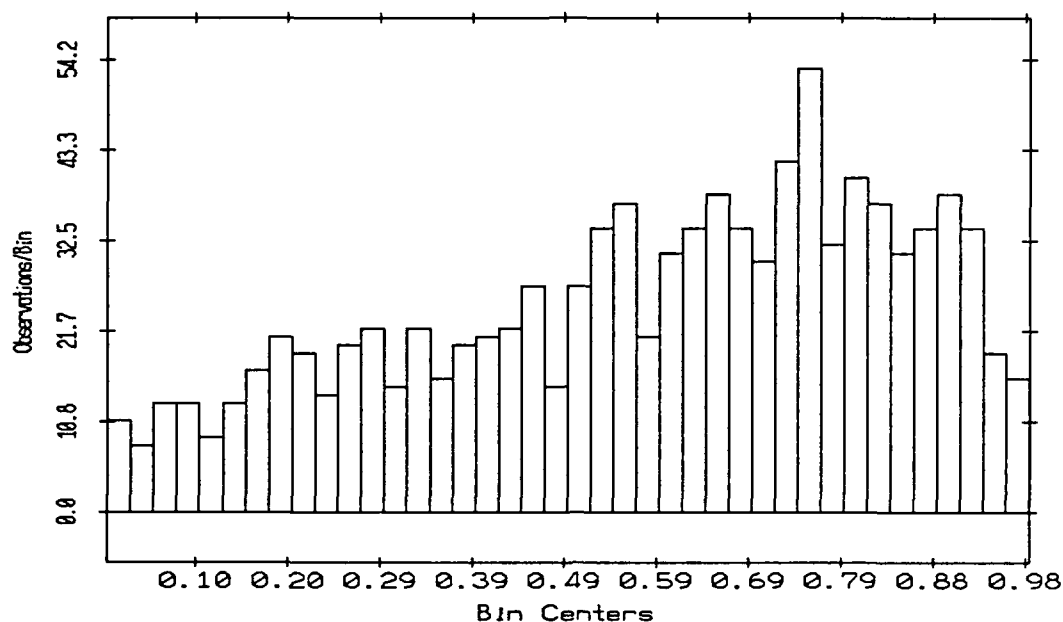


Fig. A.49: SMC estimation, using FFT method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency = 9/32

Histogram of Coherency (via FFT) at freq. = 10/32  
 Mean=0.51863, Var=0.64927e-01

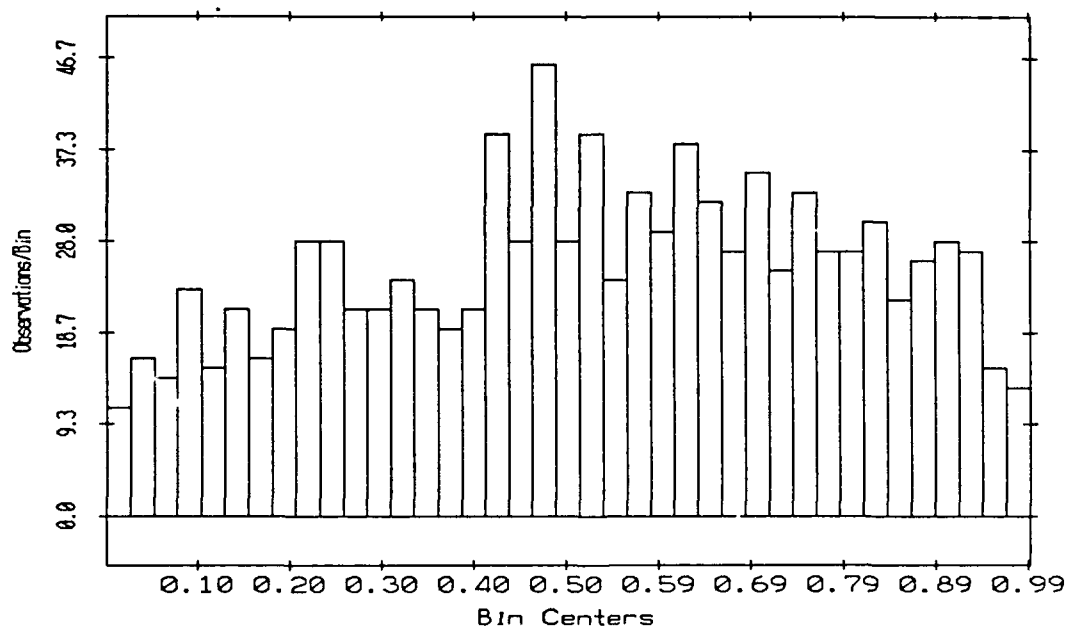


Fig. A.50: SMC estimation, using FFT method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency = 10/32



Histogram of Coherency (via FFT) at freq. =  $11/32$   
 Mean=0.46349, Var=0.65268e-01

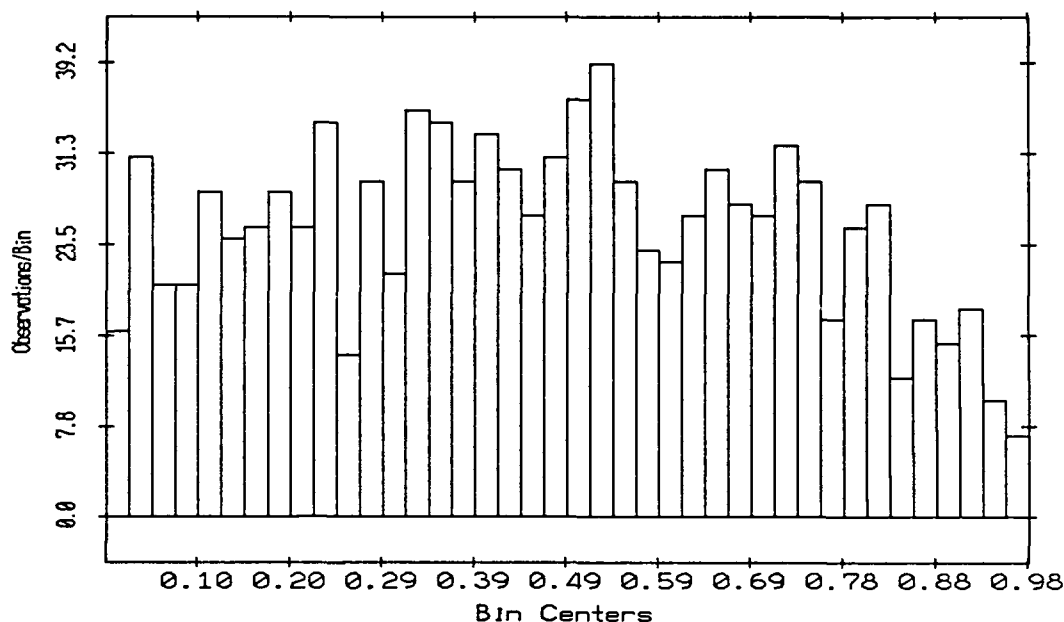


Fig. A.51: SMC estimation, using FFT method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $11/32$

Histogram of Coherency (via FFT) at freq. =  $12/32$   
 Mean=0.41317, Var=0.61149e-01

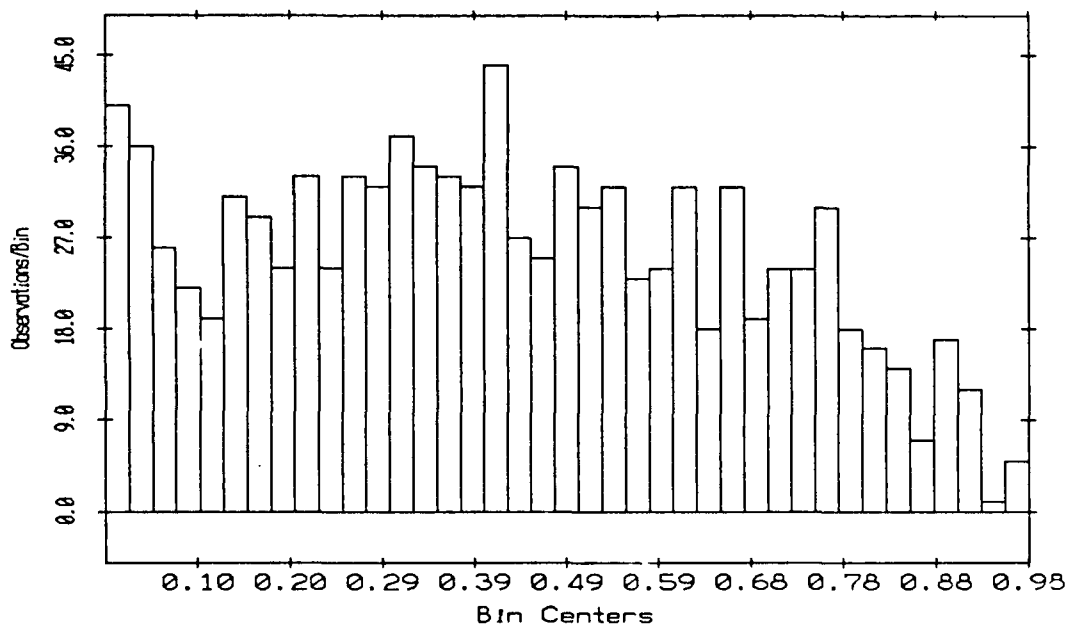


Fig. A.52: SMC estimation, using FFT method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $12/32$

## Histogram of Coherency (via FFT) at freq. = 13/32

Mean=0.39934, Var=0.63167e-01

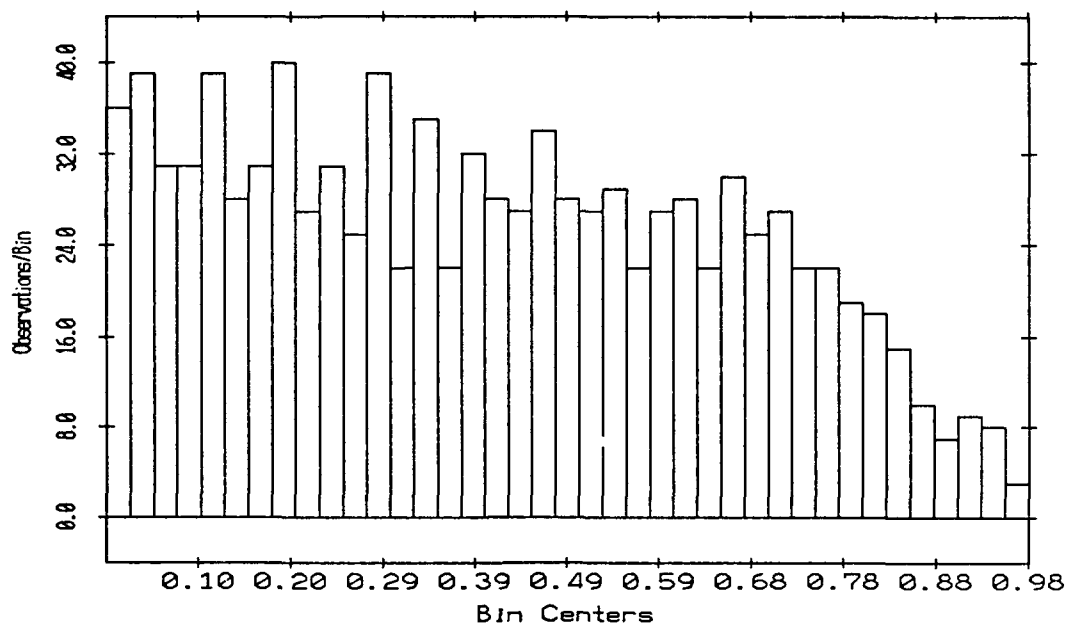


Fig. A.53: SMC estimation, using FFT method, data length 128 points,  
Histogram of estimated SMC at normalized frequency = 13/32

## Histogram of Coherency (via FFT) at freq. = 14/32

Mean=0.38437, Var=0.62063e-01

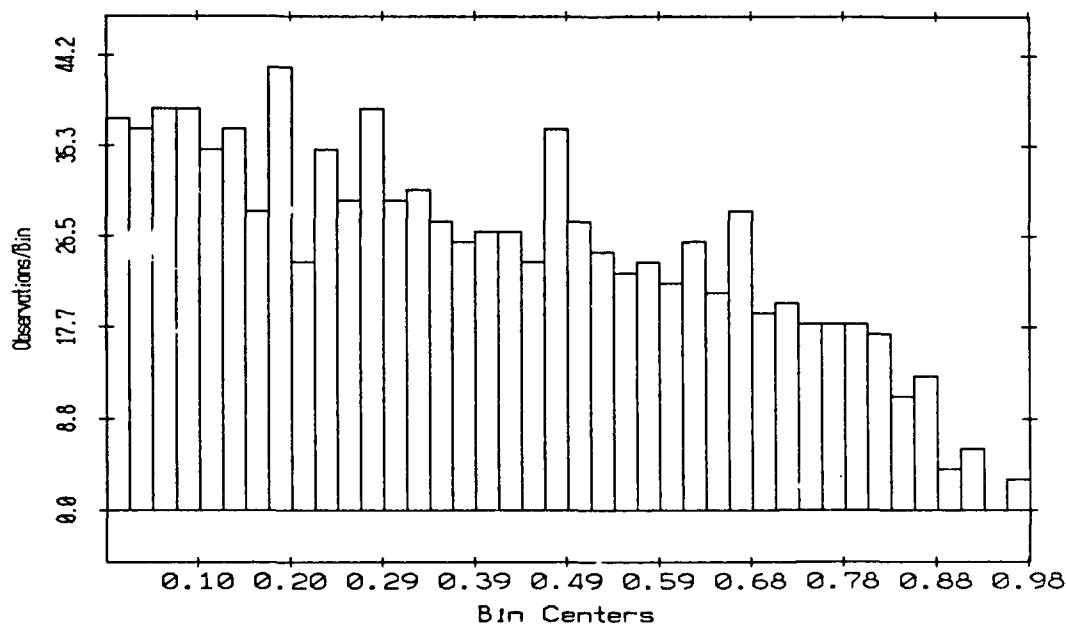


Fig. A.54: SMC estimation, using FFT method, data length 128 points,  
Histogram of estimated SMC at normalized frequency = 14/32

## Histogram of Coherency (via FFT) at freq. = 15/32

Mean=0.35435, Var=0.55124e-01

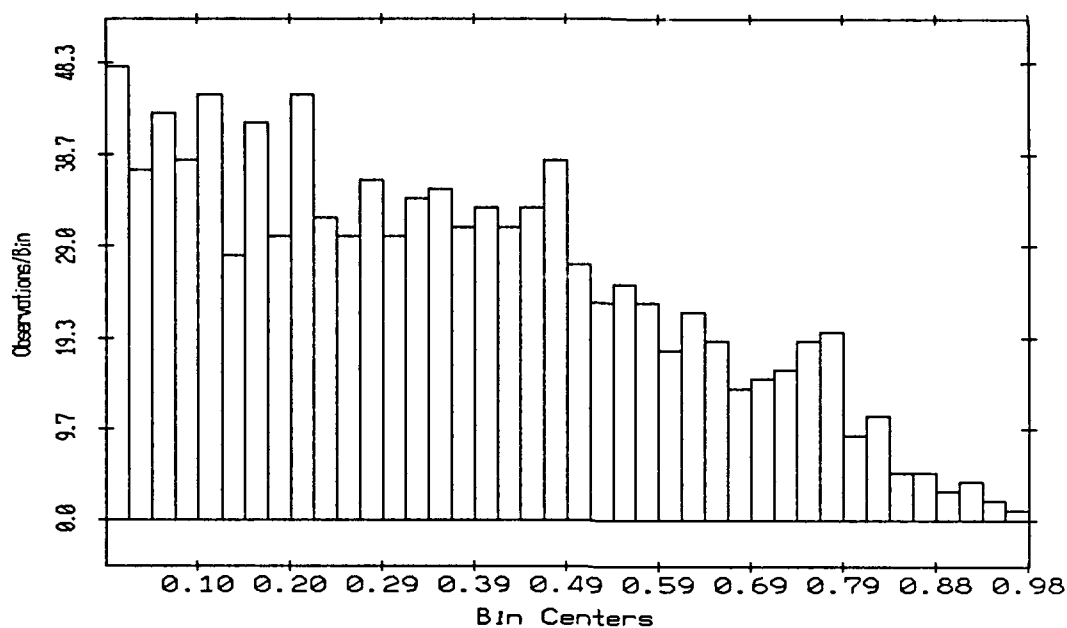


Fig. A.55 : SMC estimation, using FFT method, data length 128 points,  
Histogram of estimated SMC at normalized frequency = 15/32

## Histogram of Coherency (via FFT) at freq. = 16/32

Mean=0.35715, Var=0.60103e-01

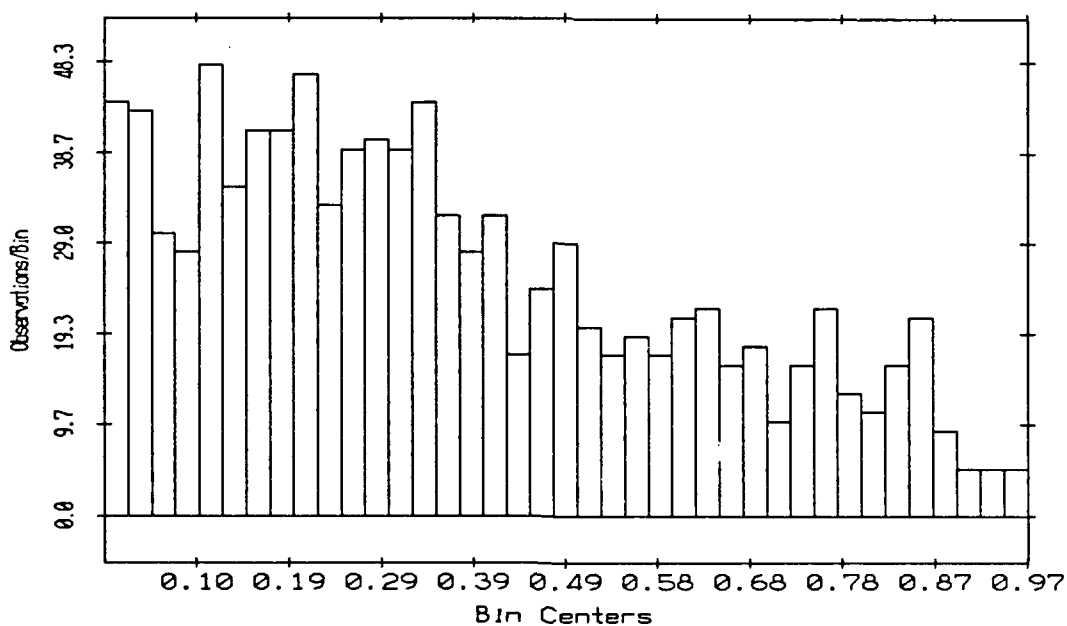


Fig. A.56 : SMC estimation, using FFT method, data length 128 points,  
Histogram of estimated SMC at normalized frequency = 16/32

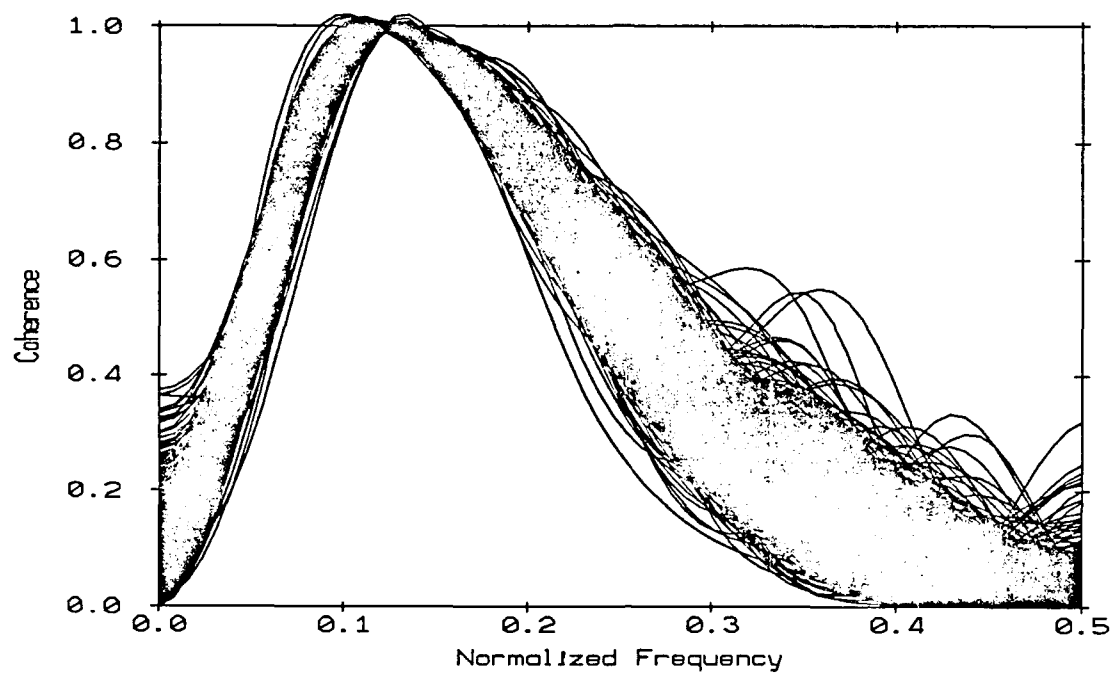


Fig. A.57 : SMC estimation, Nuttall's method, data length 512 points,

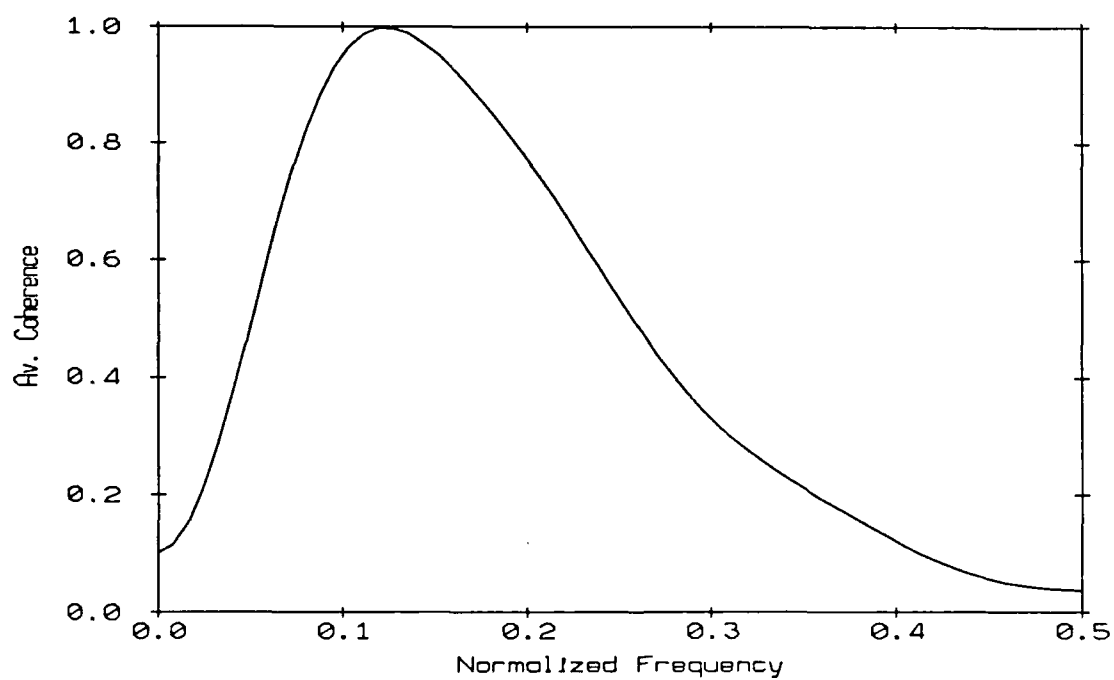


Fig. A.57.1 : SMC estimation, Nuttall's method, data length 512 points,  
Average estimated SMC over 1000 runs

SMC estimation, Nuttall's method, data length 512 points

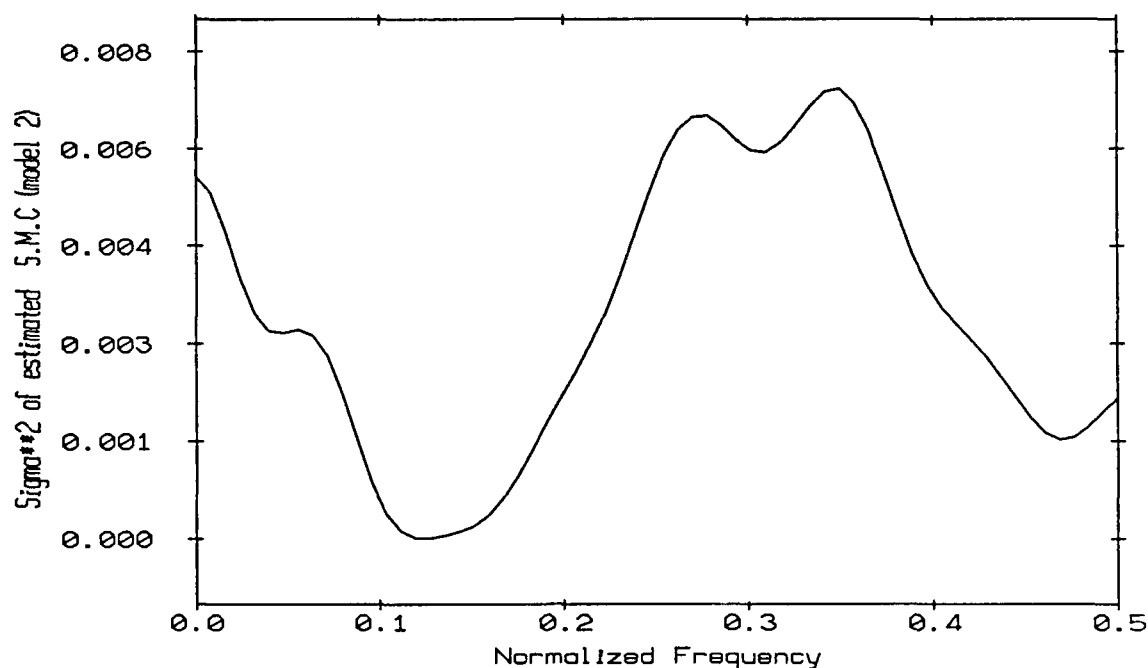


Fig. A.58 : Squared Standard Deviation, of estimated SMC, Nuttall's method  
data length 512 points, 1000 runs

SMC estimation, Nuttall's method, data length 512 points

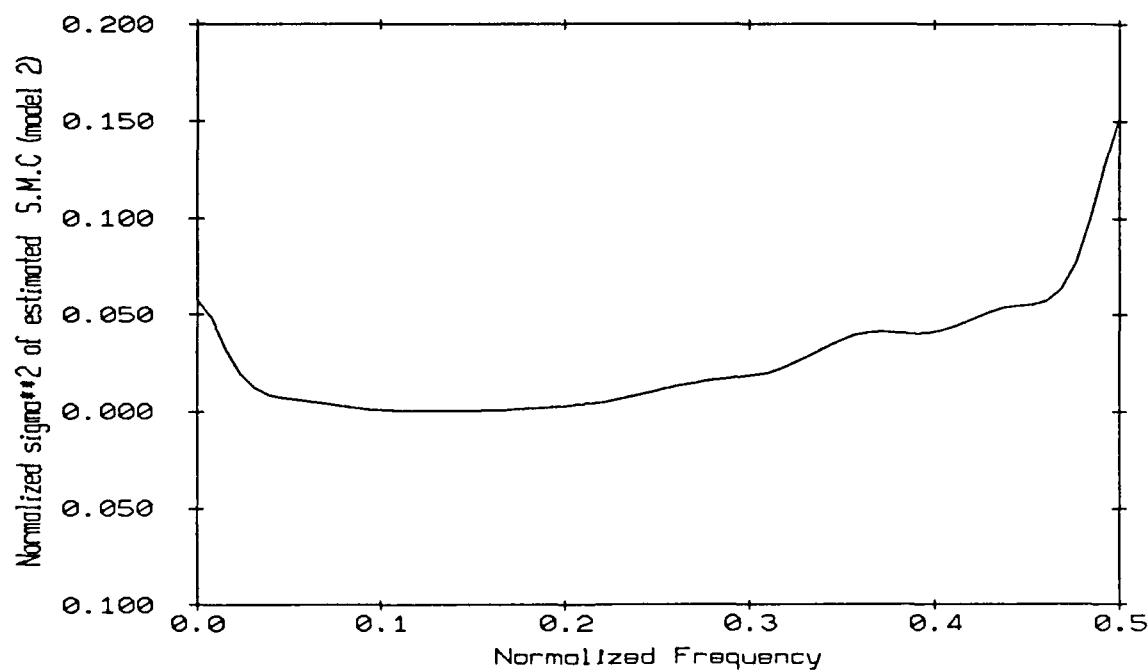


Fig. A.58.1: Normalized Squared Standard Deviation of estimated SMC, Nuttall's method  
data length 512 points, 1000 runs

Histogram of Coherency at normalized freq. =  $1/32$

Mean=0.20781, Squared Standard Deviation=0.40356e-02

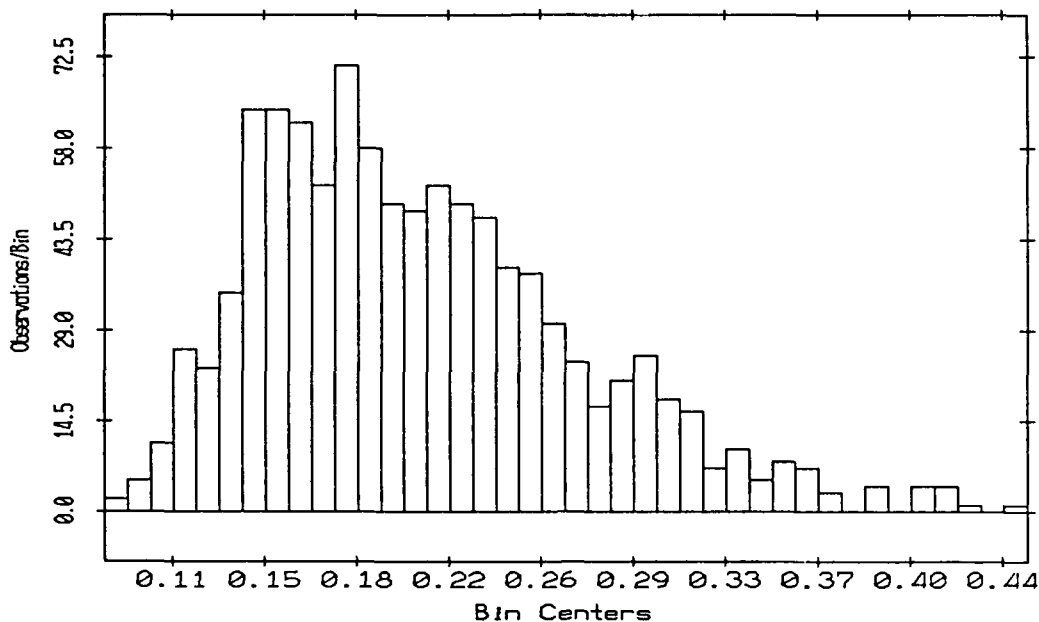


Fig. A.59: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated SMC at normalized frequency =  $1/32$

Histogram of Coherency at normalized freq. =  $2/32$

Mean=0.56090, Squared Standard Deviation=0.32260e-02

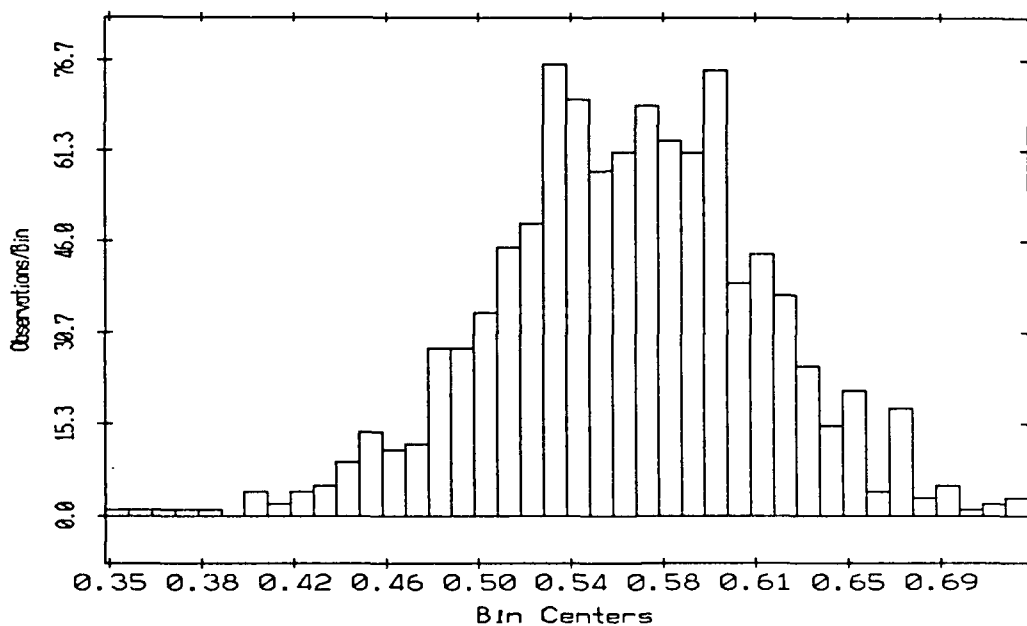


Fig. A.60: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated SMC at normalized frequency =  $2/32$

Histogram of Coherency at normalized freq. =  $3/32$

Mean=0.88129, Squared Standard Deviation=0.15348e-02

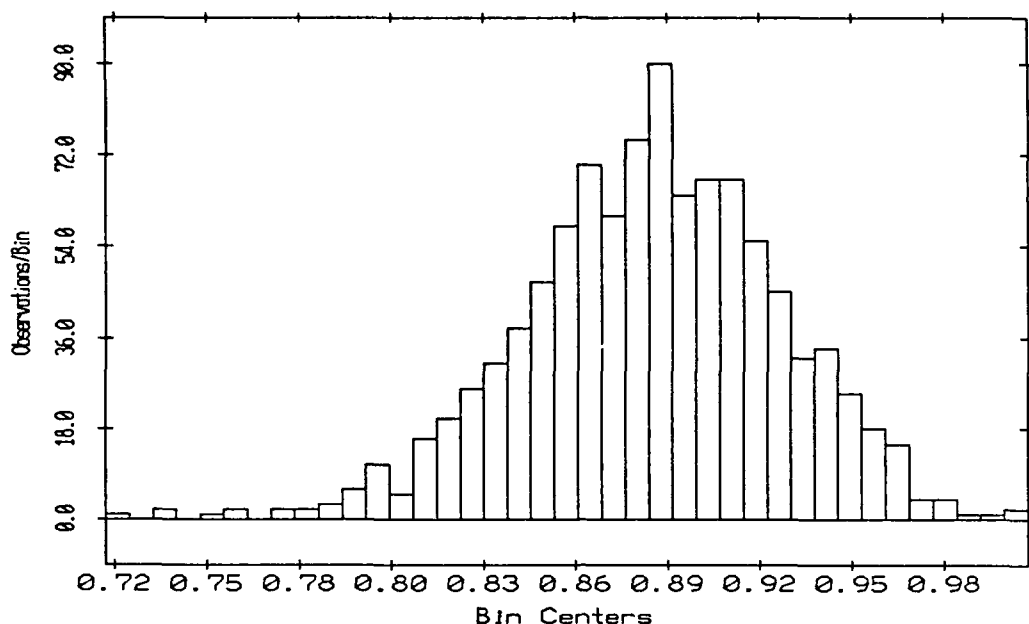


Fig. A.61: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated SMC at normalized frequency =  $3/32$

Histogram of Coherency at normalized freq. =  $4/32$

Mean=0.99800, Squared Standard Deviation=0.17916e-04

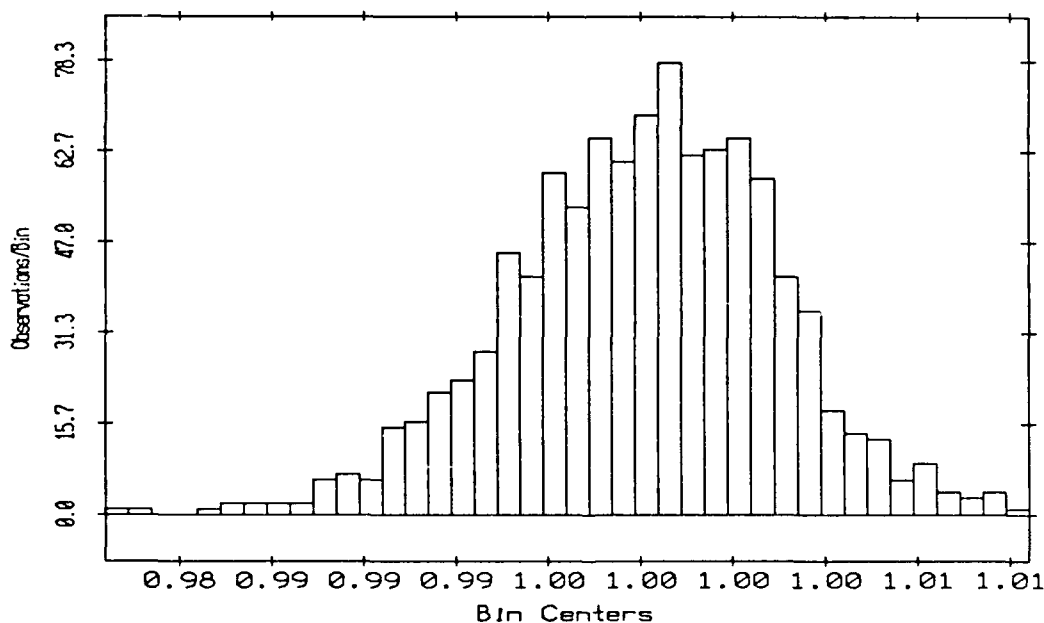


Fig. A.62: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated SMC at normalized frequency =  $4/32$

Histogram of Coherency at normalized freq. = 5/32

Mean=0.95444, Squared Standard Deviation=0.20421e-03

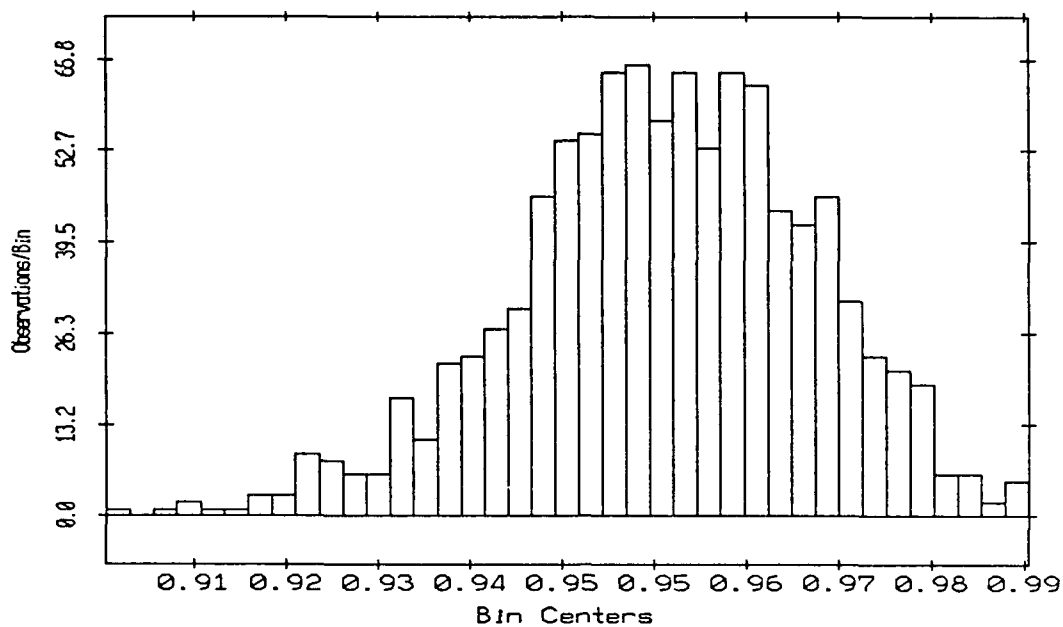


Fig. A.63: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated SMC at normalized frequency = 5/32

Histogram of Coherency at normalized freq. = 6/32

Mean=0.84423, Squared Standard Deviation=0.13608e-02

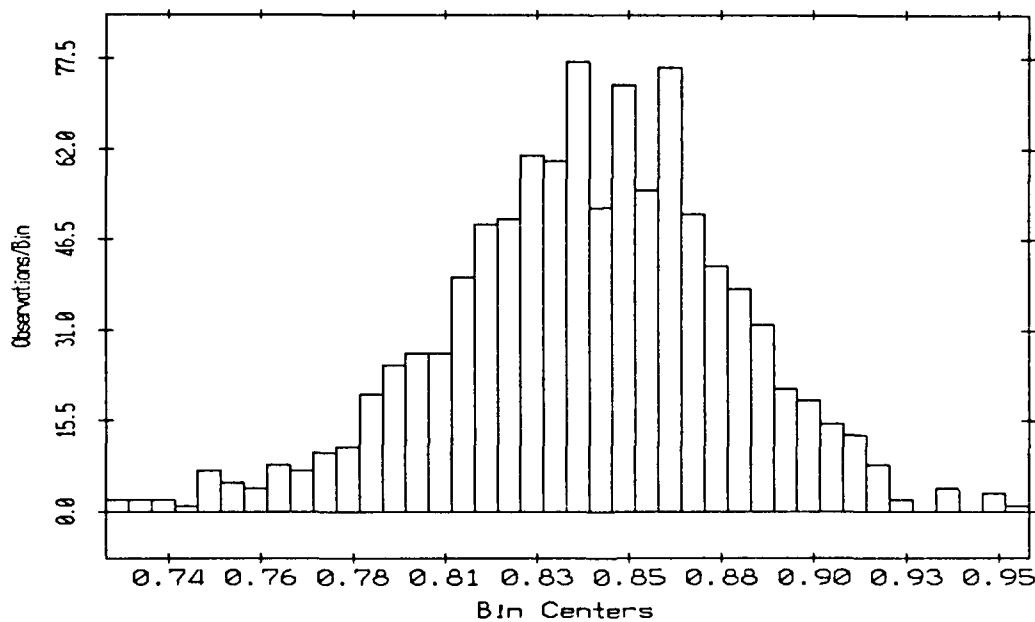


Fig. A.64: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated SMC at normalized frequency = 6/32



Histogram of Coherency at normalized freq. = 7/32

Mean=0.70663, Squared Standard Deviation=0.30001e-02

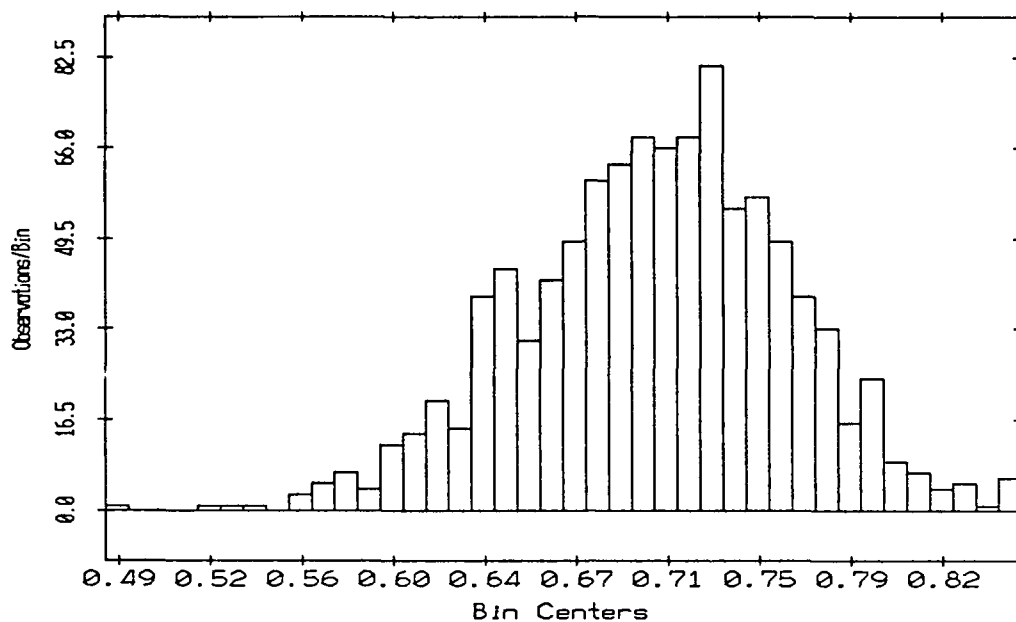


Fig. A.65: SMC estimation, Nuttall's method, data length 512 points, Histogram of estimated SMC at normalized frequency = 7/32

Histogram of Coherency at normalized freq. = 8/32

Mean=0.55274, Squared Standard Deviation=0.53004e-02

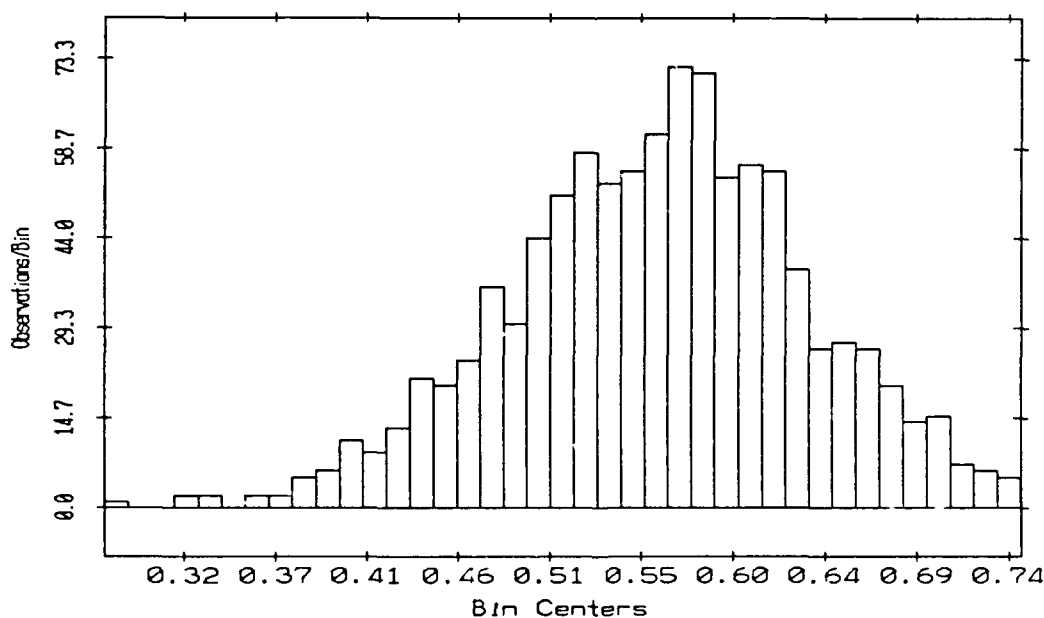


Fig. A.66: SMC estimation, Nuttall's method, data length 512 points, Histogram of estimated SMC at normalized frequency = 8/32

Histogram of Coherency at normalized freq. =  $9/32$

Mean= $0.40717$ , Squared Standard Deviation= $0.65019e-02$

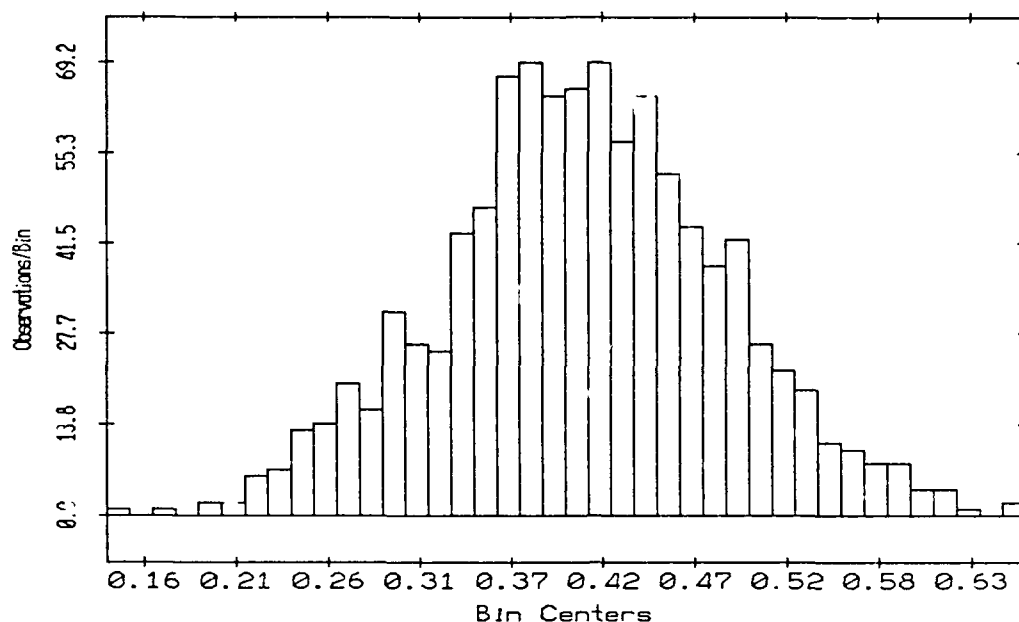


Fig. A.67: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated SMC at normalized frequency =  $9/32$

Histogram of Coherency at normalized freq. =  $10/32$

Mean= $0.30036$ , Squared Standard Deviation= $0.59597e-02$

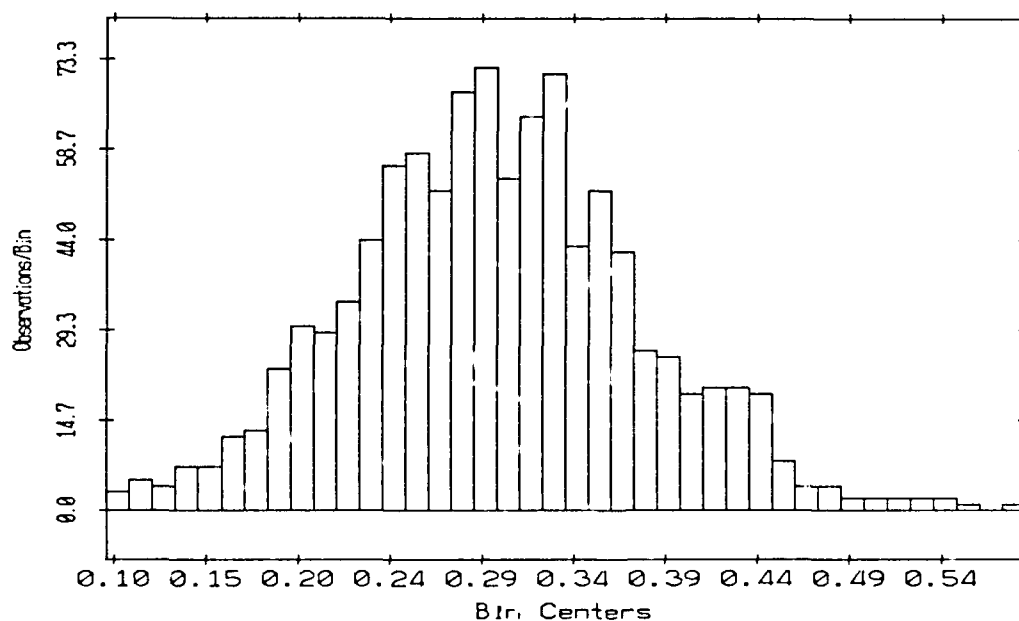


Fig. A.68: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated SMC at normalized frequency =  $10/32$

Histogram of Coherency at normalized freq. = 11/32  
 Mean=0.22773, Squared Standard Deviation=0.68822e-02

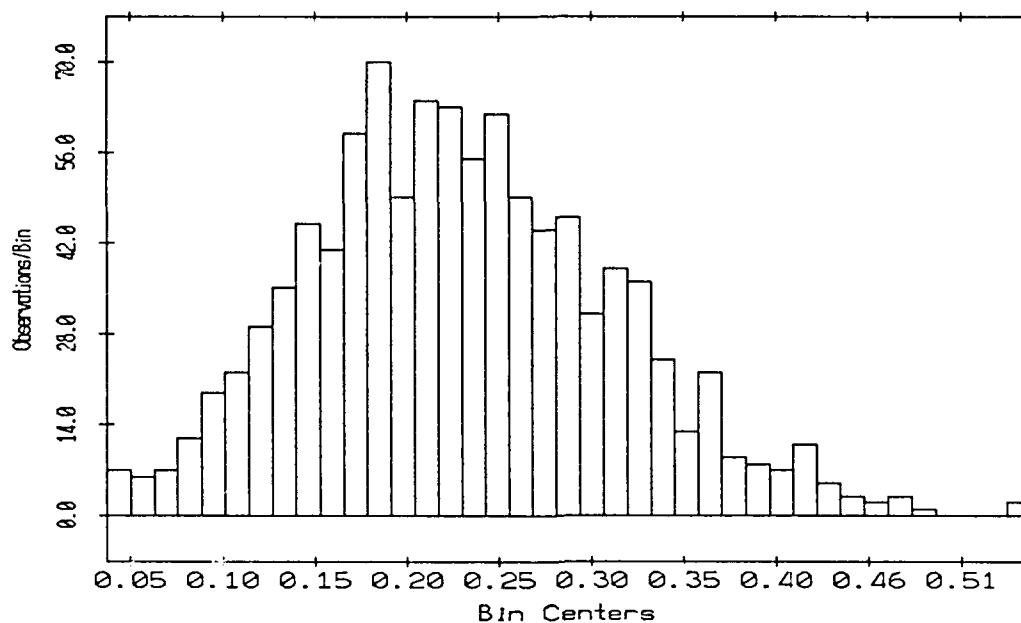


Fig. A.69: SMC estimation, Nuttall's method, data length 512 points,  
 Histogram of estimated SMC at normalized frequency = 11/32

Histogram of Coherency at normalized freq. = 12/32  
 Mean=0.16689, Squared Standard Deviation=0.56618e-02

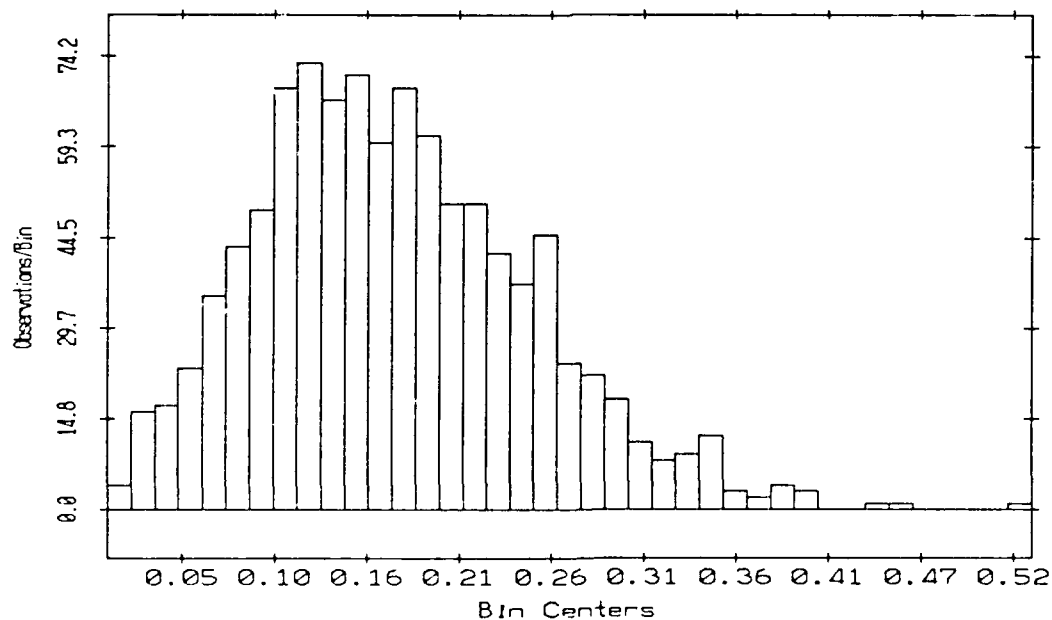


Fig. A.70: SMC estimation, Nuttall's method, data length 512 points,  
 Histogram of estimated SMC at normalized frequency = 12/32

## Histogram of Coherency at normalized freq. = 13/32

Mean=0.11230, Squared Standard Deviation=0.35522e-02

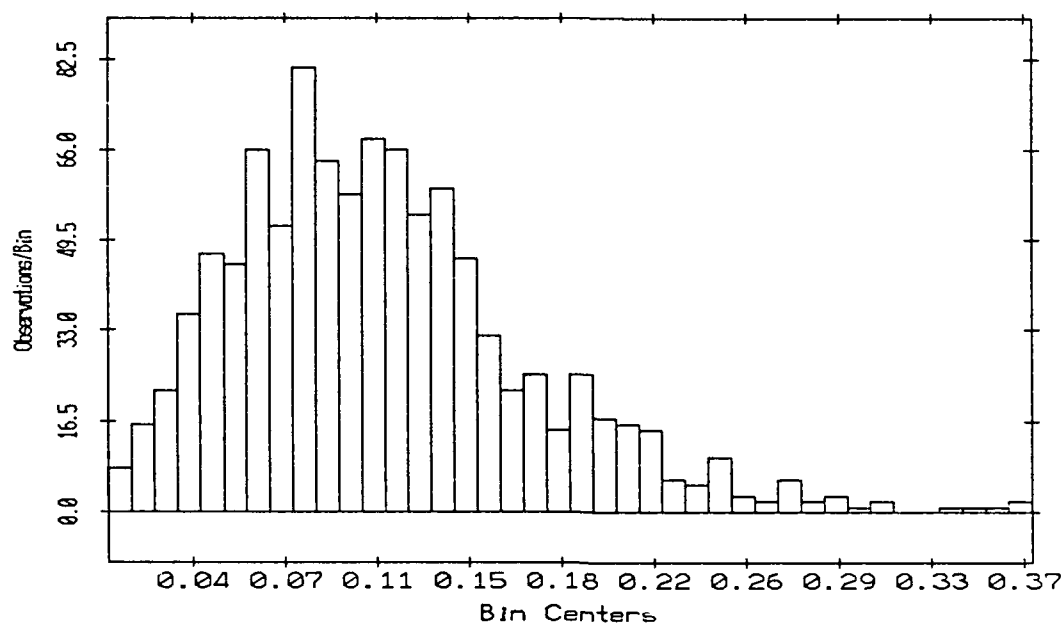


Fig. A.71: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated SMC at normalized frequency = 13/32

## Histogram of Coherency at normalized freq. = 14/32

Mean=0.68639e-01, Squared Standard Deviation=0.25034e-02

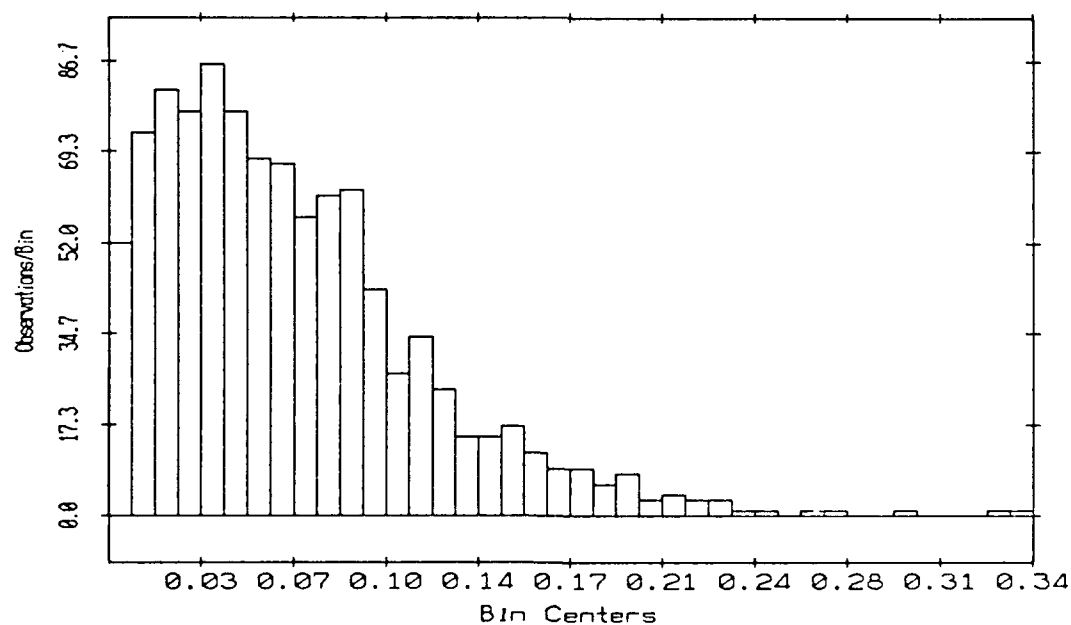


Fig. A.72: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated SMC at normalized frequency = 14/32

Histogram of Coherency at normalized freq. = 15/32  
 Mean=0.45600e-01, Squared Standard Deviation=0.15243e-02

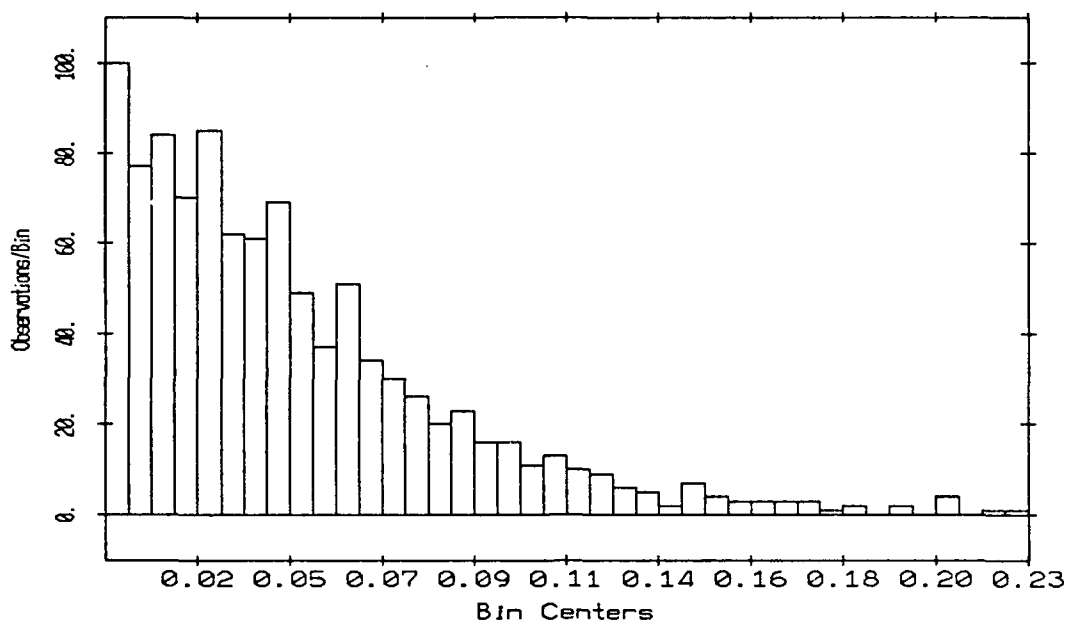


Fig. A.73 : SMC estimation, Nuttall's method, data length 512 points,  
 Histogram of estimated SMC at normalized frequency = 15/32

Histogram of Coherency at normalized freq. = 16/32  
 Mean=0.38250e-01, Squared Standard Deviation=0.21616e-02

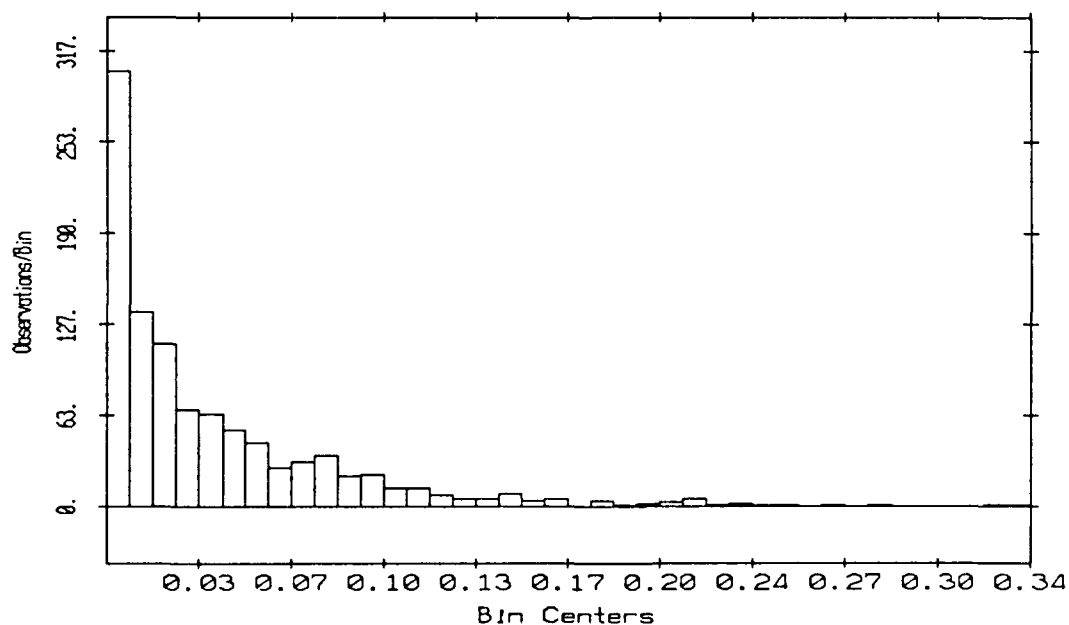


Fig. A.74 : SMC estimation, Nuttall's method, data length 512 points,  
 Histogram of estimated SMC at normalized frequency = 16/32

## Histogram of A1

Mean=-0.33873e-02, Standard Deviation=0.27179e-01

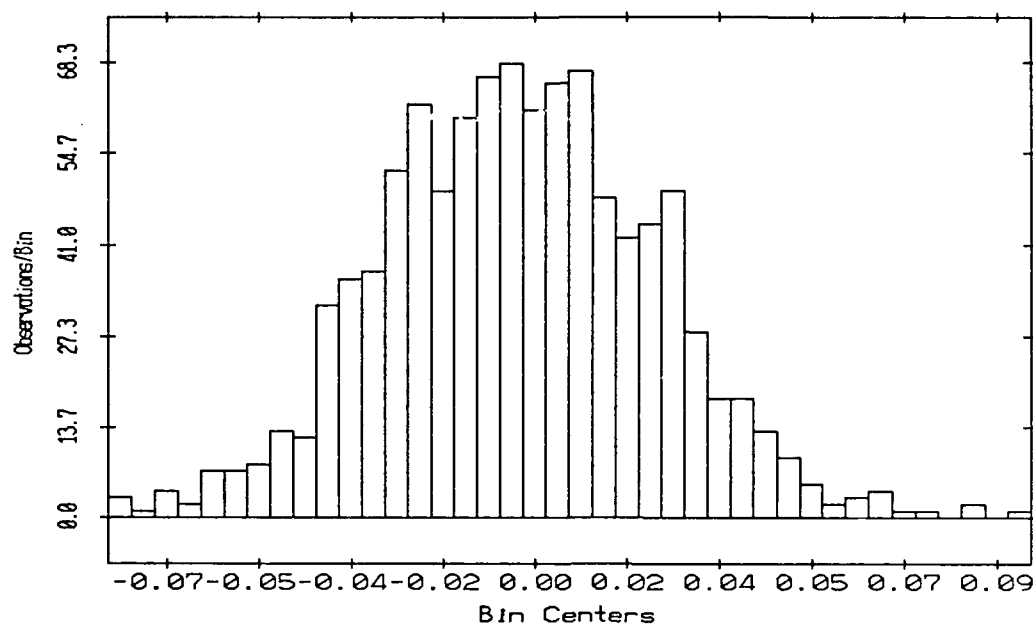


Fig. A.75 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A2'

## Histogram of A2

Mean=-0.31799e-02, Standard Deviation=0.32773e-01

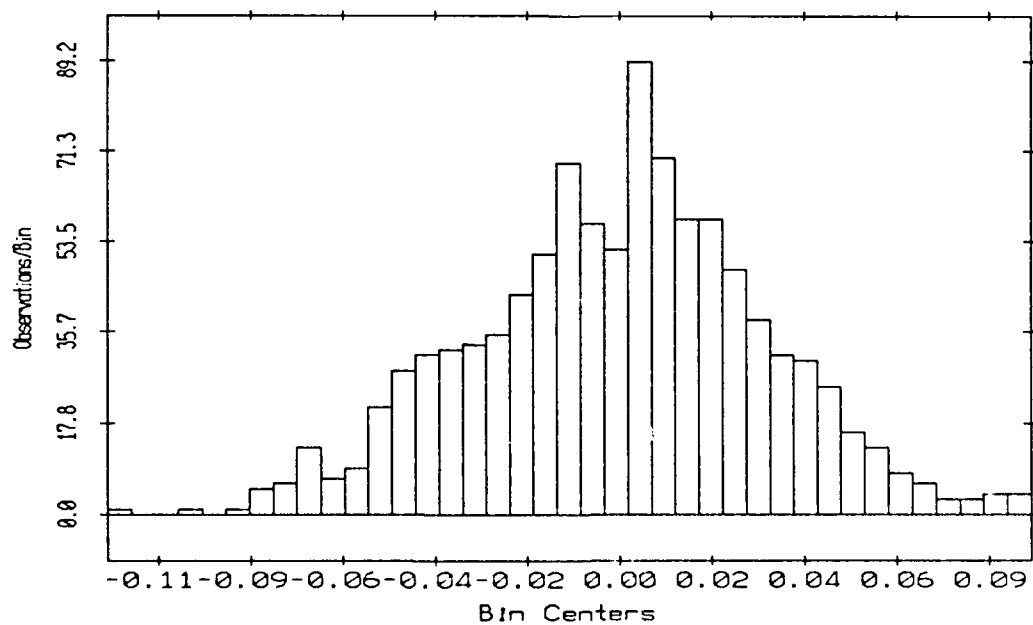


Fig. A.76 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A1'

## Histogram of A3

Mean=-0.14976e-01, Standard Deviation=0.34077e-01

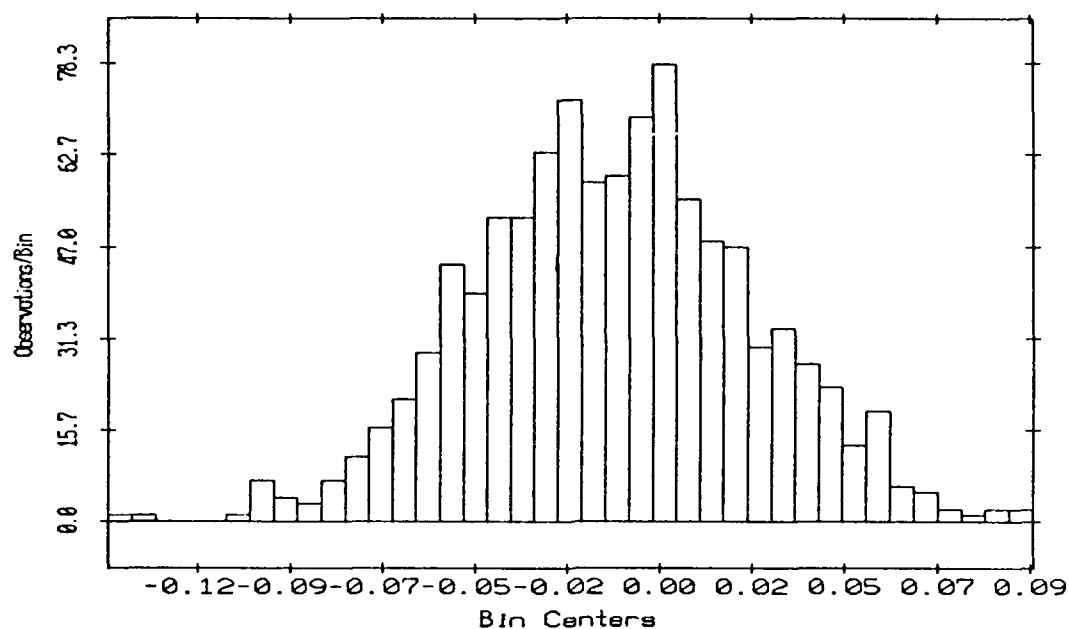


Fig. A.77 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A4'

## Histogram of A4

Mean=-0.44590e-01, Standard Deviation=0.34920e-01

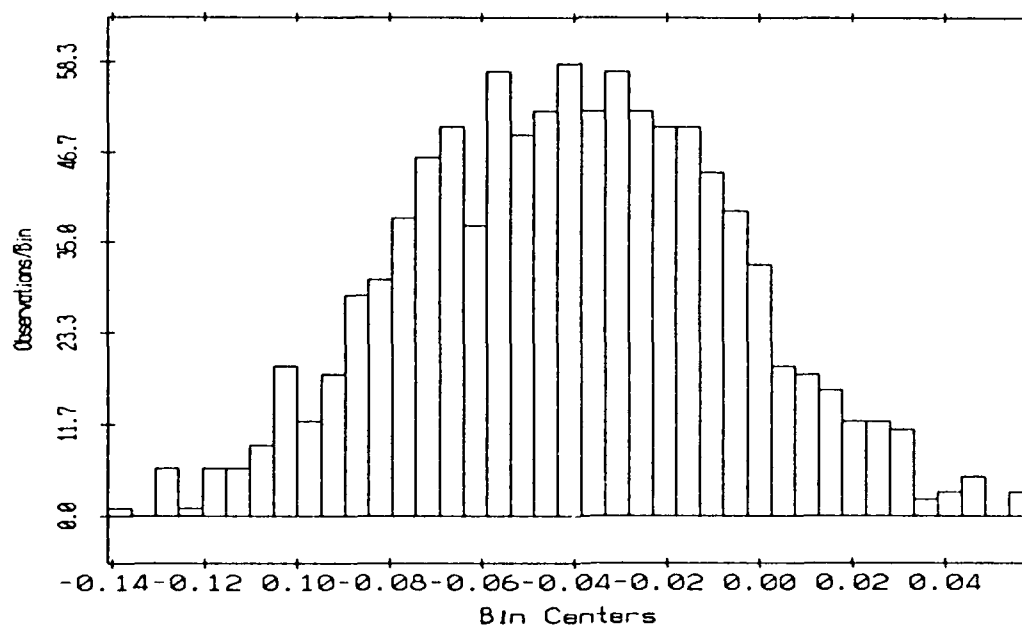


Fig. A.78 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A3'

## Histogram of A5

Mean=-0.12934, Standard Deviation=0.35209e-01

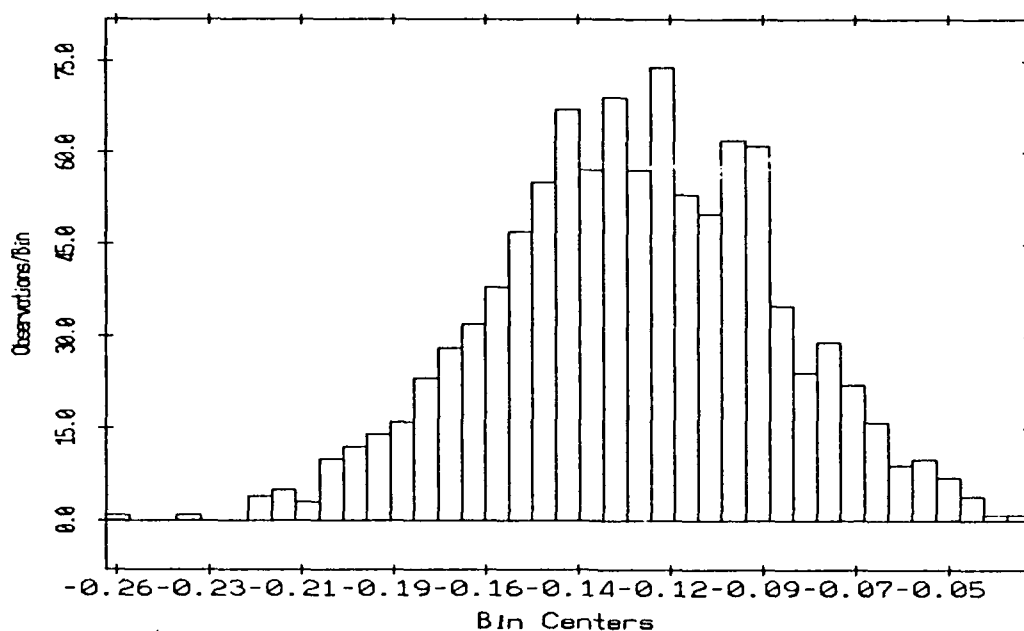


Fig. A.79 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A6'

## Histogram of A6

Mean=-0.38943, Standard Deviation=0.41950e-01

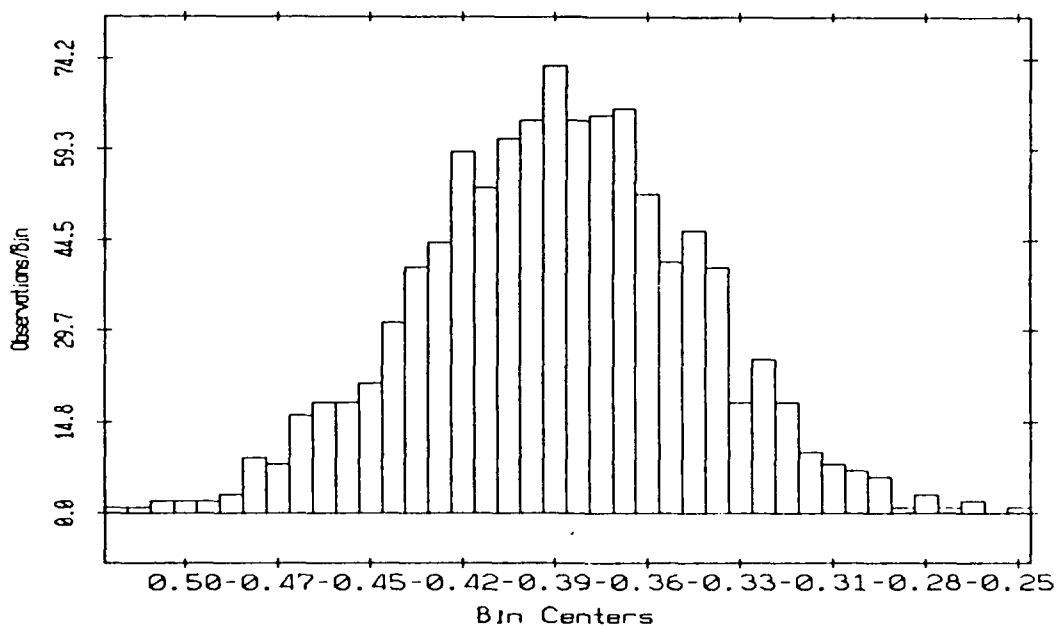


Fig. A.80 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A5'



## Histogram of A7

Mean=0.16501, Standard Deviation=0.42072e-01

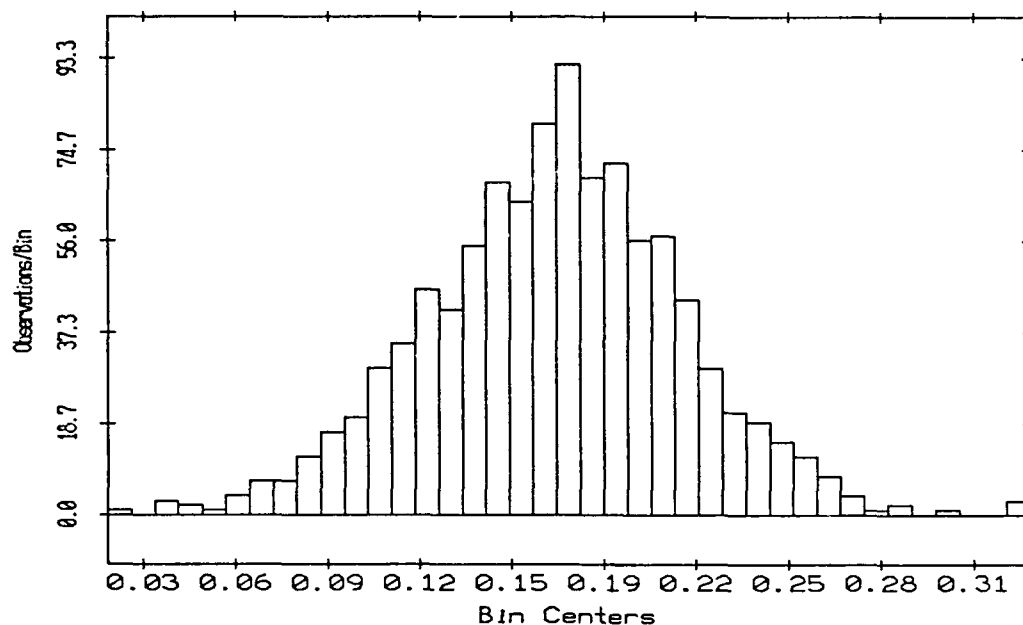


Fig. A.81 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A8'

## Histogram of A8

Mean=0.43380, Standard Deviation=0.42375e-01

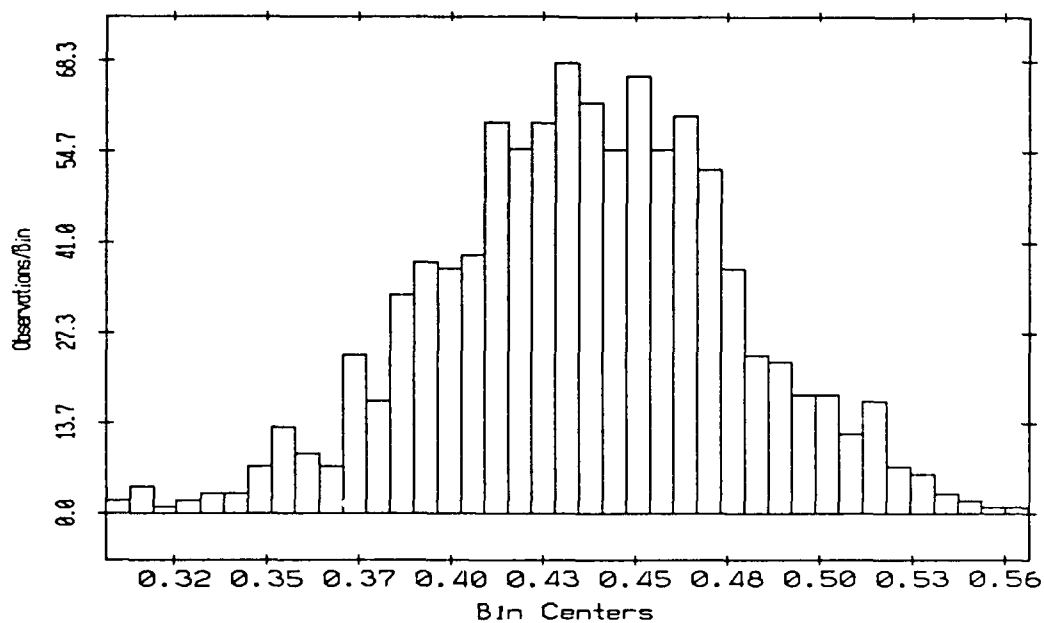


Fig. A.82 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A7'

## Histogram of A9

Mean=0.14513, Standard Deviation=0.35208e-01

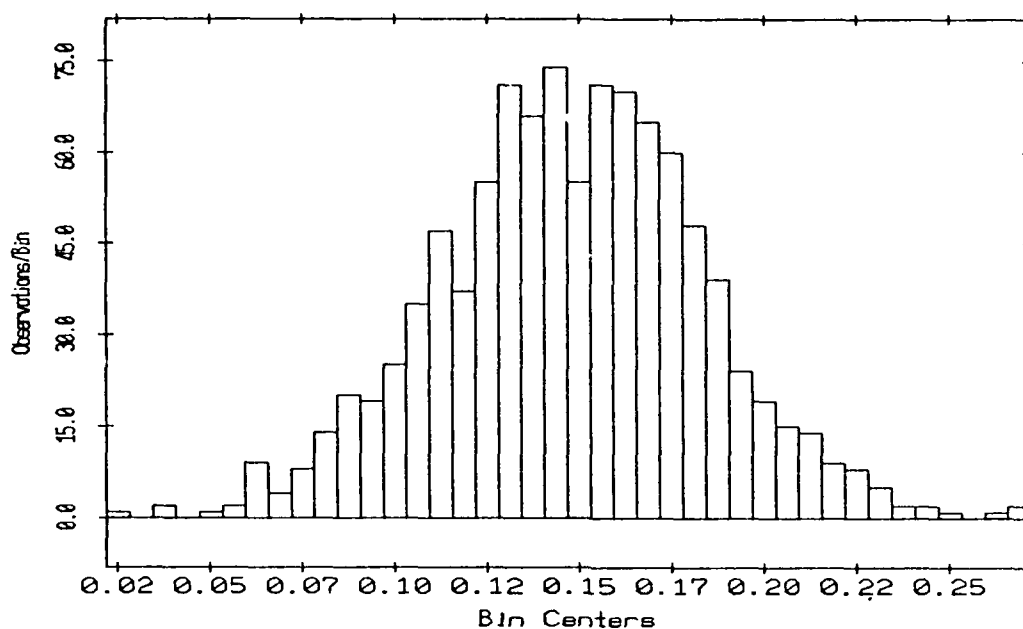


Fig. A.83: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A10'

## Histogram of A10

Mean=0.45962e-01, Standard Deviation=0.34167e-01

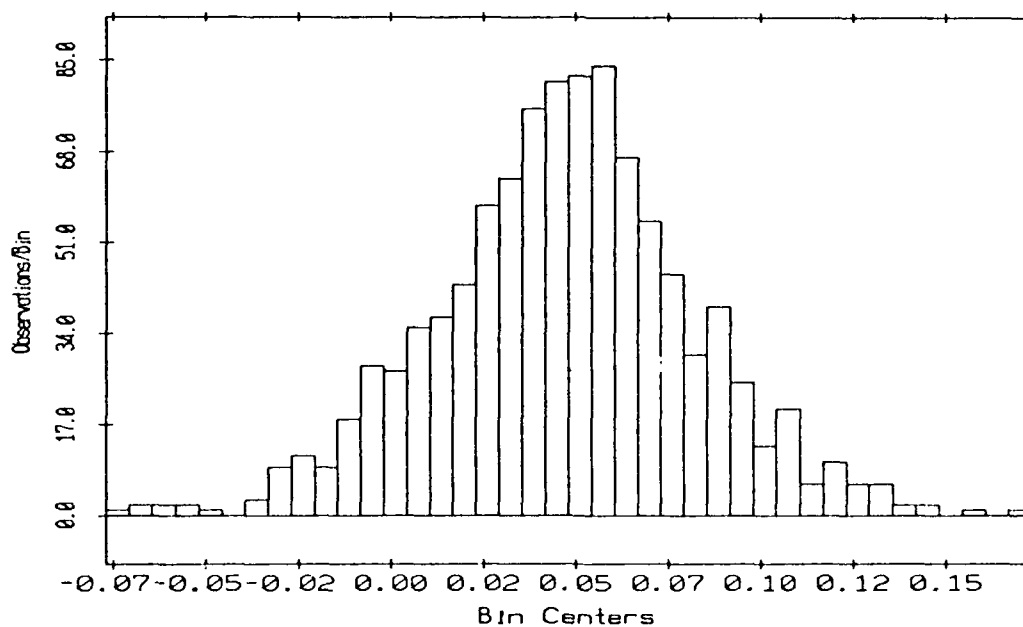


Fig. A.84: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A9'

## Histogram of A11

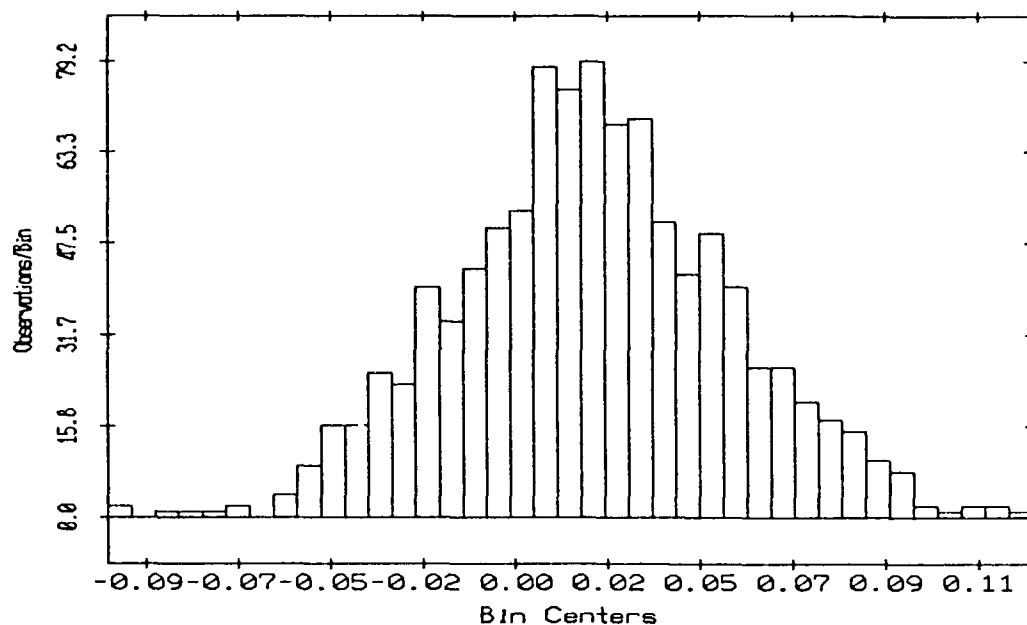
Mean= $0.15874 \times 10^{-1}$ , Standard Deviation= $0.33145 \times 10^{-1}$ 

Fig. A.85 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A12'

## Histogram of A12

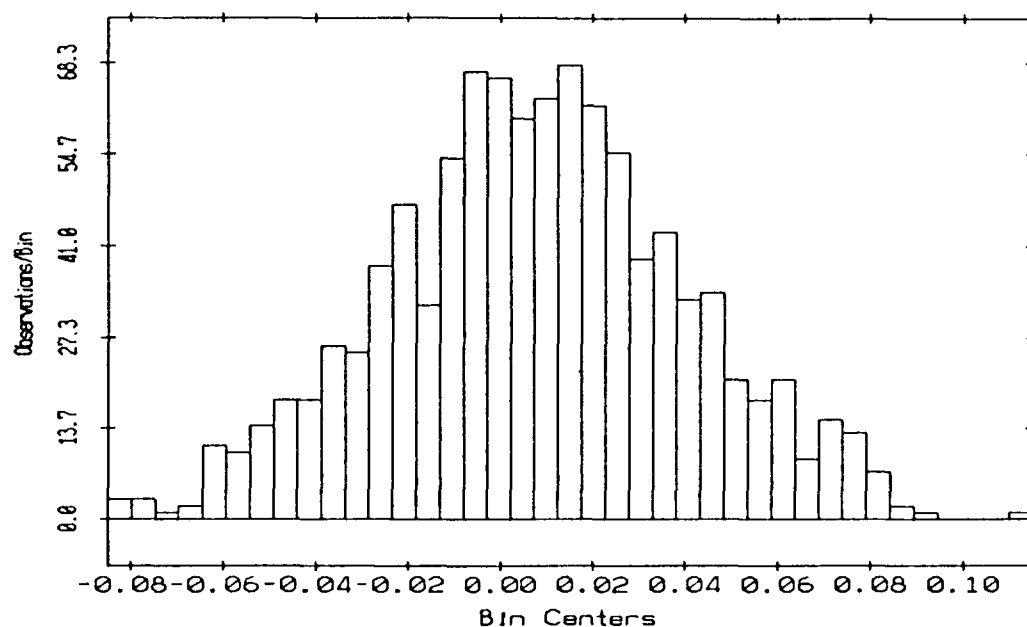
Mean= $0.55921 \times 10^{-2}$ , Standard Deviation= $0.32301 \times 10^{-1}$ 

Fig. A.86 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A11'

## Histogram of A13

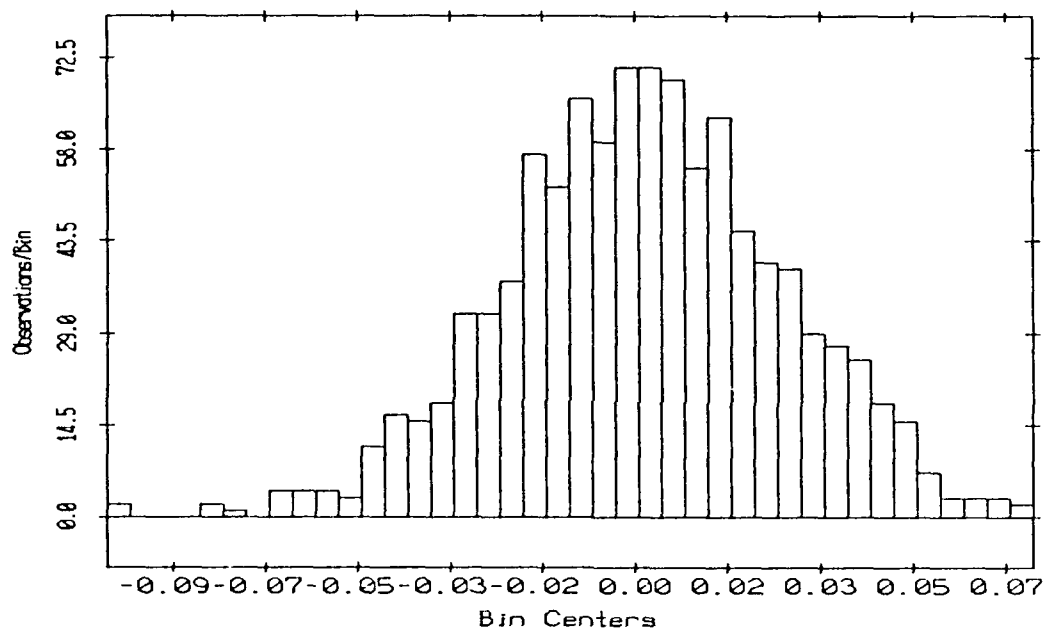
Mean= $0.15489\text{e-}02$ , Standard Deviation= $0.26147\text{e-}01$ 

Fig. A.87: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'A13'

## Histogram of B1

Mean=0.18435e-01, standard deviation=0.28718e-01

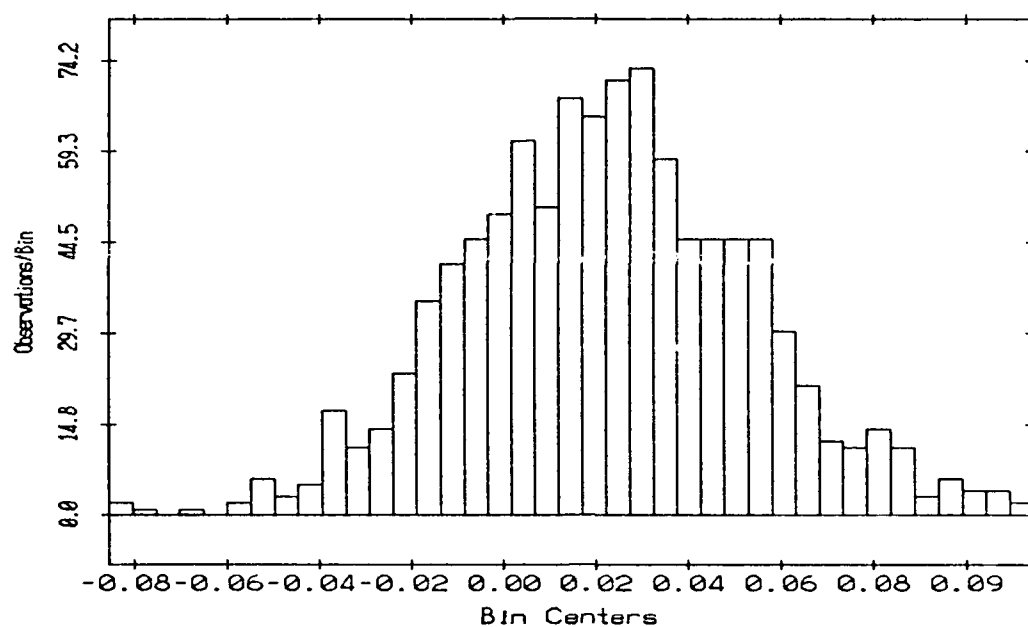


Fig. A.88 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B2'

## Histogram of B2

Mean=0.21003e-01, standard deviation=0.29494e-01

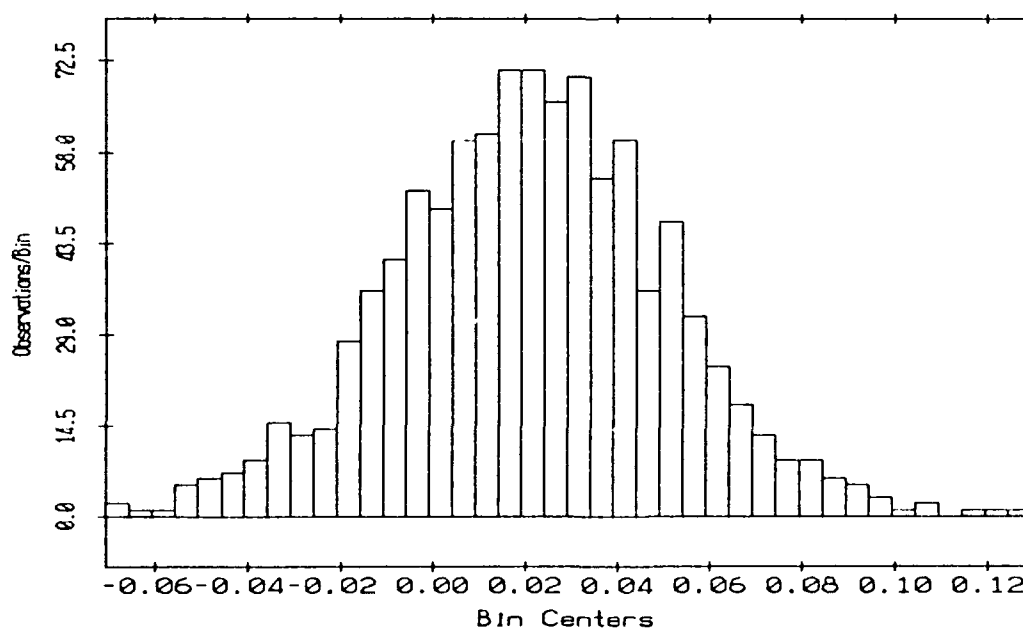


Fig. A.89 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B1'

## Histogram of B3

Mean=0.50869e-01, standard deviation=0.28749e-01

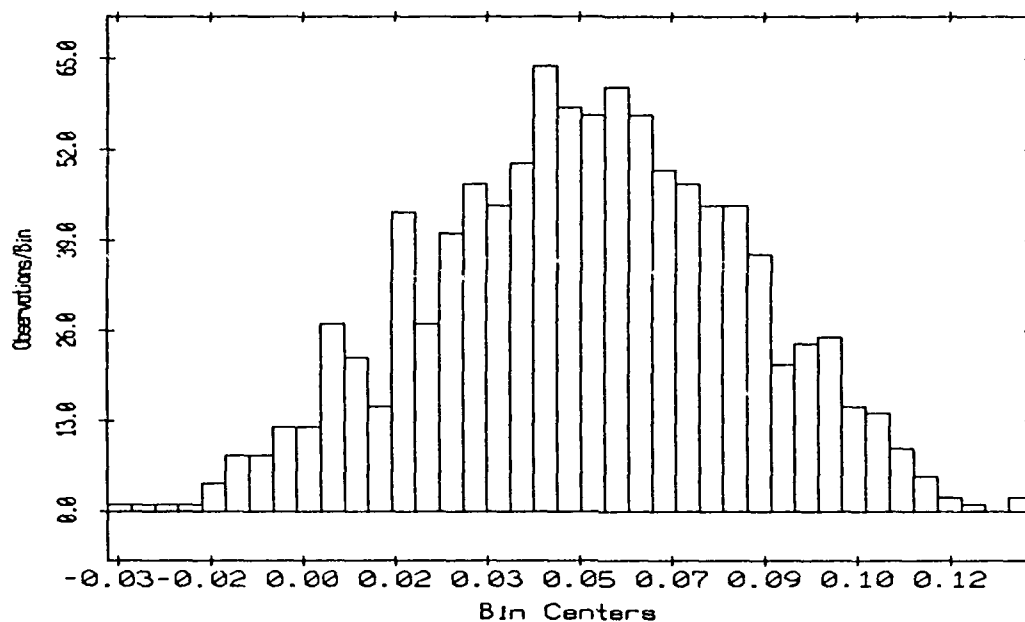


Fig. A.90 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B4'

## Histogram of B4

Mean=0.10775, standard deviation=0.31620e-01

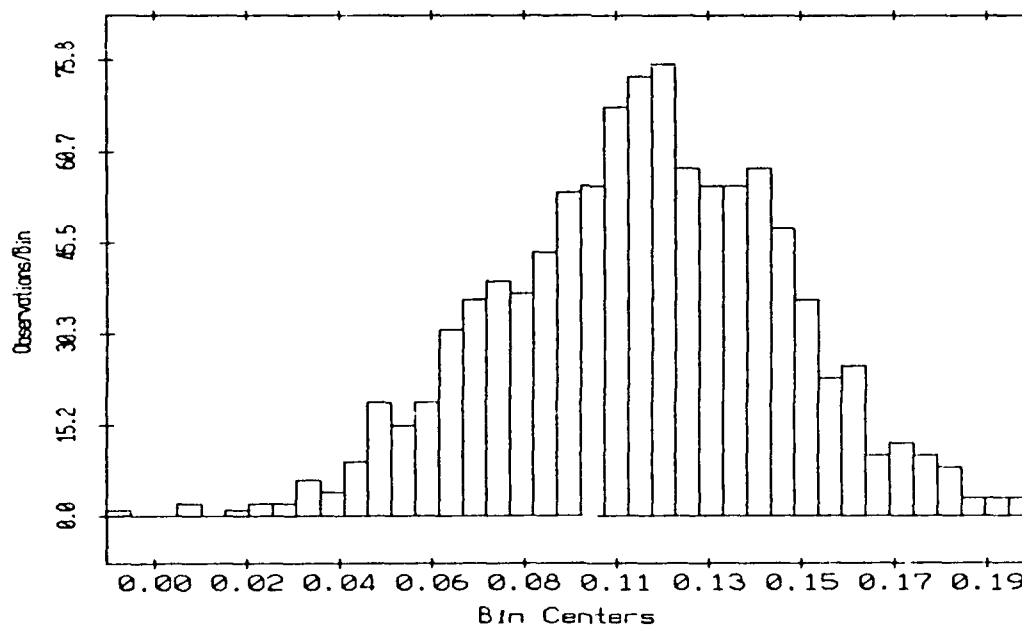


Fig. A.91 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B3'

## Histogram of B5

Mean=0.22547, standard deviation=0.33770e-01

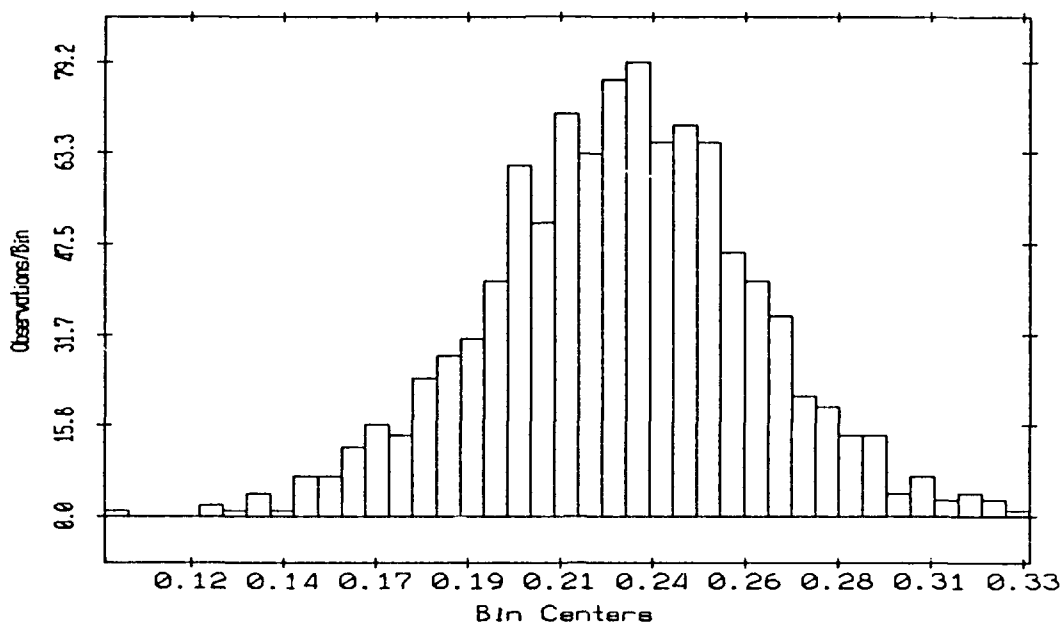


Fig. A.92: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B6'

## Histogram of B6

Mean=0.46214, standard deviation=0.39211e-01

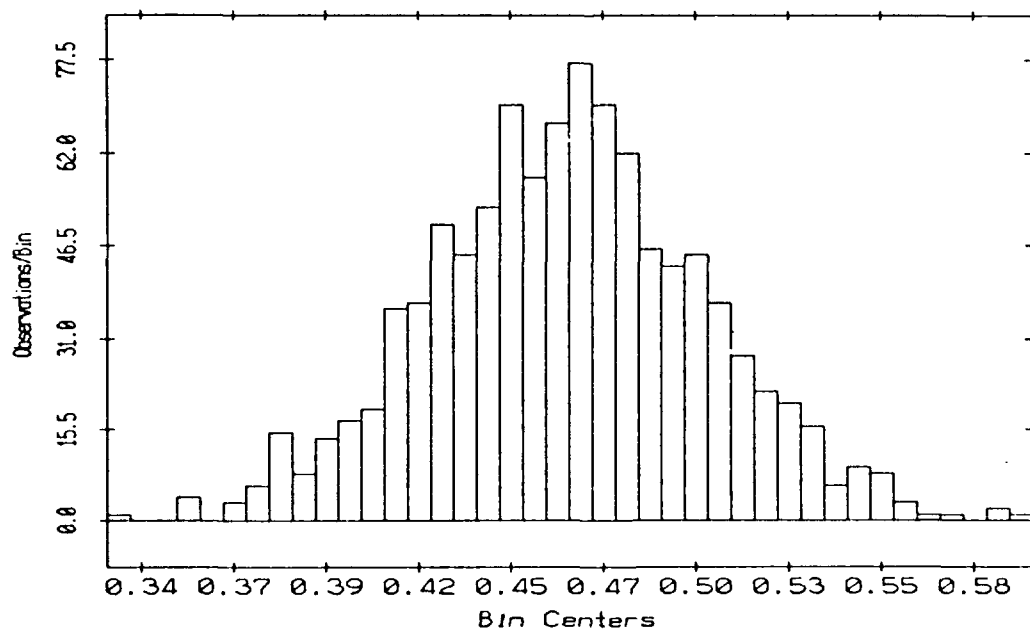


Fig. A.93: SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B5'

## Histogram of B7

Mean=0.17790, standard deviation=0.36951e-01

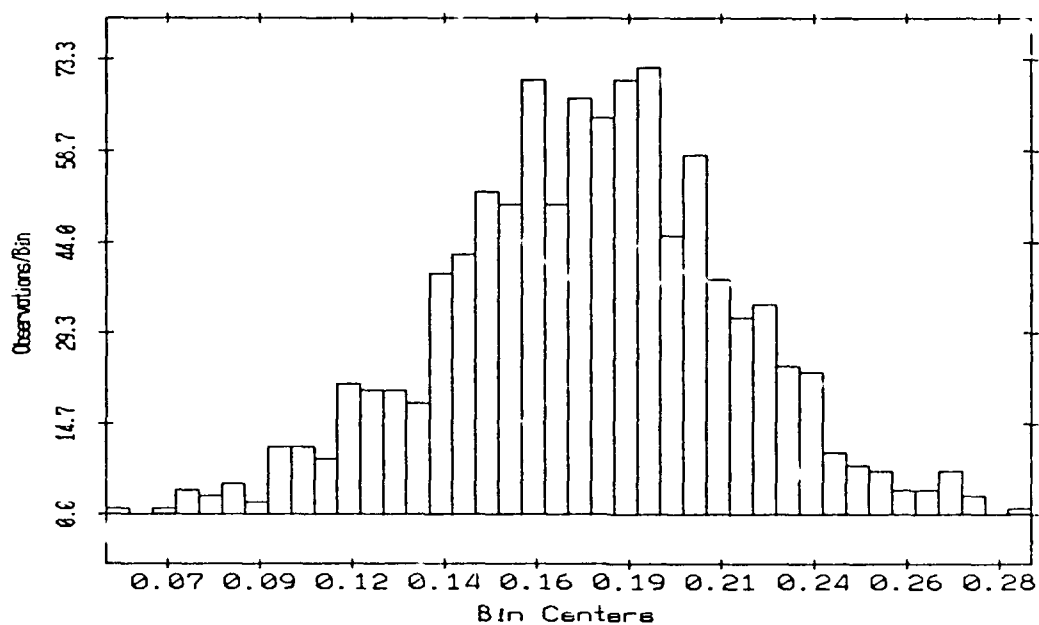


Fig. A.94 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B8'

## Histogram of B8

Mean=-0.35241, standard deviation=0.37902e-01

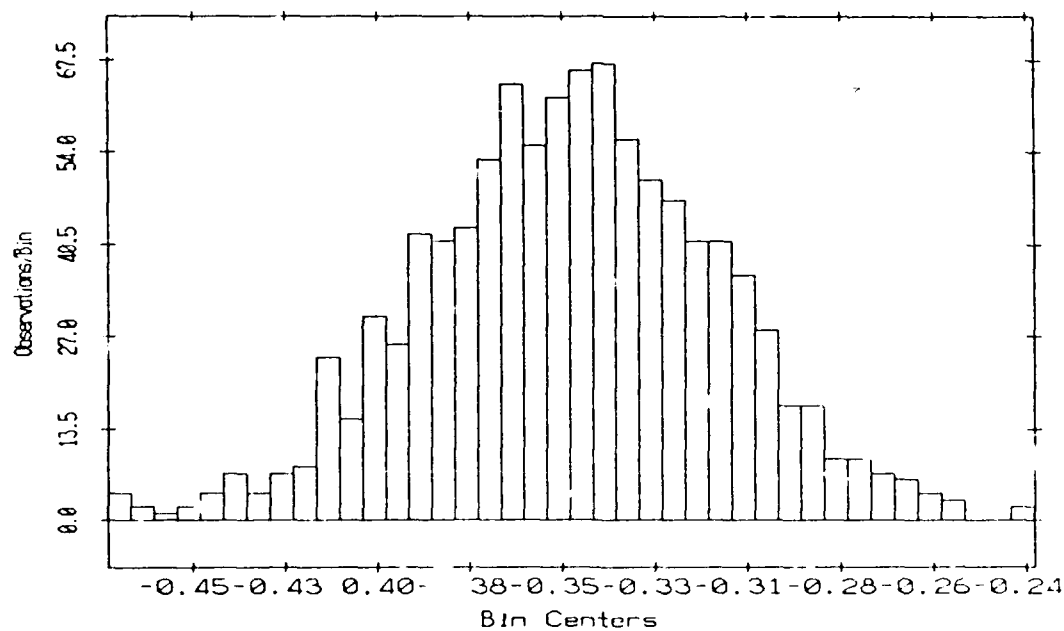


Fig. A.95 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B7'



## Histogram of B9

Mean=-0.17174, standard deviation=0.33043e-01

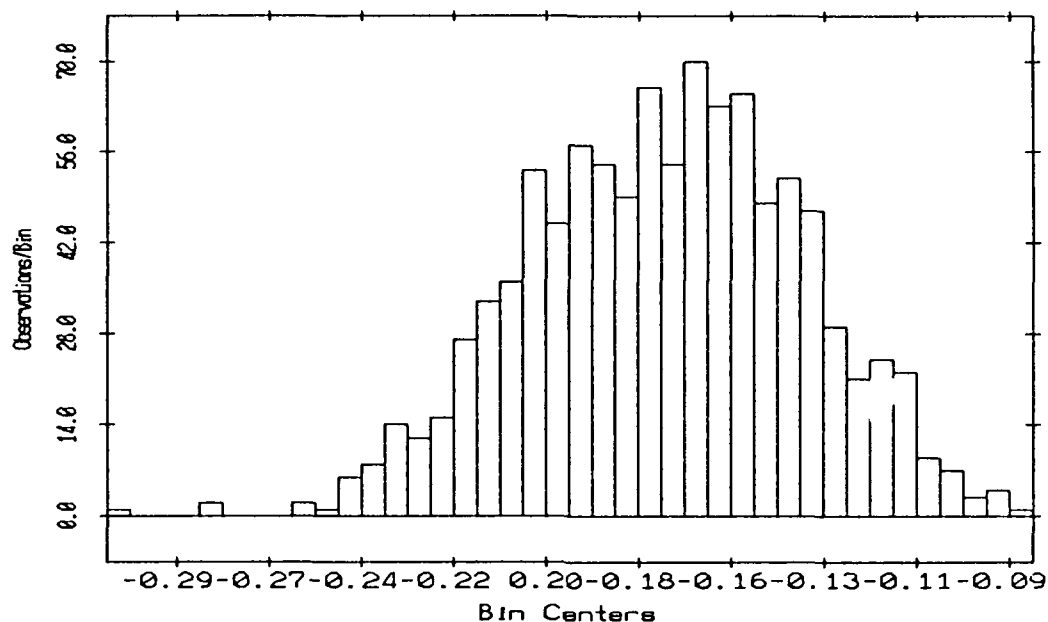


Fig. A.96 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B10'

## Histogram of B10

Mean=-0.83983e-01, standard deviation=0.31503e-01

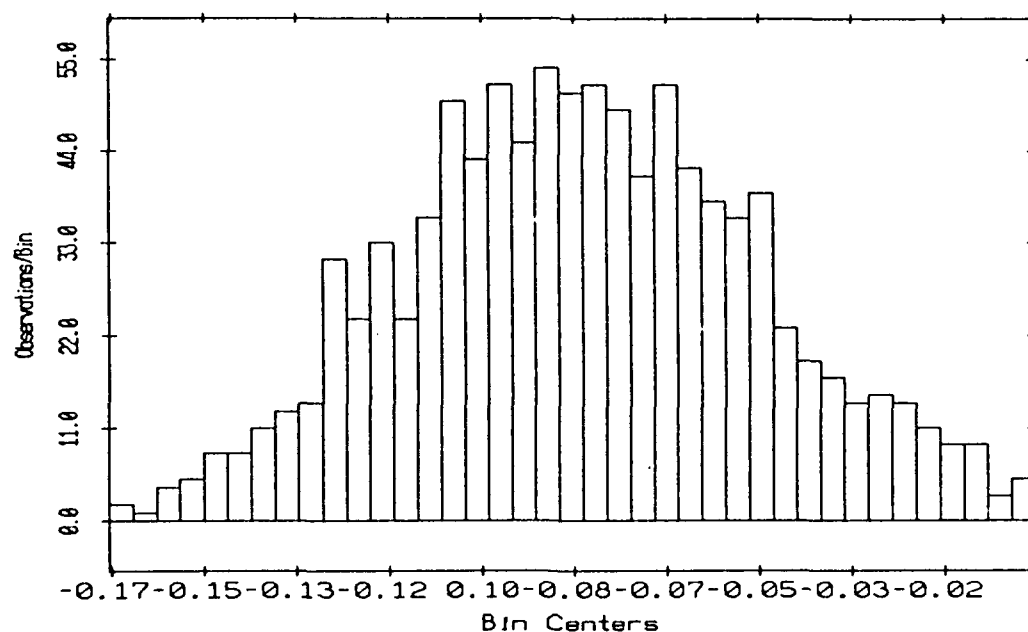


Fig. A.97 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B9'

## Histogram of B11

Mean=-0.39225e-01, standard deviation=0.29499e-01

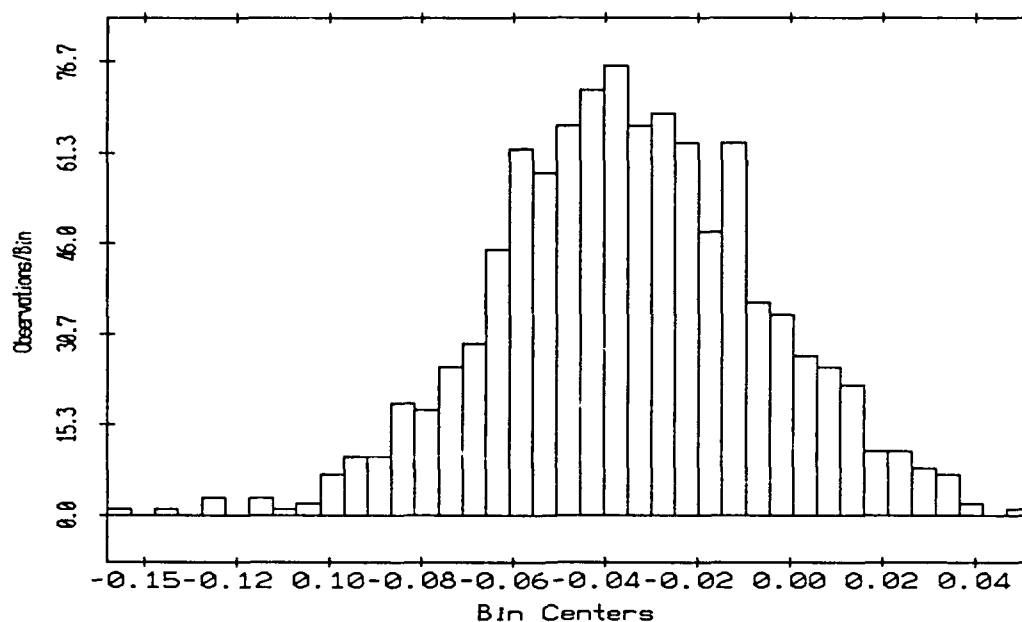


Fig. A.98 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B12'

## Histogram of B12

Mean=-0.14721e-01, standard deviation=0.28769e-01

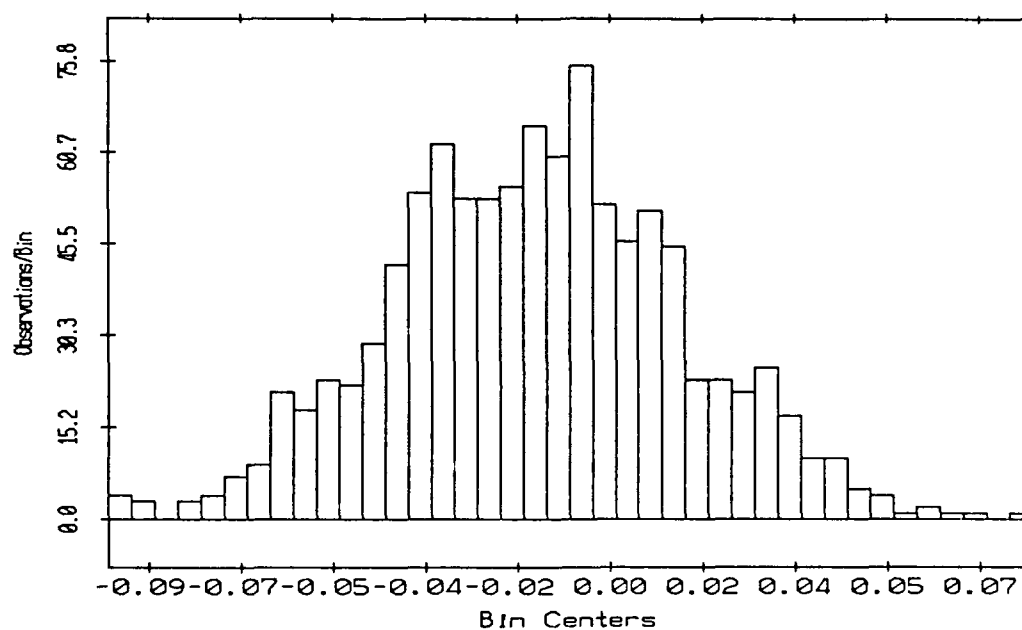


Fig. A.99 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B11'

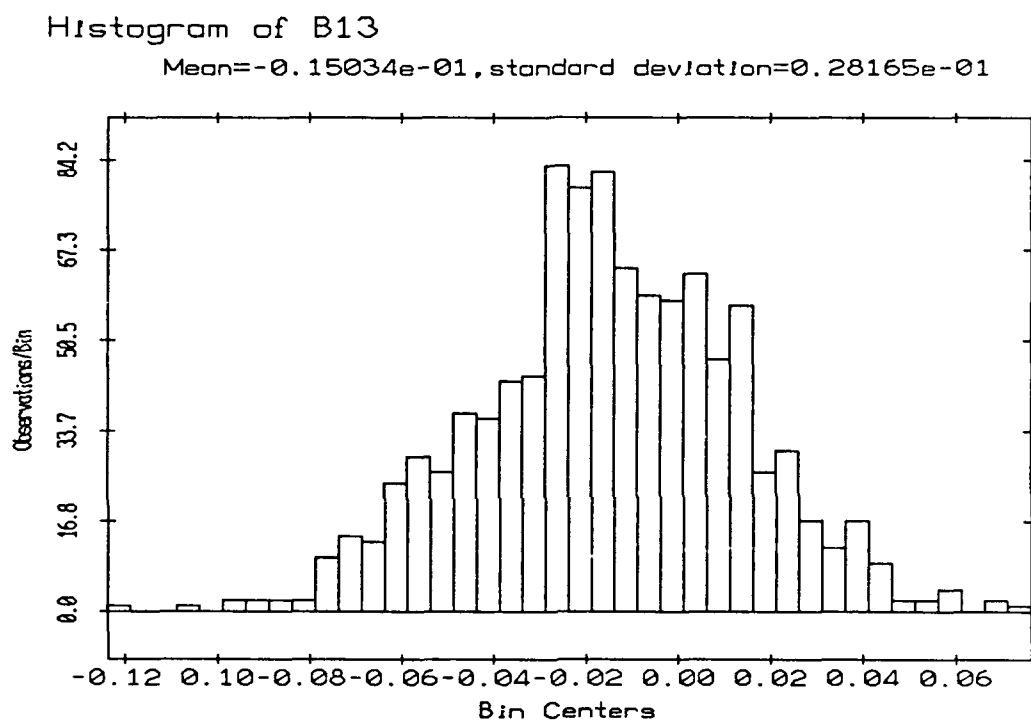


Fig. A.100 : SMC estimation, Nuttall's method, data length 512 points,  
Histogram of estimated model coefficient 'B13'

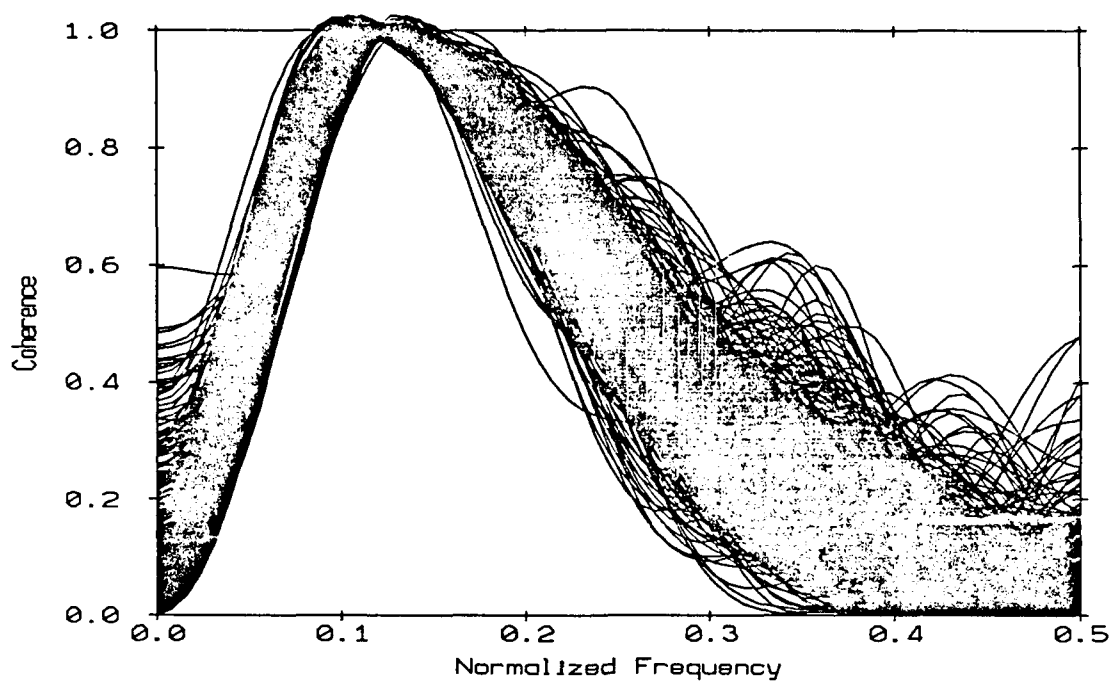


Fig. A.101 : SMC estimation, Nuttall's method, data length 256 points,

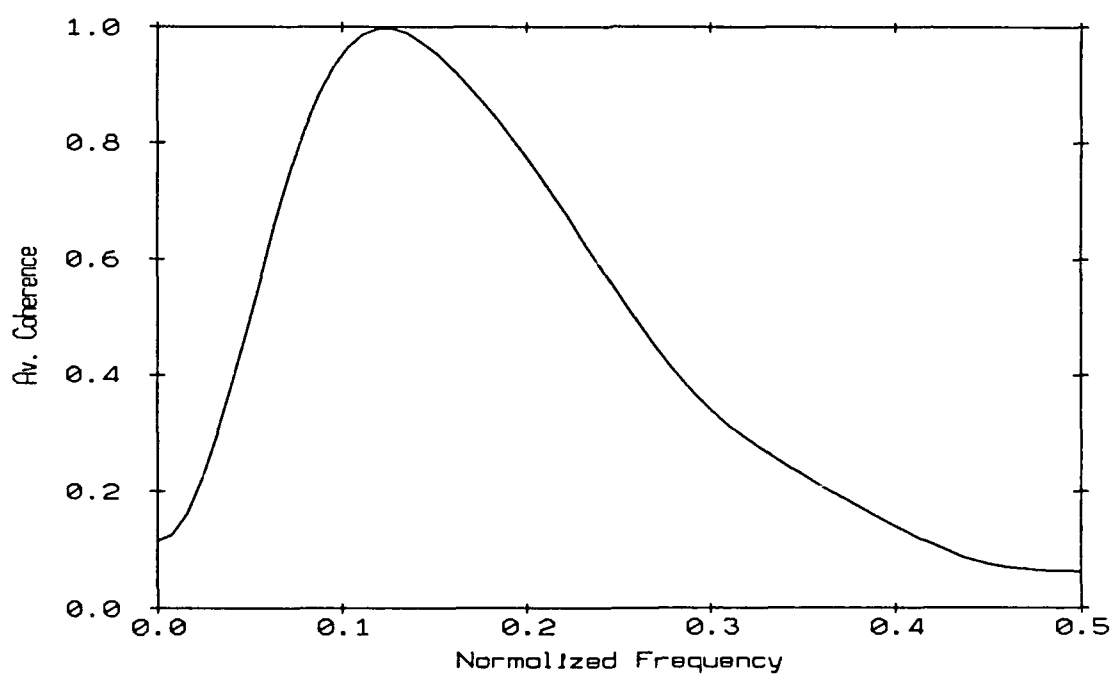


Fig. A.101.1 : SMC estimation, Nuttall's method, data length 256 points,  
Average estimated SMC over 1000 runs

SMC estimation, Nuttall's method, data length 256 points

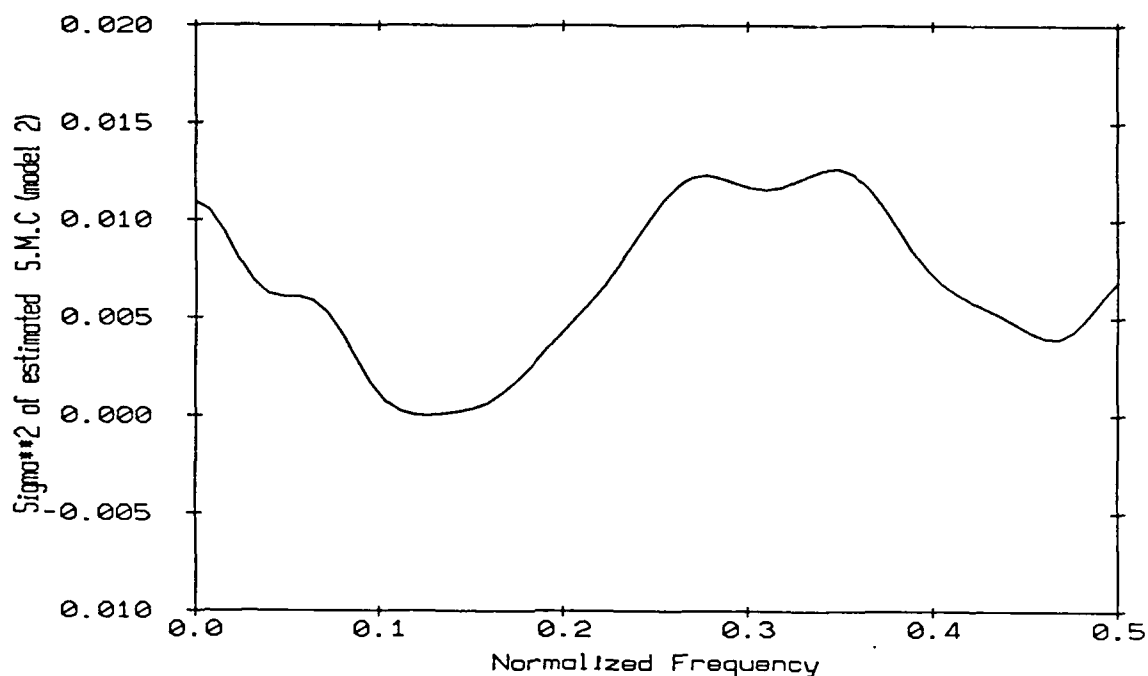


Fig. A.102 : Squared Standard Deviation, of estimated SMC, Nuttall's method  
data length 256 points, 1000 runs

SMC estimation, Nuttall's method, data length 256 points

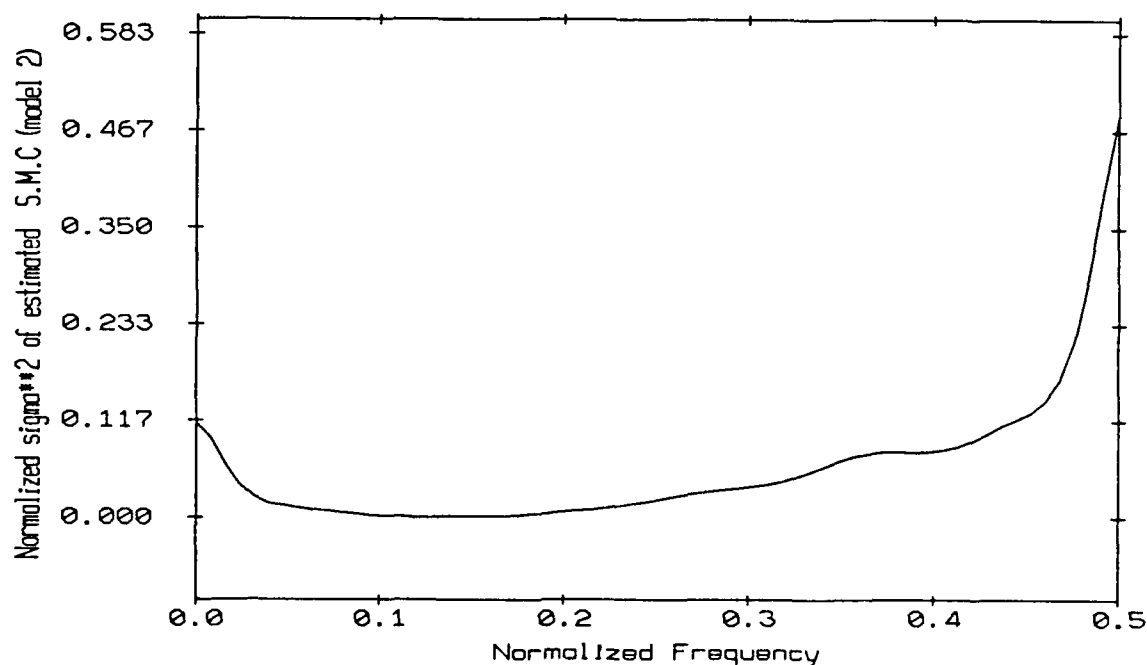


Fig. A.102.1: Normalized Squared Standard Deviation of estimated SMC, Nuttall's method  
data length 256 points, 1000 runs

## Histogram of Coherency at normalized freq. = 1/32

Mean=0.21940, Standard Deviation=0.80411e-02

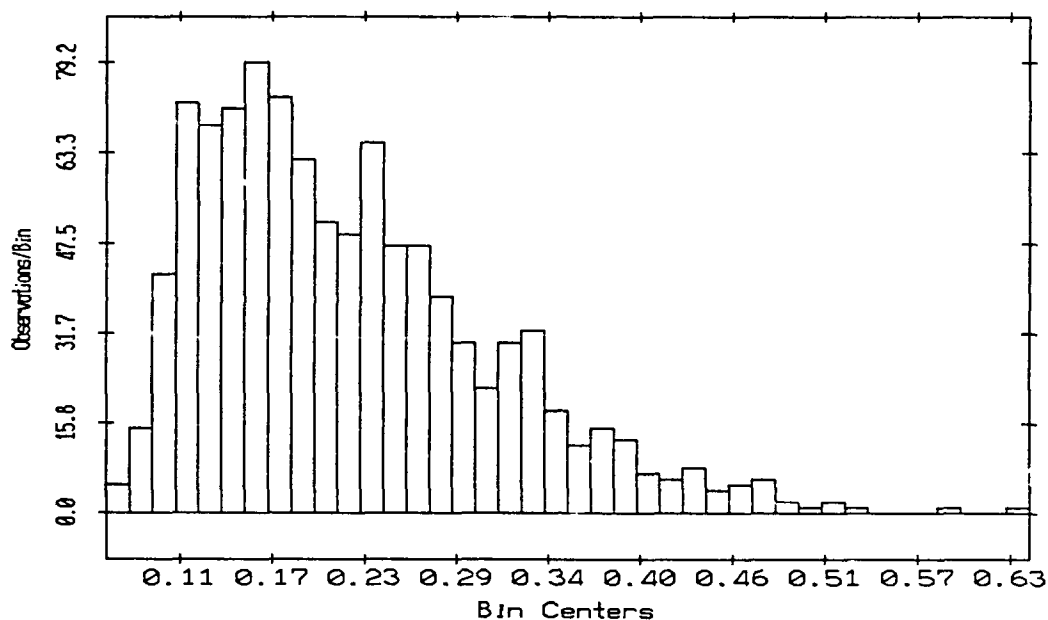


Fig. A.103 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated SMC at normalized frequency = 1/32

## Histogram of Coherency at normalized freq. = 2/32

Mean=0.56618, Standard Deviation=0.61044e-02

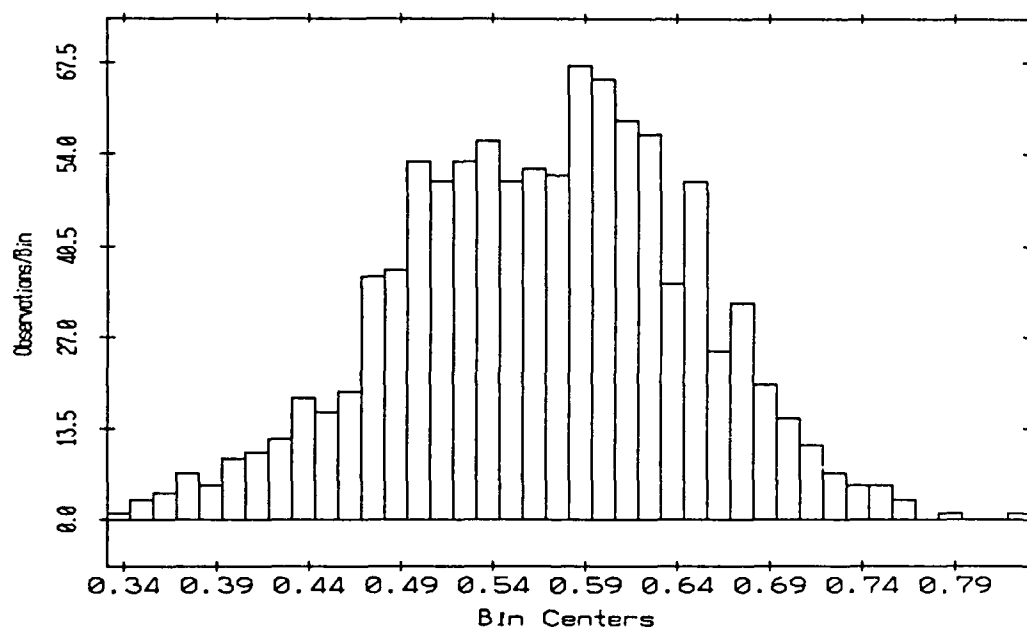


Fig. A.104 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated SMC at normalized frequency = 2/32

Histogram of Coherency at normalized freq. =  $3/32$   
 Mean=0.88267, Standard Deviation=0.29254e-02

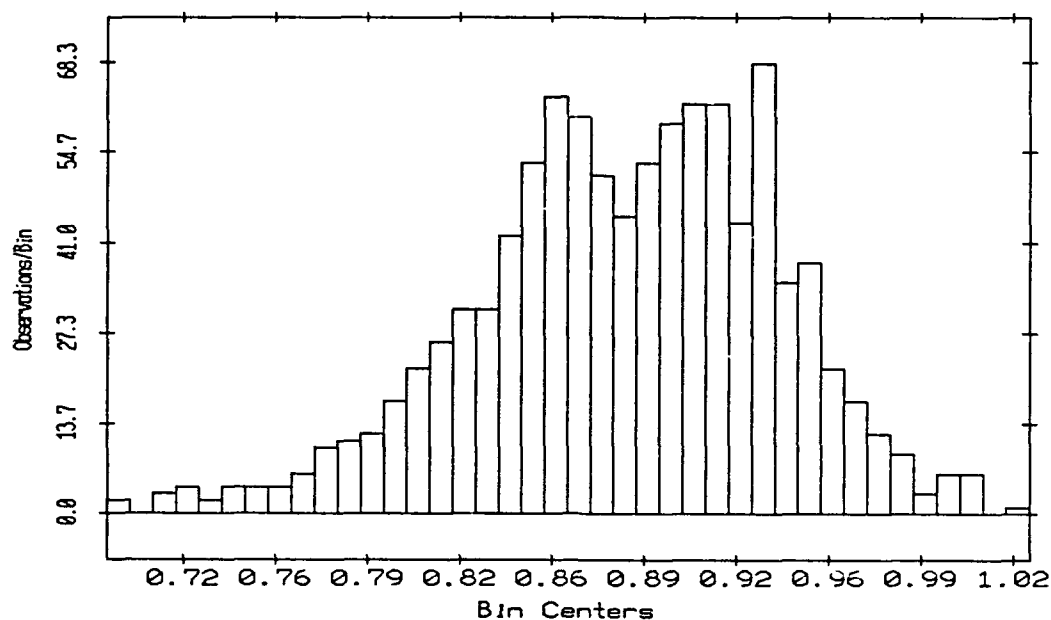


Fig. A.105 : SMC estimation, Nuttall's method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $3/32$

Histogram of Coherency at normalized freq. =  $4/32$   
 Mean=0.99781, Standard Deviation=0.39533e-04

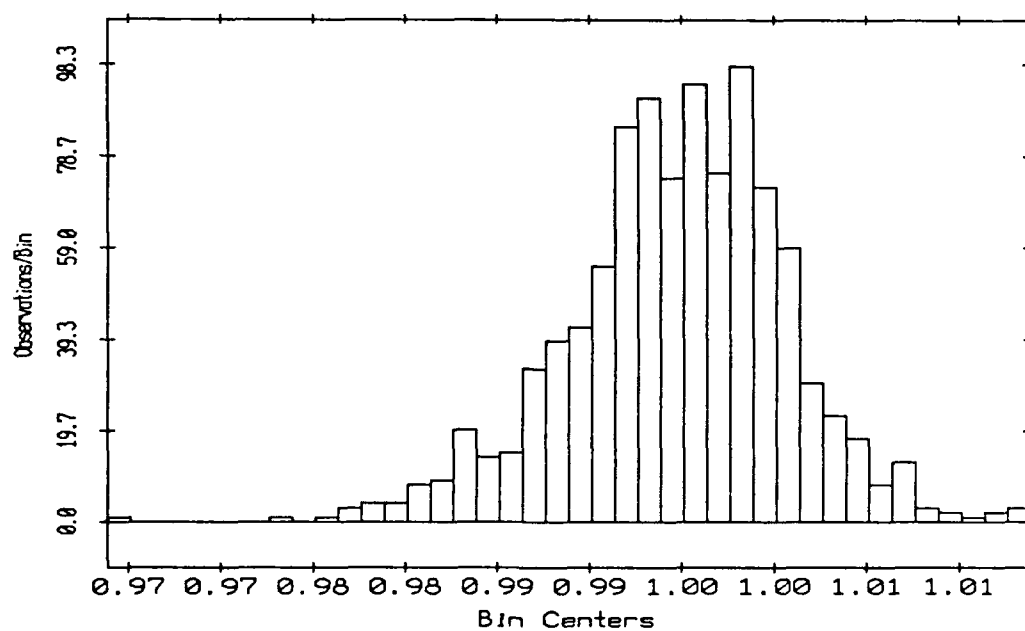


Fig. A.106 : SMC estimation, Nuttall's method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $4/32$

Histogram of Coherency at normalized freq. =  $5/32$   
 Mean=0.95476, Standard Deviation=0.37391e-03

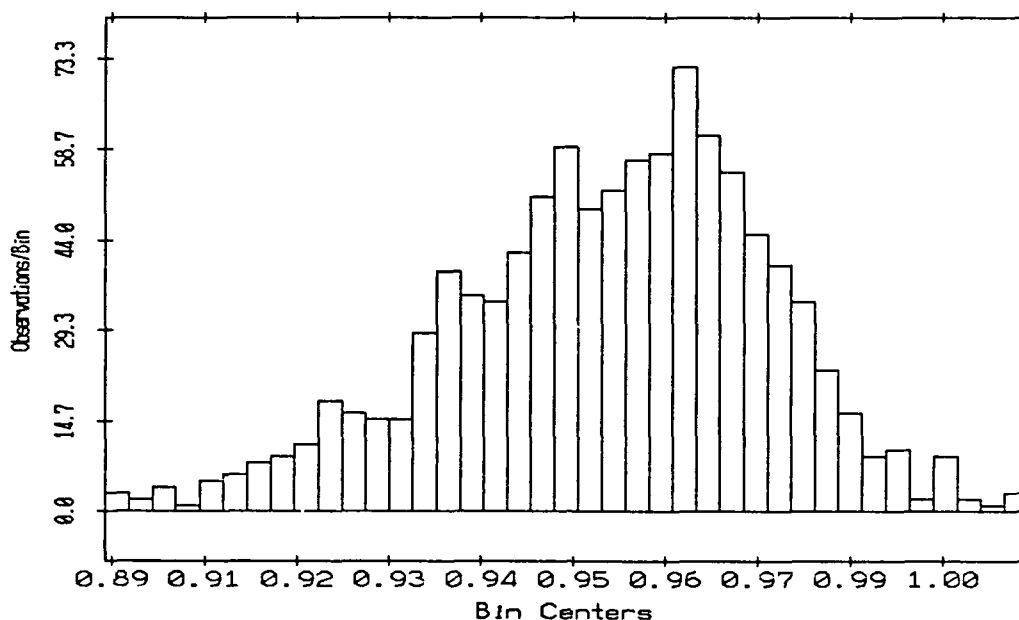


Fig. A.107 : SMC estimation, Nuttall's method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $5/32$

Histogram of Coherency at normalized freq. =  $6/32$   
 Mean=0.84568, Standard Deviation=0.25183e-02

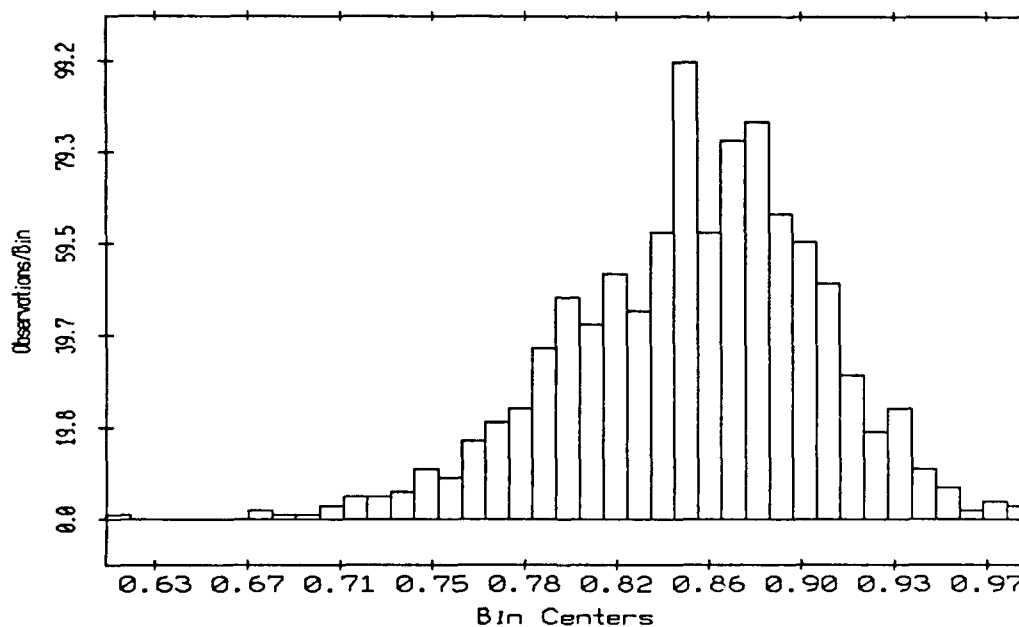


Fig. A.108 : SMC estimation, Nuttall's method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $6/32$



Histogram of Coherency at normalized freq. =  $7/32$   
 Mean= $0.70948$ , Standard Deviation= $0.58033e-02$

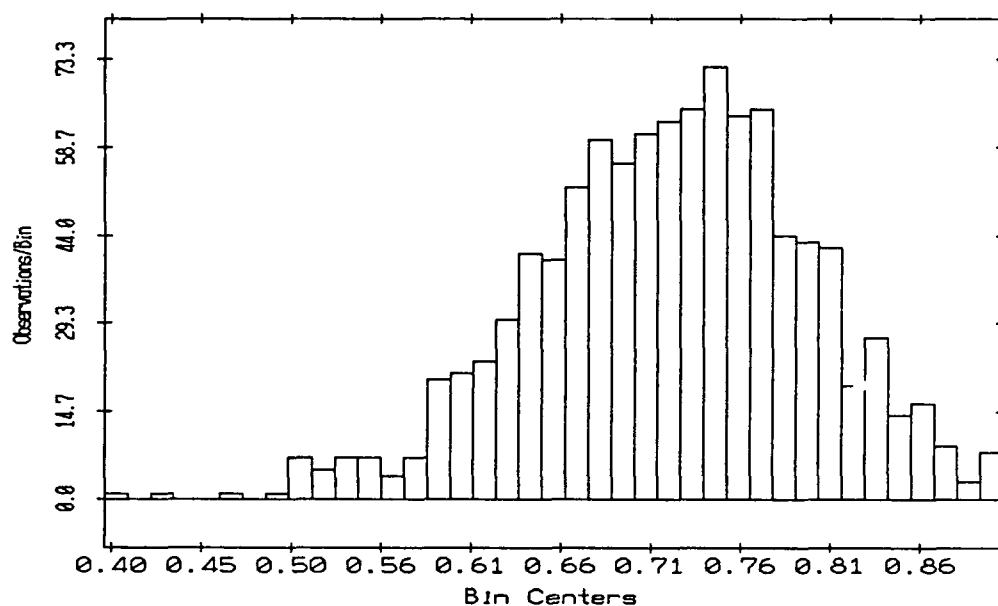


Fig. A.109 : SMC estimation, Nuttall's method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $7/32$

Histogram of Coherency at normalized freq. =  $8/32$   
 Mean= $0.55781$ , Standard Deviation= $0.99694e-02$

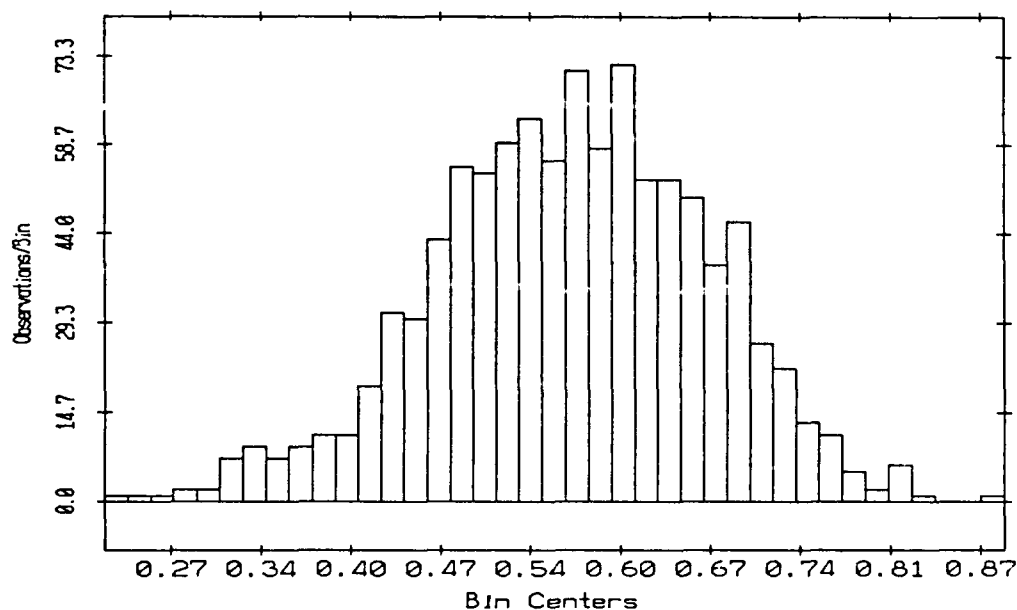


Fig. A.110 : SMC estimation, Nuttall's method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency =  $8/32$

Histogram of Coherency at normalized freq. = 9/32  
 Mean=0.41450, Standard Deviation=0.12344e-01

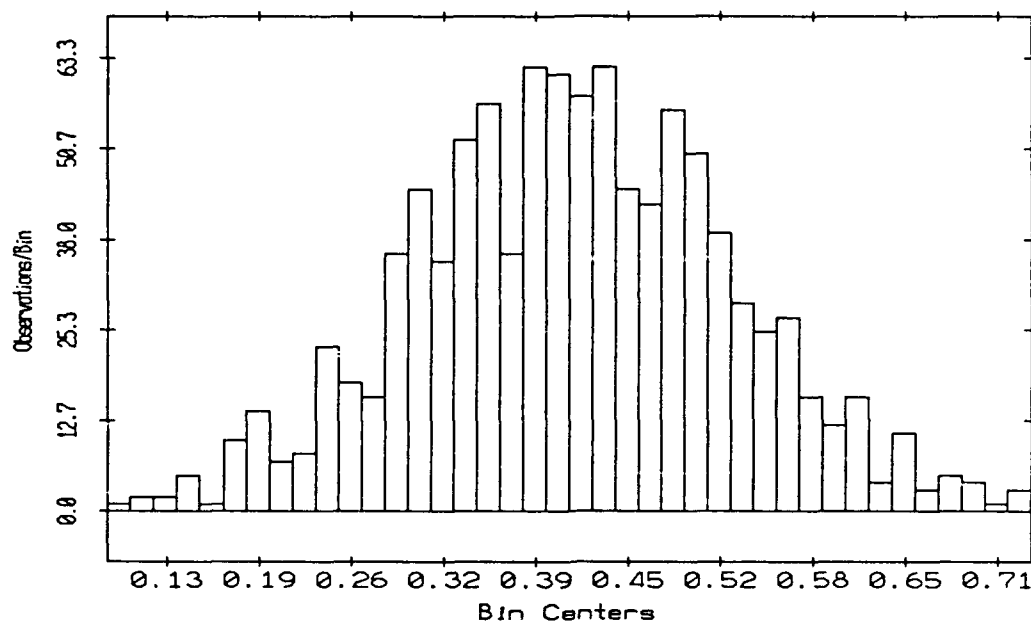


Fig. A.111 : SMC estimation, Nuttall's method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency = 9/32

Histogram of Coherency at normalized freq. = 10/32  
 Mean=0.31220, Standard Deviation=0.11597e-01

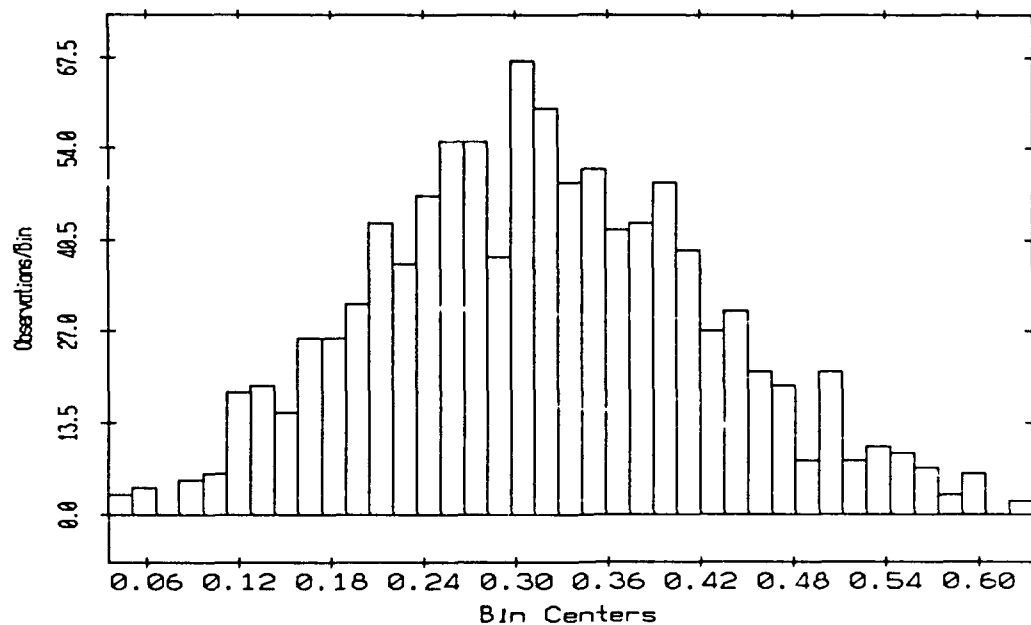


Fig. A.112 : SMC estimation, Nuttall's method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency = 10/32

## Histogram of Coherency at normalized freq. = 11/32

Mean=0.24358, Standard Deviation=0.12626e-01

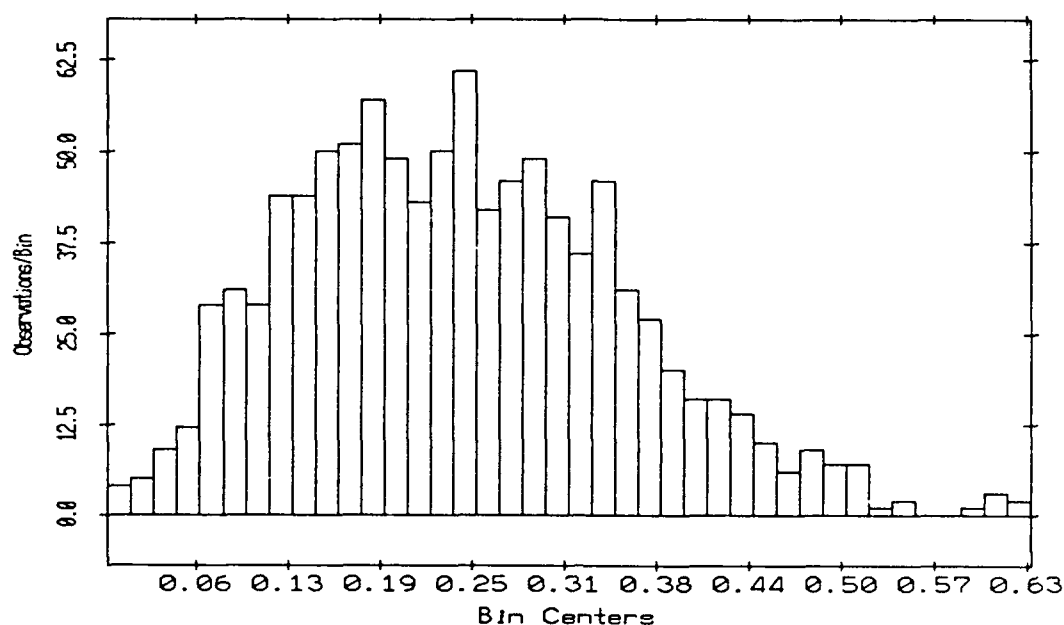


Fig. A.113 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated SMC at normalized frequency = 11/32

## Histogram of Coherency at normalized freq. = 12/32

Mean=0.18550, Standard Deviation=0.10737e-01

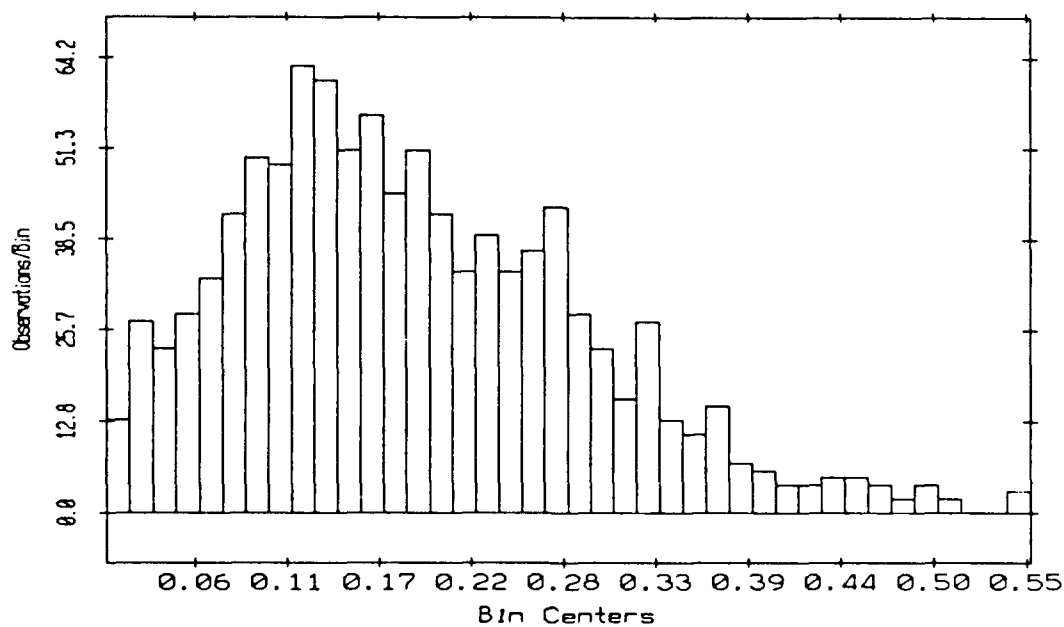


Fig. A.114 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated SMC at normalized frequency = 12/32

Histogram of Coherency at normalized freq. = 13/32  
 Mean=0.13277, Standard Deviation=0.67788e-02

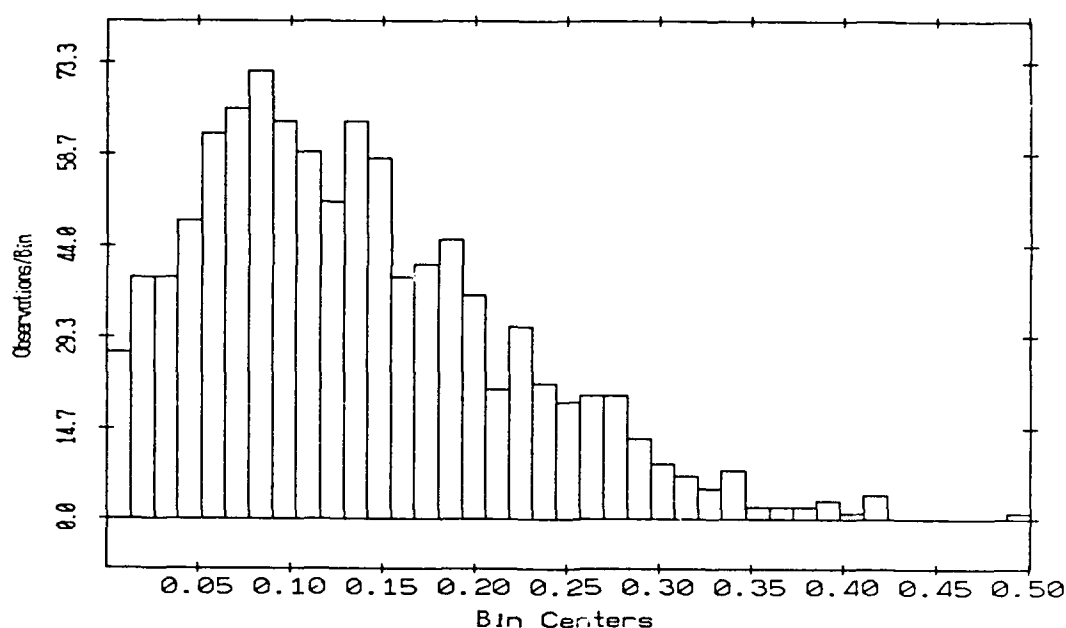


Fig. A.115 : SMC estimation, Nuttall's method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency = 13/32

Histogram of Coherency at normalized freq. = 14/32  
 Mean=0.87783e-01, Standard Deviation=0.51150e-02

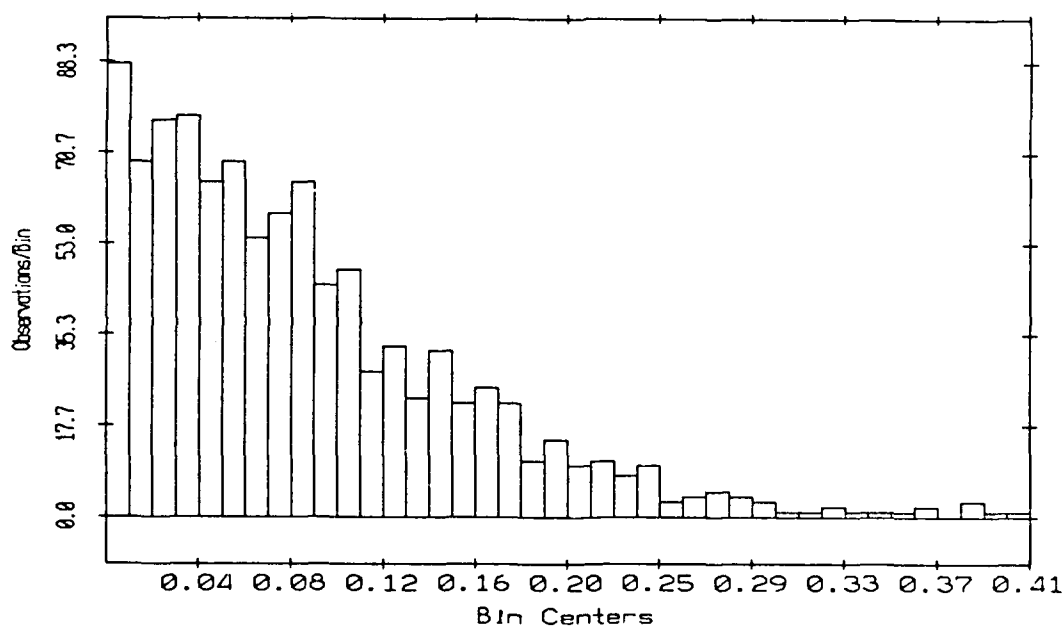


Fig. A.116 : SMC estimation, Nuttall's method, data length 256 points,  
 Histogram of estimated SMC at normalized frequency = 14/32

## Histogram of Coherency at normalized freq. = 15/32

Mean=0.67151e-01, Standard Deviation=0.39357e-02

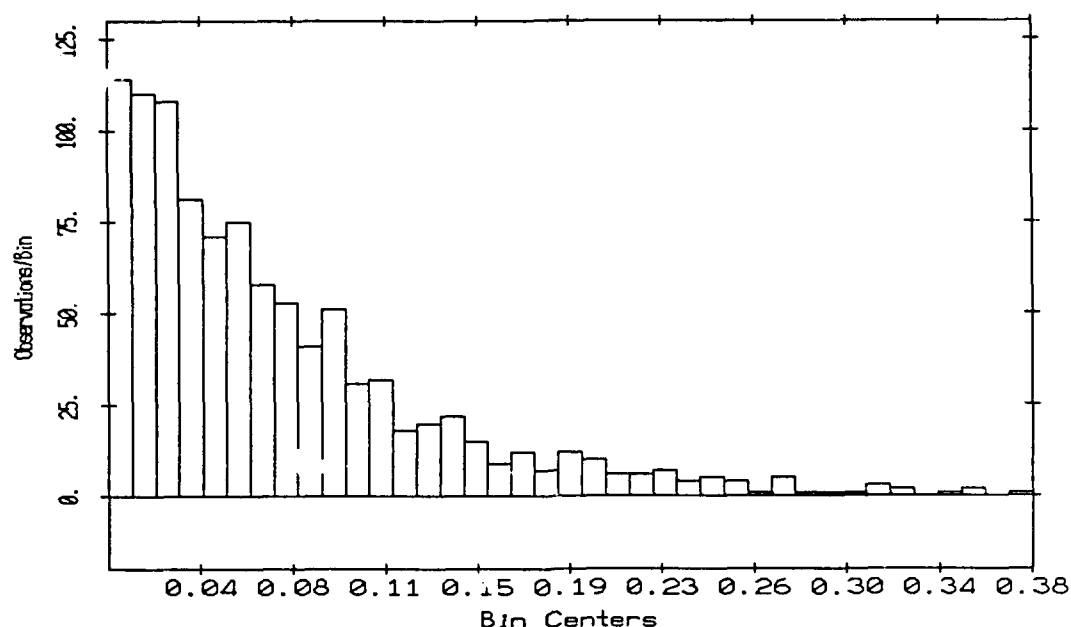


Fig. A.117 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated SMC at normalized frequency = 15/32

## Histogram of Coherency at normalized freq. = 16/32

Mean=0.63681e-01, Standard Deviation=0.68501e-02

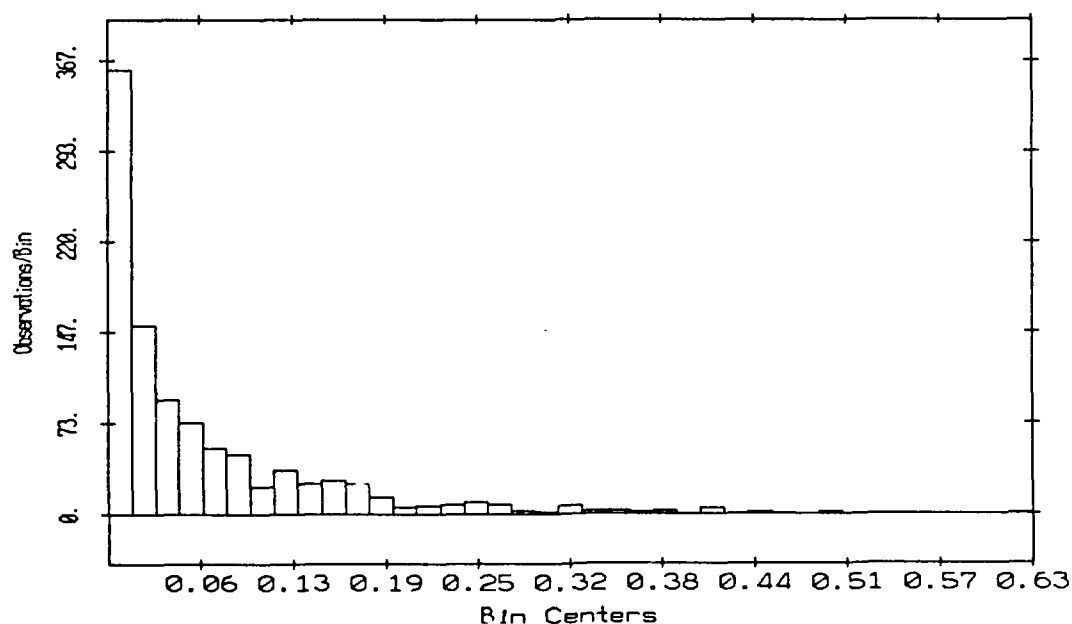


Fig. A.118 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated SMC at normalized frequency = 16/32

## Histogram of A1

Mean=-0.43680e-02, Standard Deviation=0.35356e-01

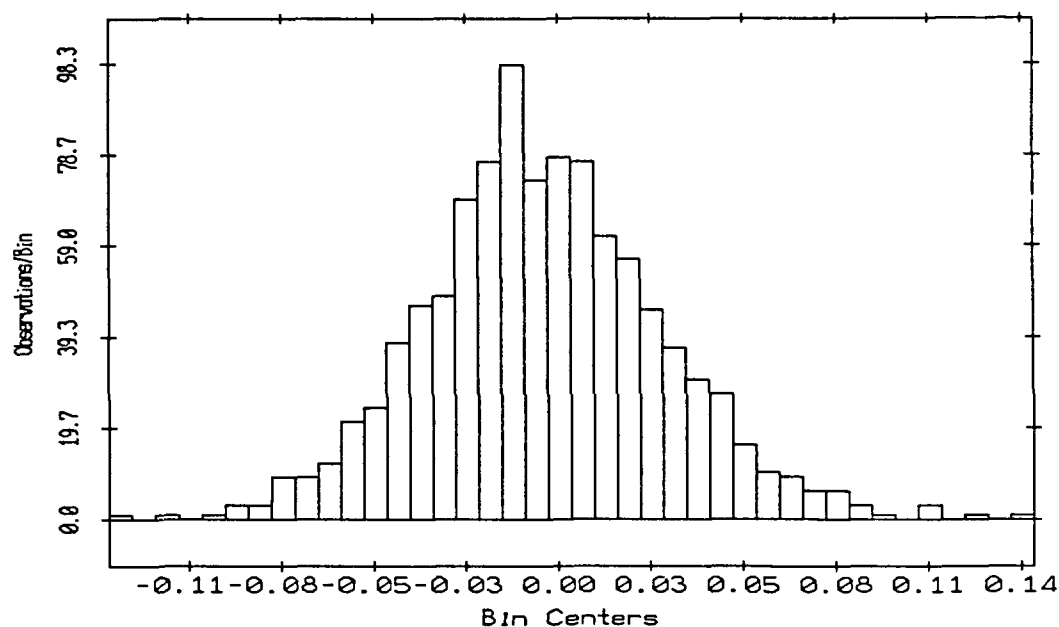


Fig. A.119 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A2'

## Histogram of A2

Mean=-0.35929e-02, Standard Deviation=0.44659e-01

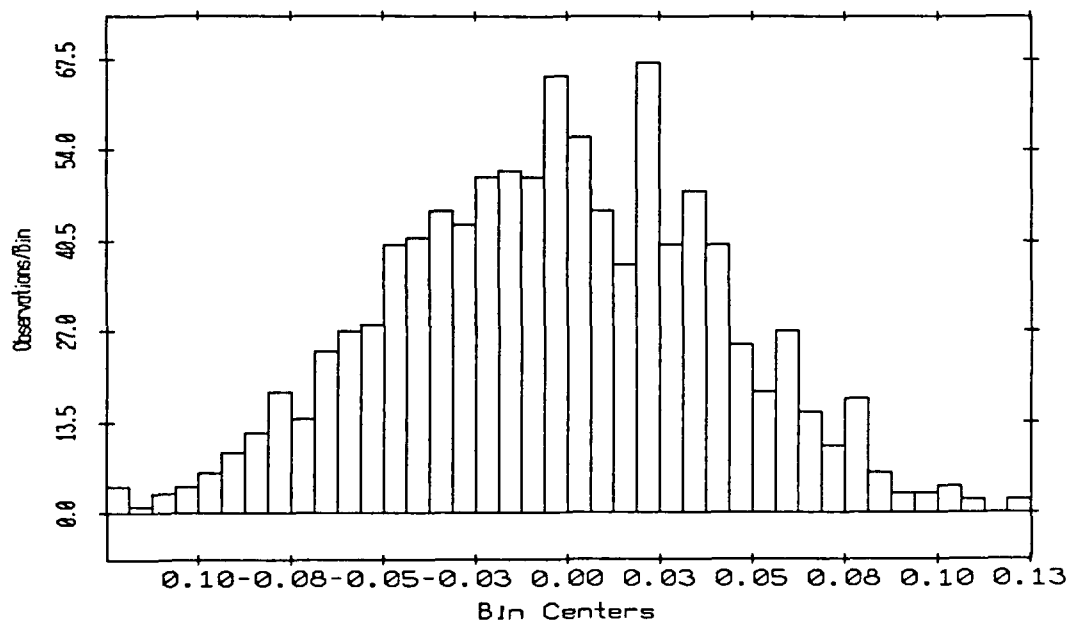


Fig. A.120 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A1'

## Histogram of A3

Mean=-0.15421e-01, Standard Deviation=0.45672e-01

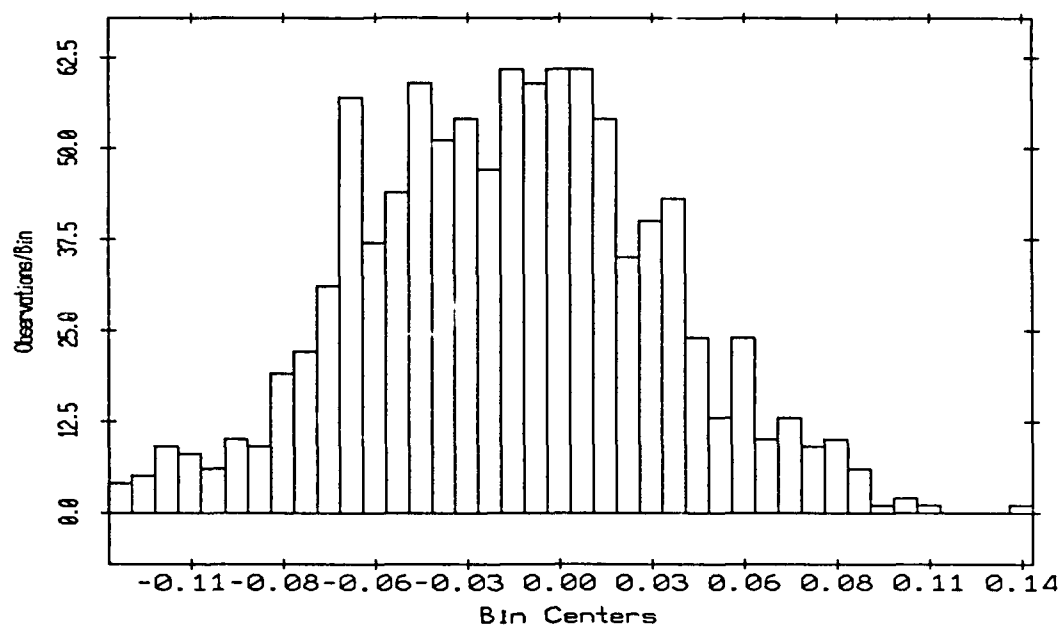


Fig. A.121 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A4'

## Histogram of A4

Mean=-0.45589e-01, Standard Deviation=0.46792e-01

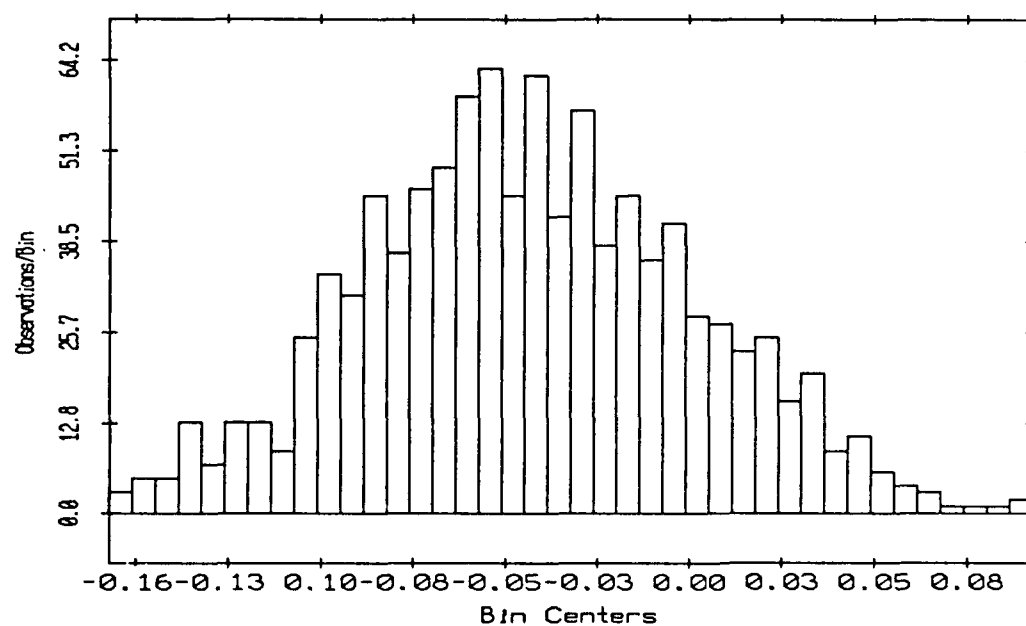


Fig. A.122 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A3'

## Histogram of A5

Mean=-0.13075, Standard Deviation=0.48664e-01

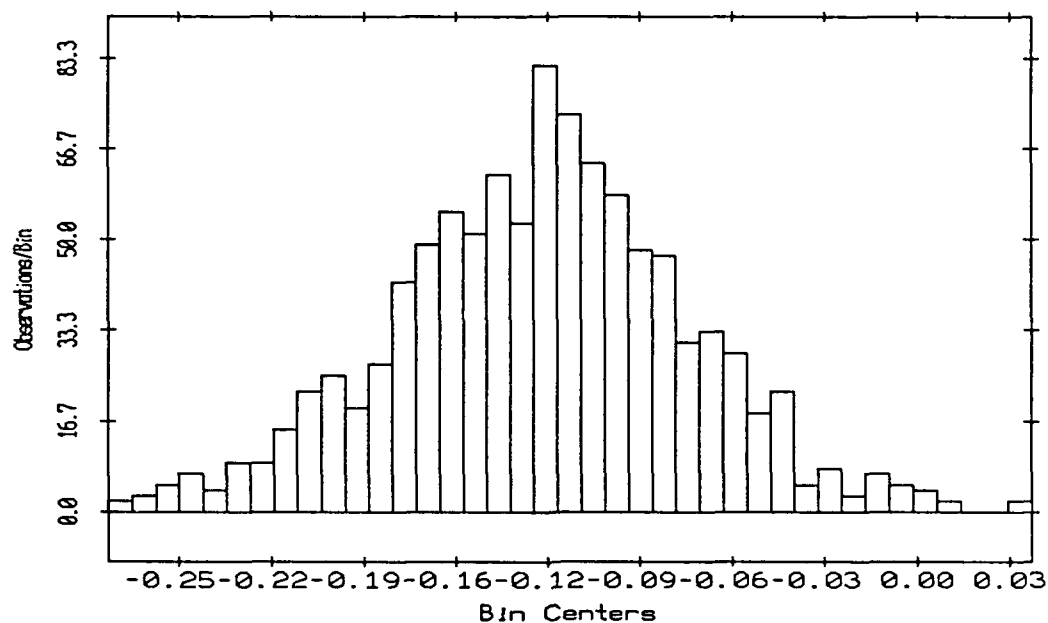


Fig. A.123 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A6'

## Histogram of A6

Mean=-0.38799, Standard Deviation=0.59865e-01

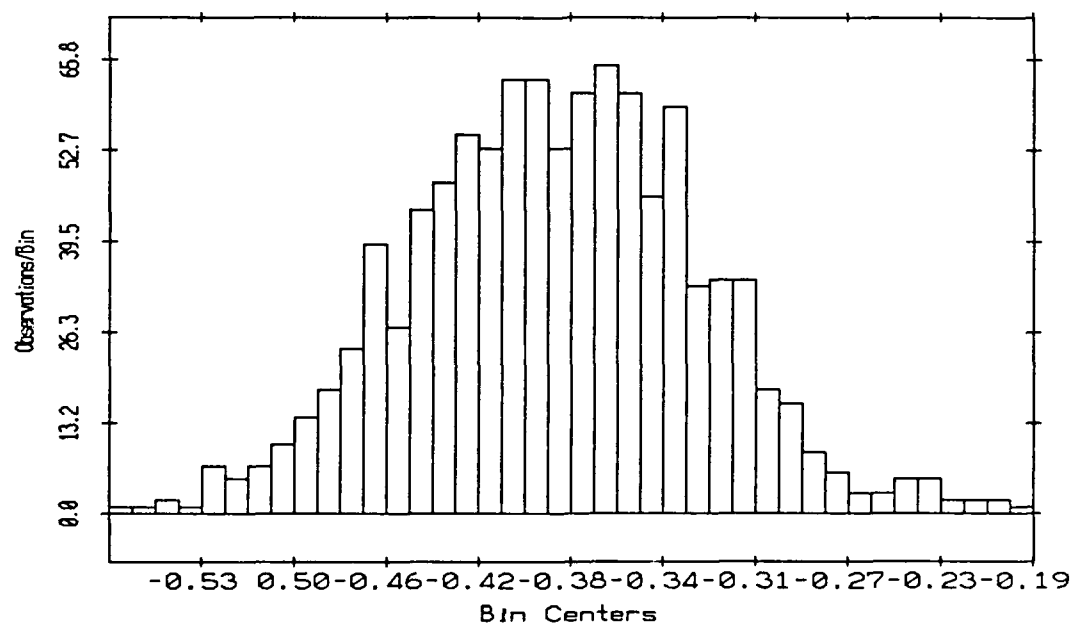


Fig. A.124 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A5'



## Histogram of A7

Mean=0.16593, Standard Deviation=0.60747e-01

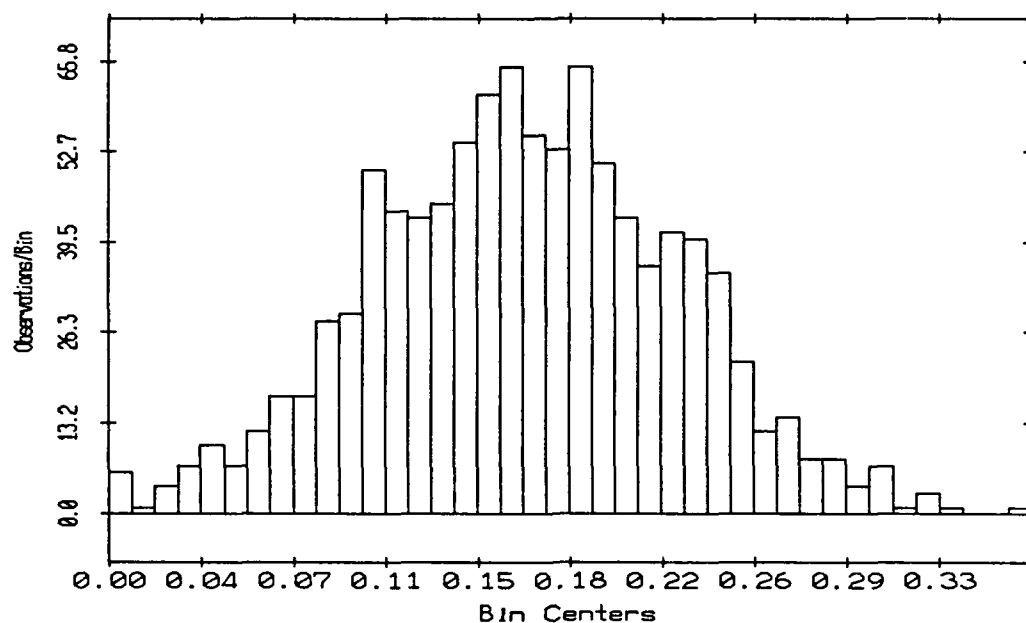


Fig. A.125 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A8'

## Histogram of A8

Mean=0.43224, Standard Deviation=0.59424e-01

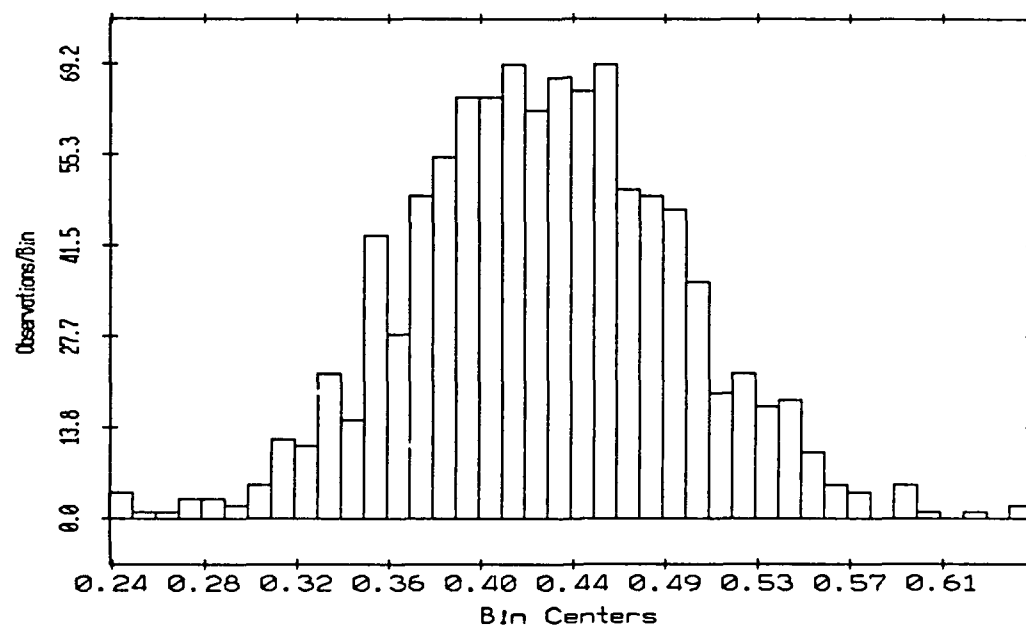


Fig. A.126 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A7'

## Histogram of A9

Mean=0.14539, Standard Deviation=0.50574e-01

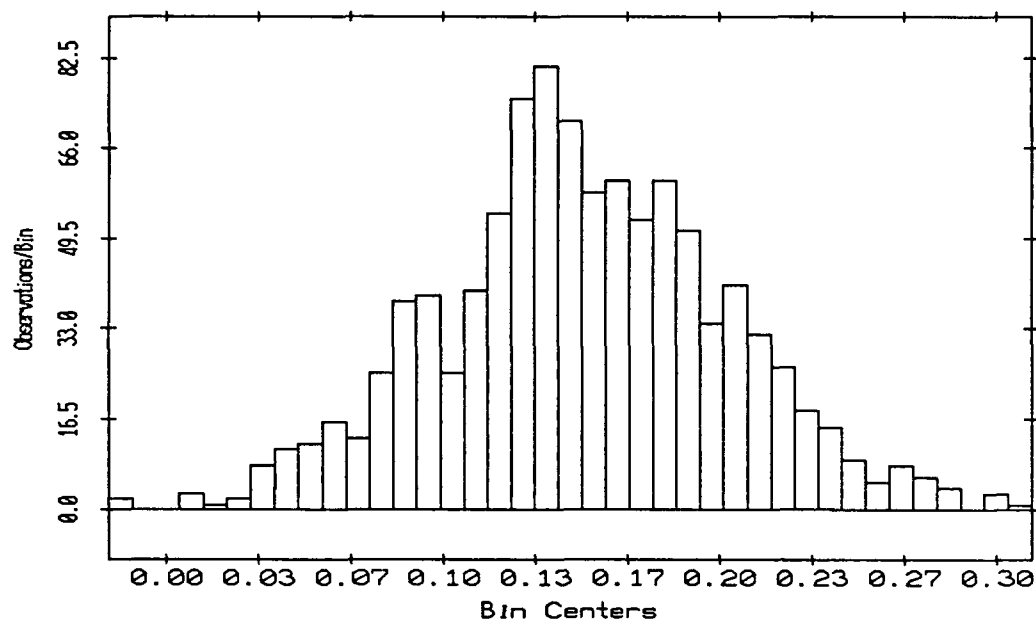


Fig. A.127 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A10'

## Histogram of A10

Mean=0.49067e-01, Standard Deviation=0.48060e-01

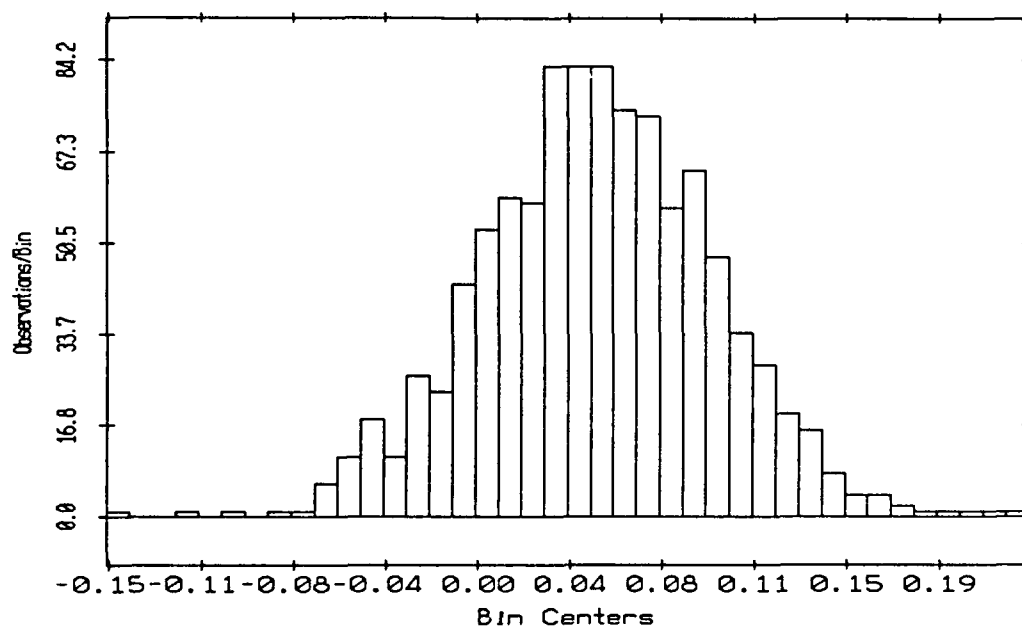


Fig. A.128 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A9'

## Histogram of A11

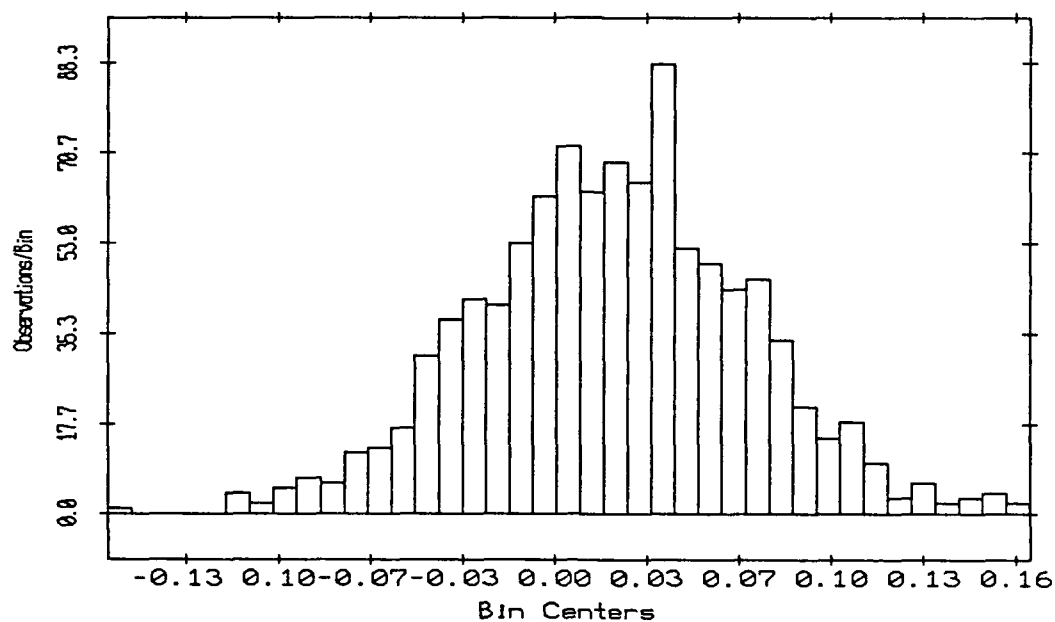
Mean= $0.16069e-01$ , Standard Deviation= $0.47125e-01$ 

Fig. A.129 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A12'

## Histogram of A12

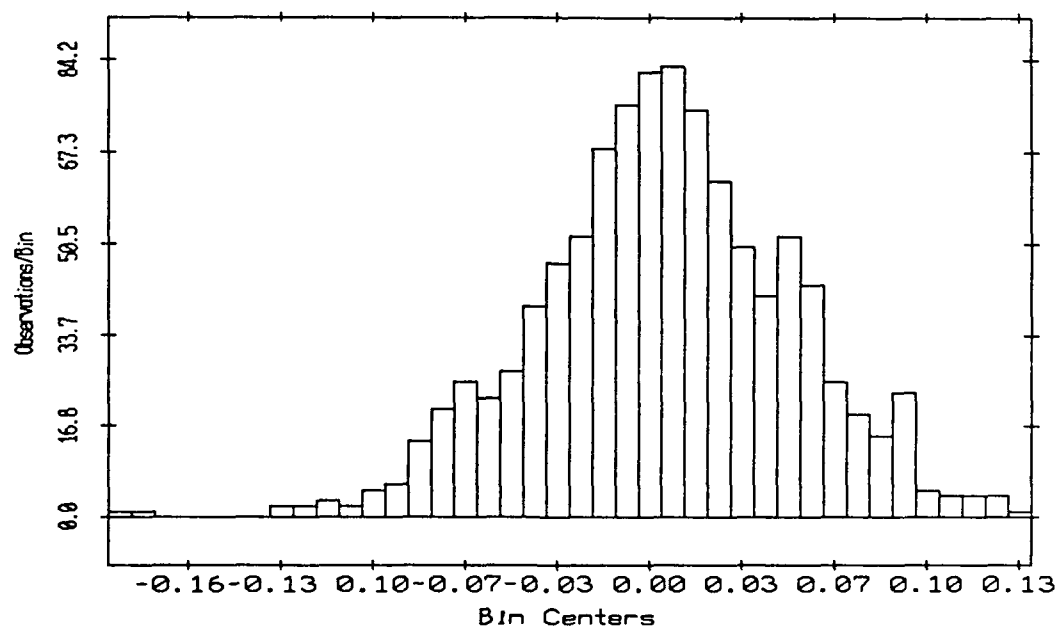
Mean= $0.50536e-02$ , Standard Deviation= $0.44996e-01$ 

Fig. A.130 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A11'

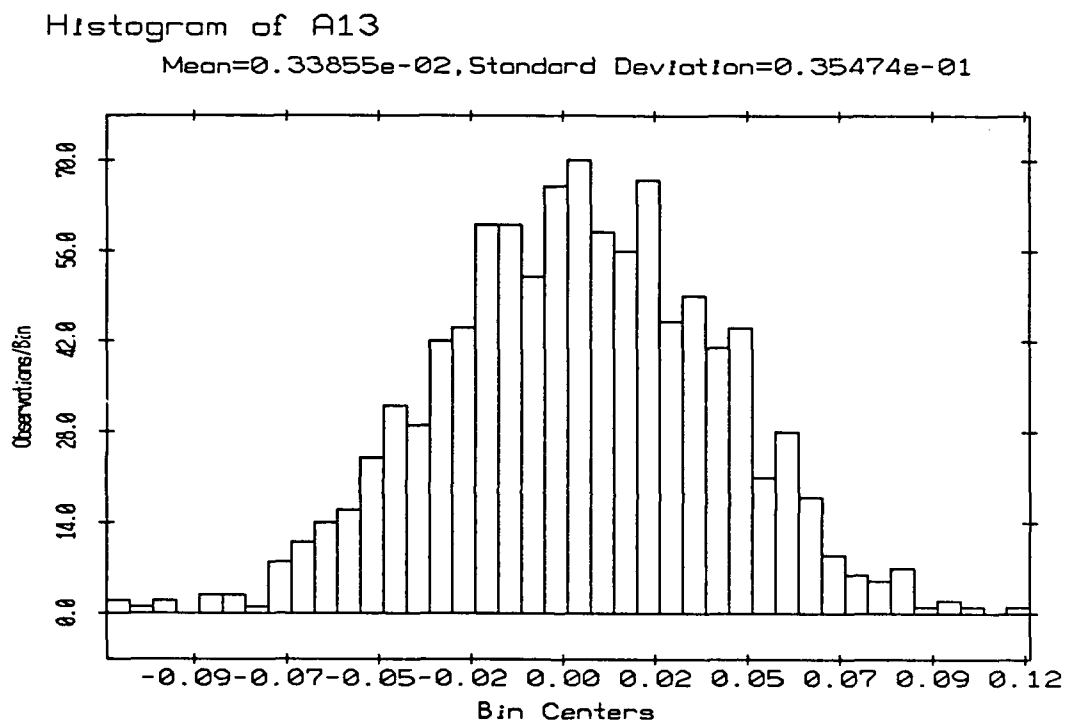


Fig. A.131 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'A13'

## Histogram of B1

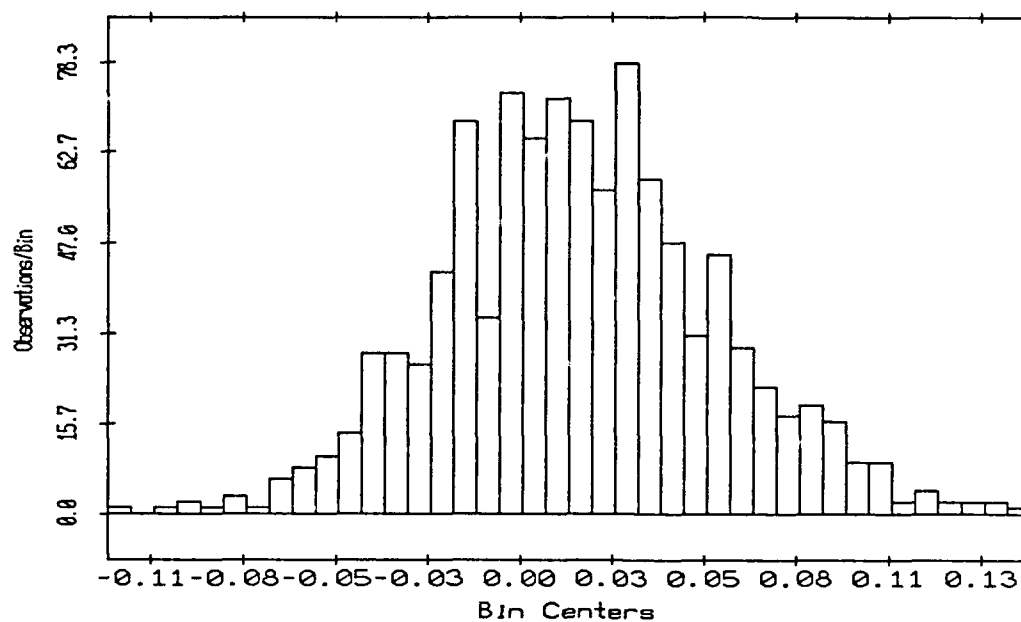
Mean= $0.17169 \times 10^{-1}$ , standard deviation= $0.39444 \times 10^{-1}$ 

Fig. A.132 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B2'

## Histogram of B2

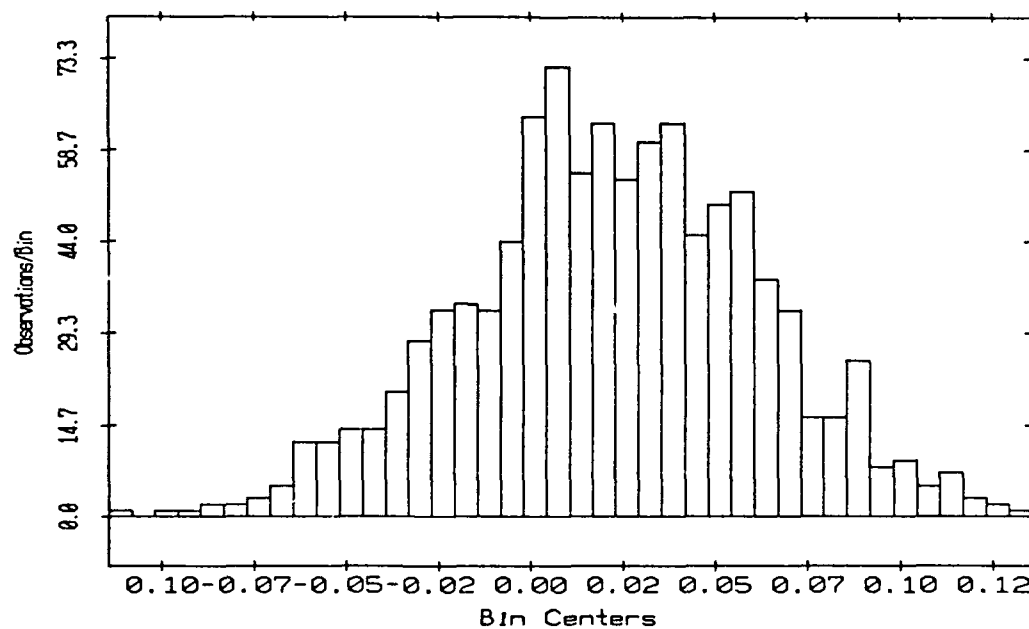
Mean= $0.21526 \times 10^{-1}$ , standard deviation= $0.39560 \times 10^{-1}$ 

Fig. A.133 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B1'

## Histogram of B3

Mean=0.48461e-01, standard deviation=0.41551e-01

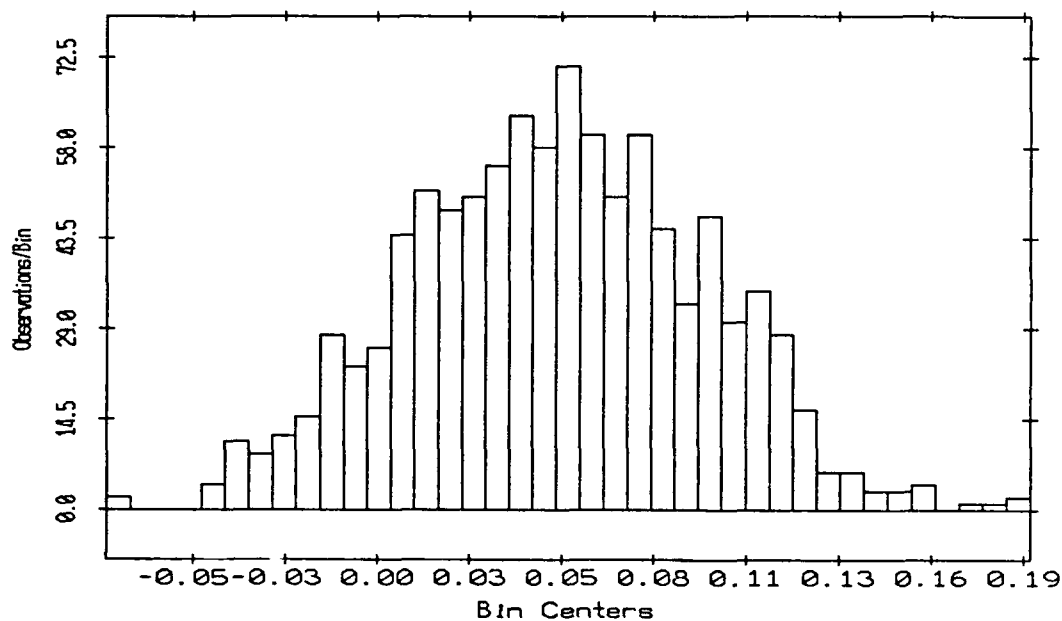


Fig. A.134 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B4'

## Histogram of B4

Mean=0.10704, standard deviation=0.44986e-01

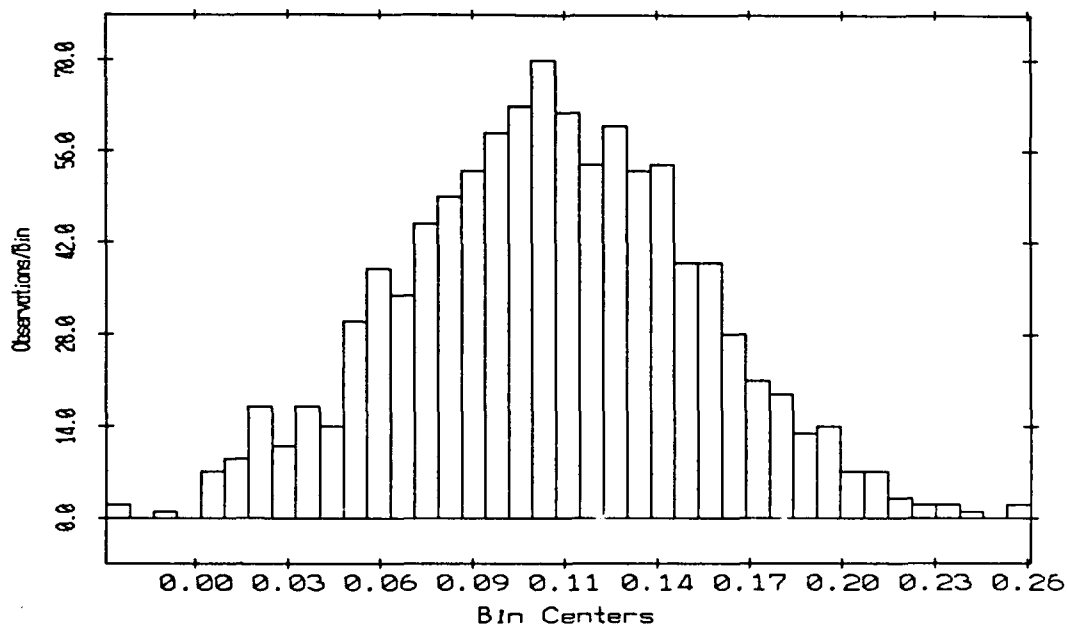


Fig. A.135 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B3'

## Histogram of B5

Mean=0.22284, standard deviation=0.46659e-01

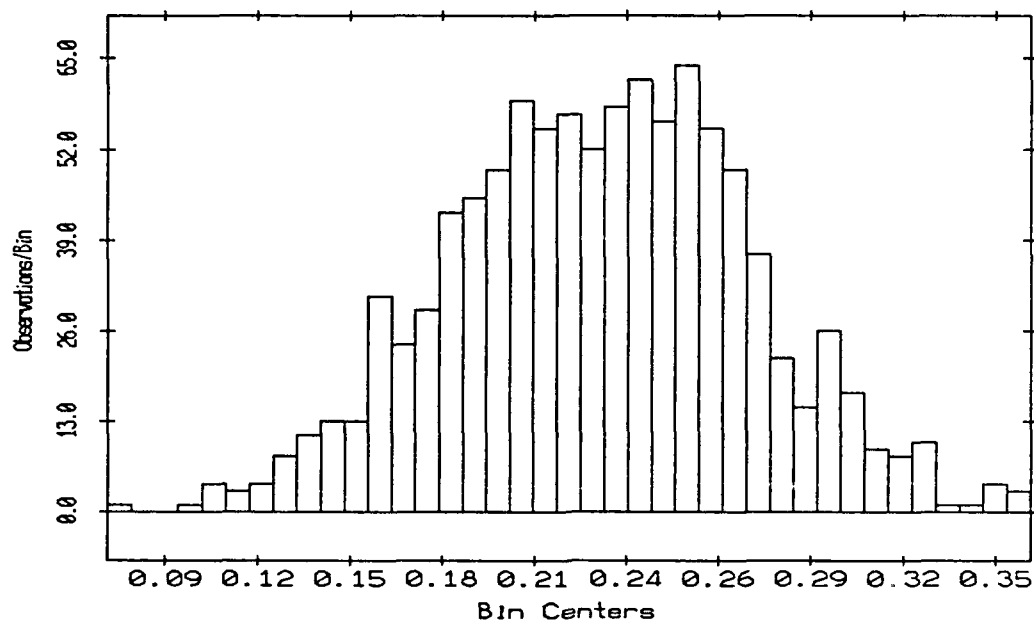


Fig. A.136 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B6'

## Histogram of B6

Mean=0.46243, standard deviation=0.56436e-01

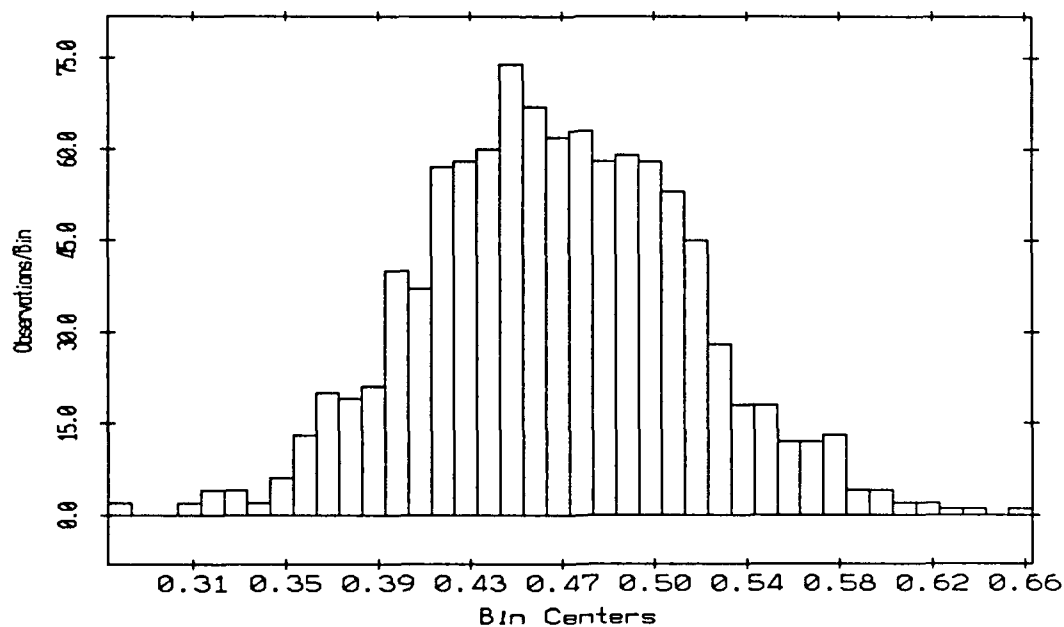


Fig. A.137 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B5'

## Histogram of B7

Mean=0.17843, standard deviation=0.53715e-01

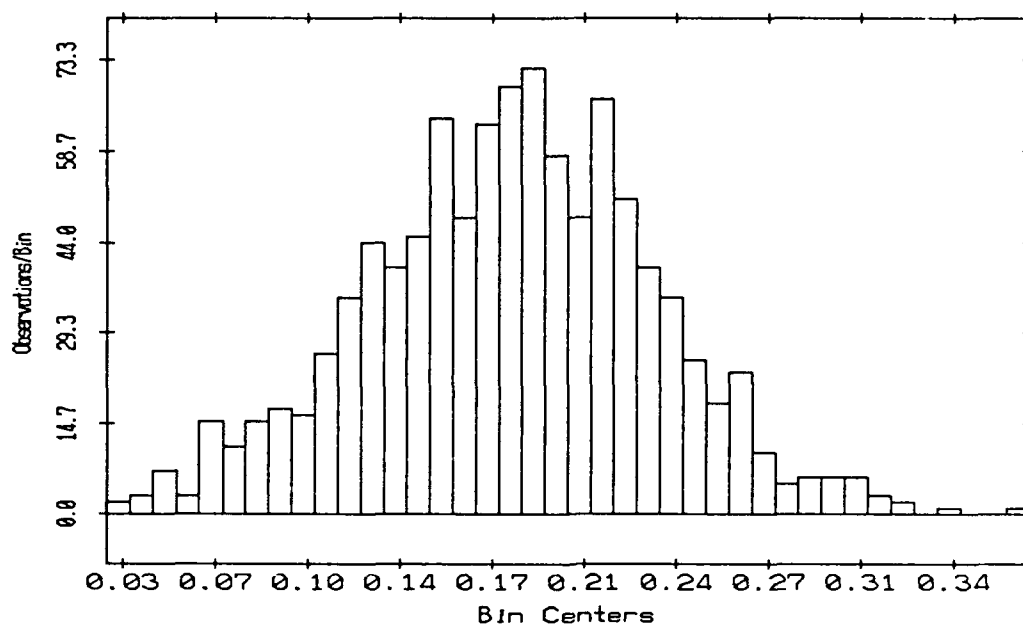


Fig. A.138 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B8'

## Histogram of B8

Mean=-0.35516, standard deviation=0.56201e-01

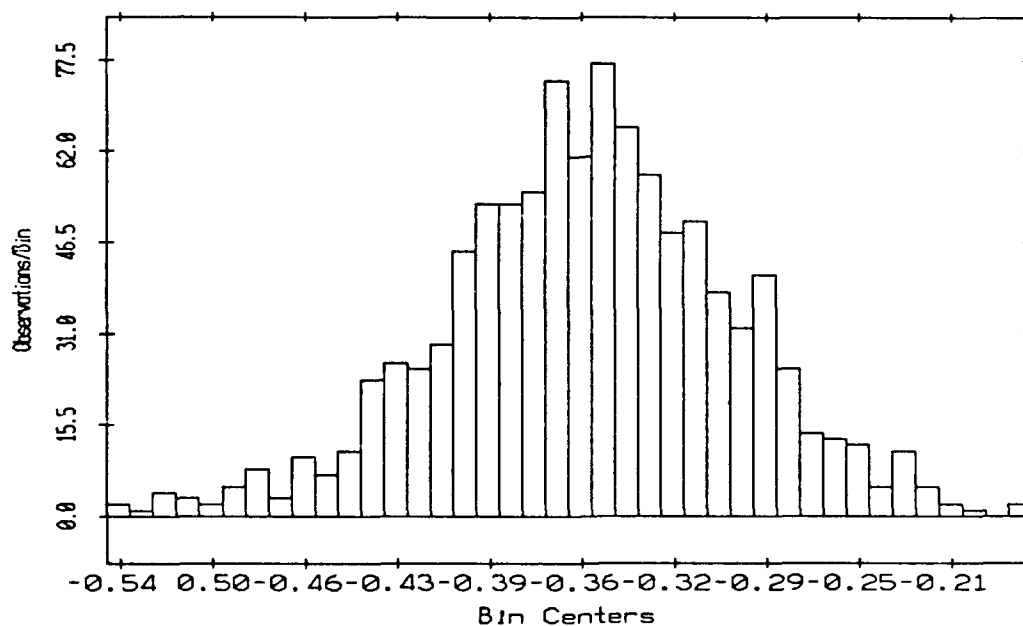


Fig. A.139 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B7'



## Histogram of B9

Mean=-0.17181, standard deviation=0.46984e-01

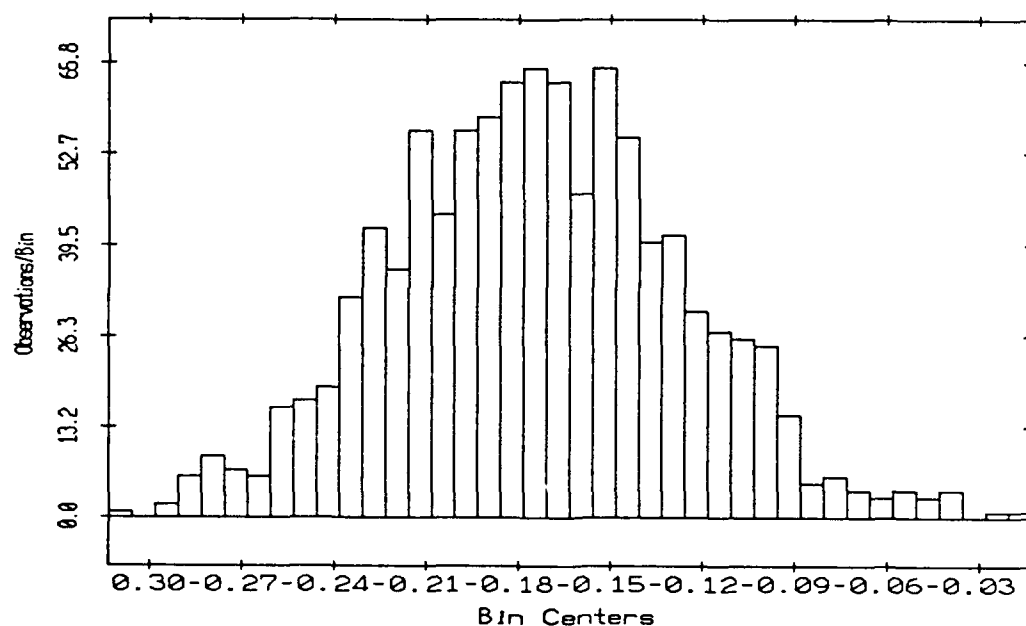


Fig. A.140 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B10'

## Histogram of B10

Mean=-0.82649e-01, standard deviation=0.44575e-01

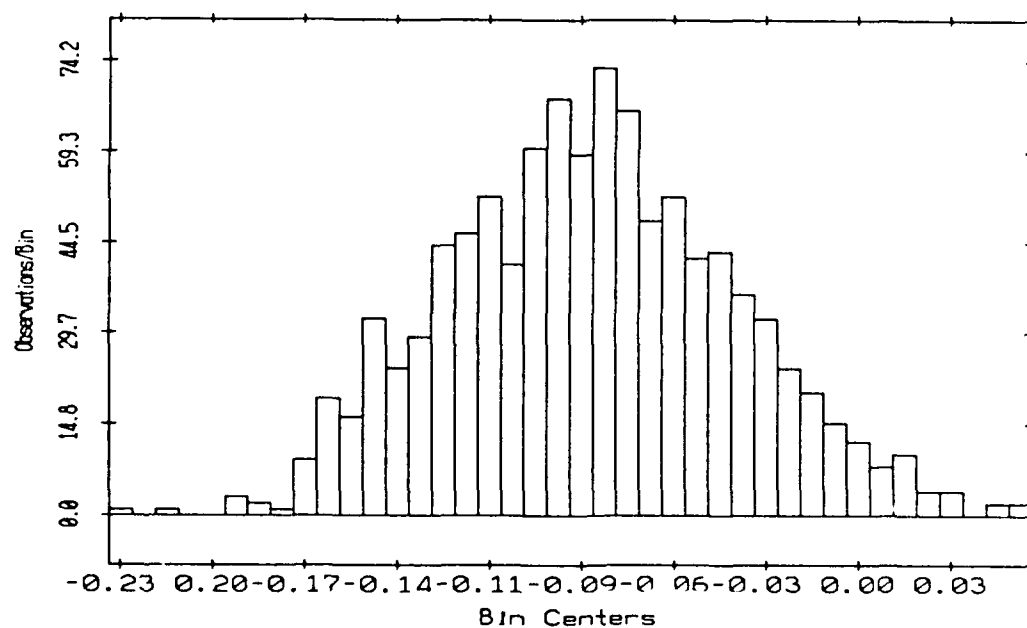


Fig. A.141 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B9'

## Histogram of B11

Mean=-0.39088e-01, standard deviation=0.40278e-01

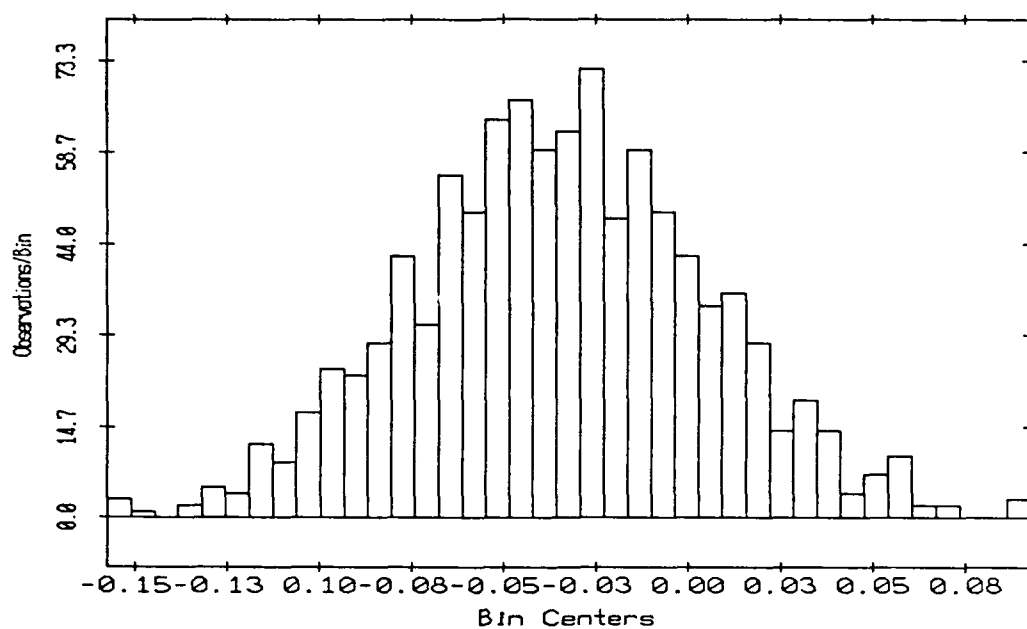


Fig. A.142 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B12'

## Histogram of B12

Mean=-0.14826e-01, standard deviation=0.40031e-01

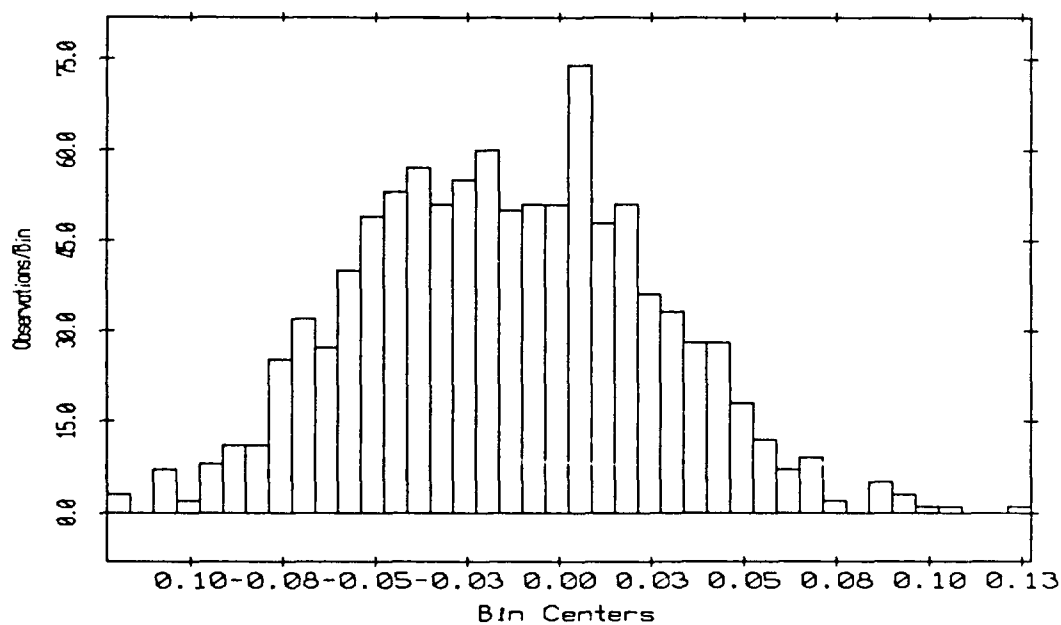


Fig. A.143 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B11'

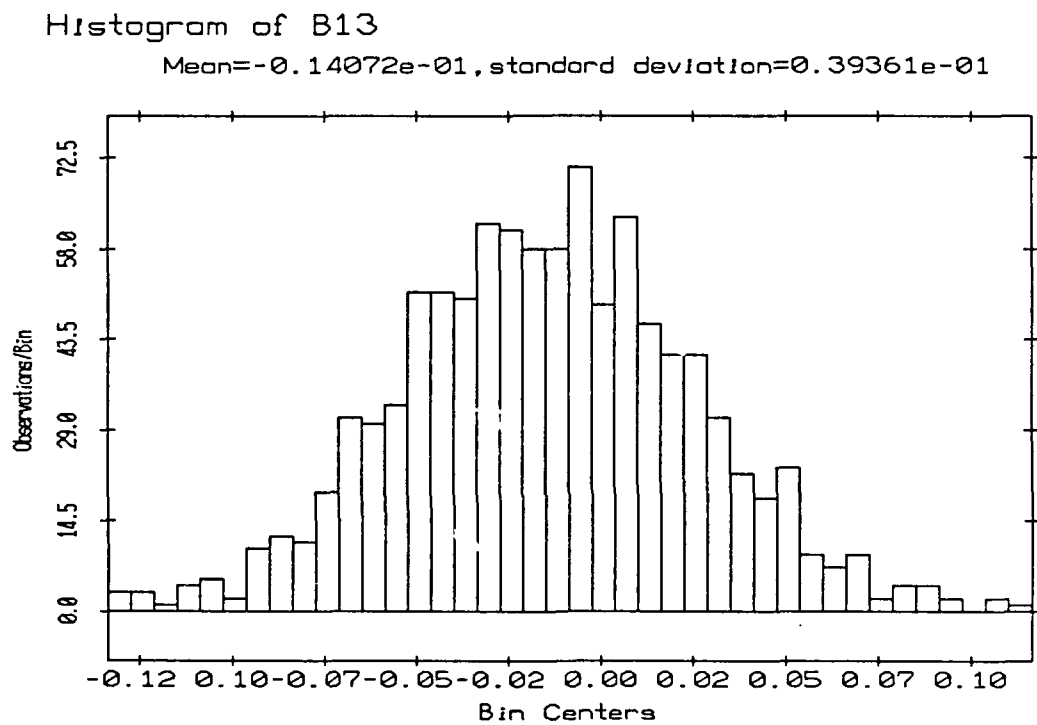


Fig. A.144 : SMC estimation, Nuttall's method, data length 256 points,  
Histogram of estimated model coefficient 'B13'

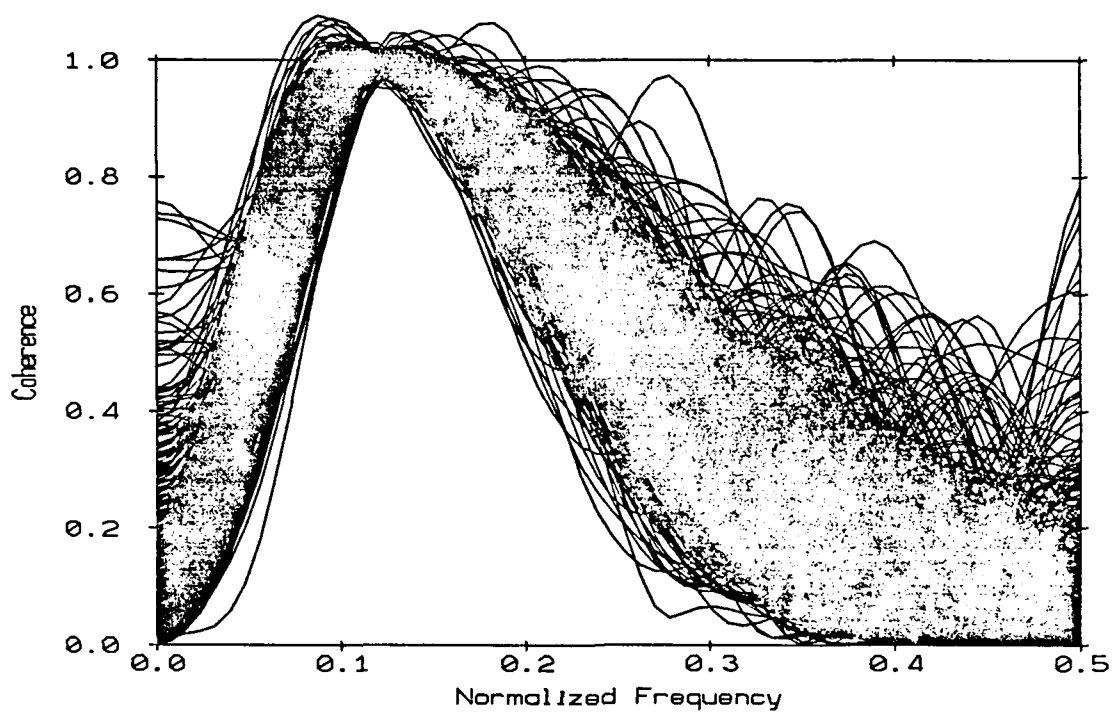


Fig. A.145 : SMC estimation, Nuttall's method, data length 128 points,

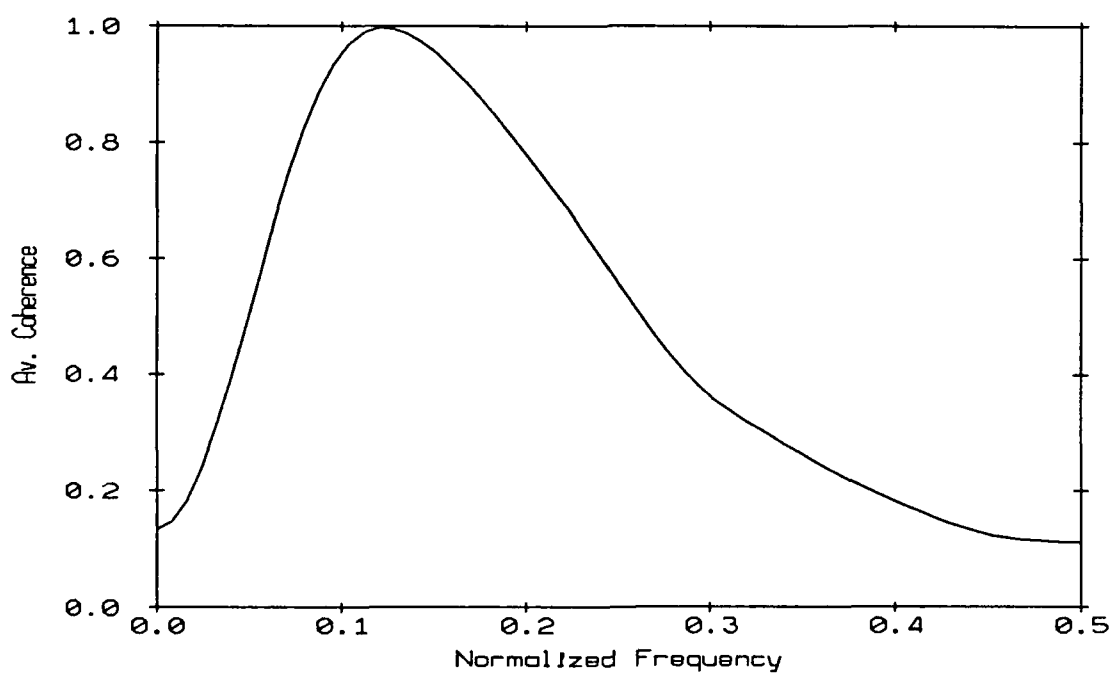


Fig. A.145.1 : SMC estimation, Nuttall's method, data length 128 points,  
Average estimated SMC over 1000 runs

SMC estimation, Nuttall's method, data length 128 points

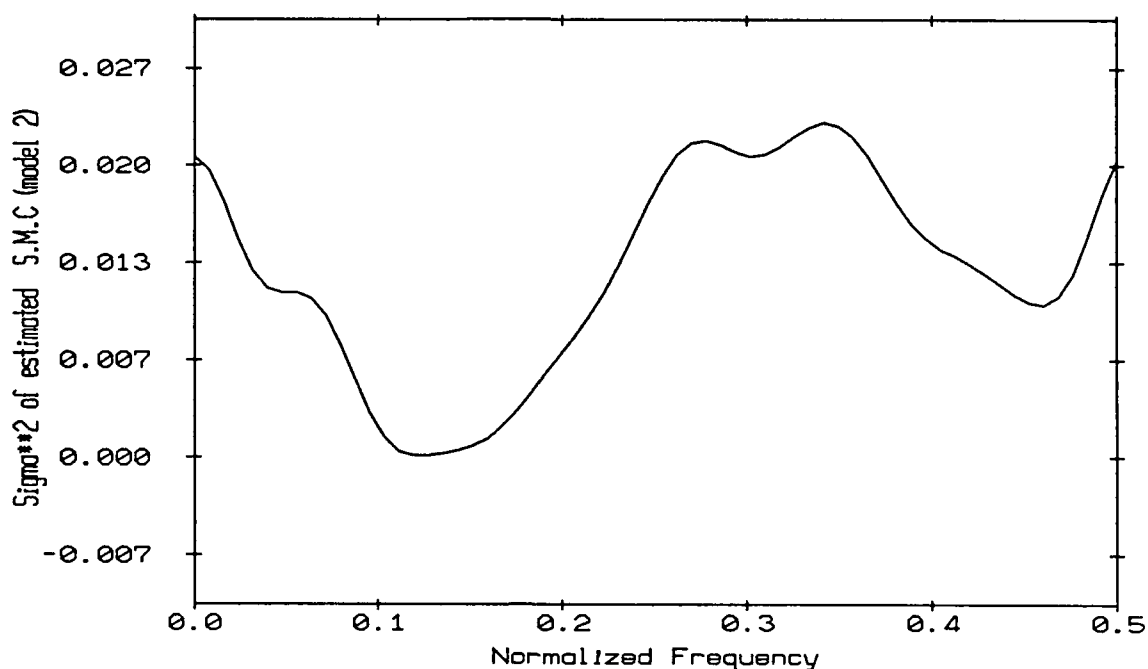


Fig. A.146 : Squared Standard Deviation, of estimated SMC, Nuttall's method  
data length 128 points, 1000 runs

SMC estimation, Nuttall's method, data length 128 points

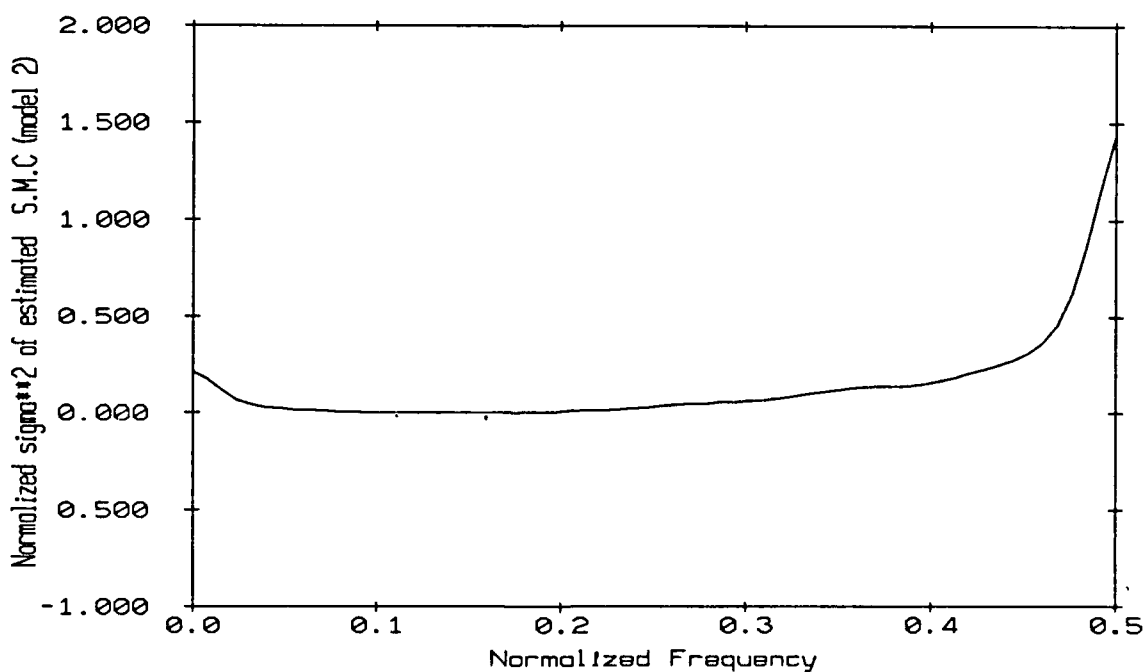


Fig. A.146.1: Normalized Squared Standard Deviation of estimated SMC, Nuttall's method  
data length 128 points, 1000 runs

Histogram of Coherency at normalized freq. =  $1/32$ 

Mean=0.23572, Standard Deviation=0.12243

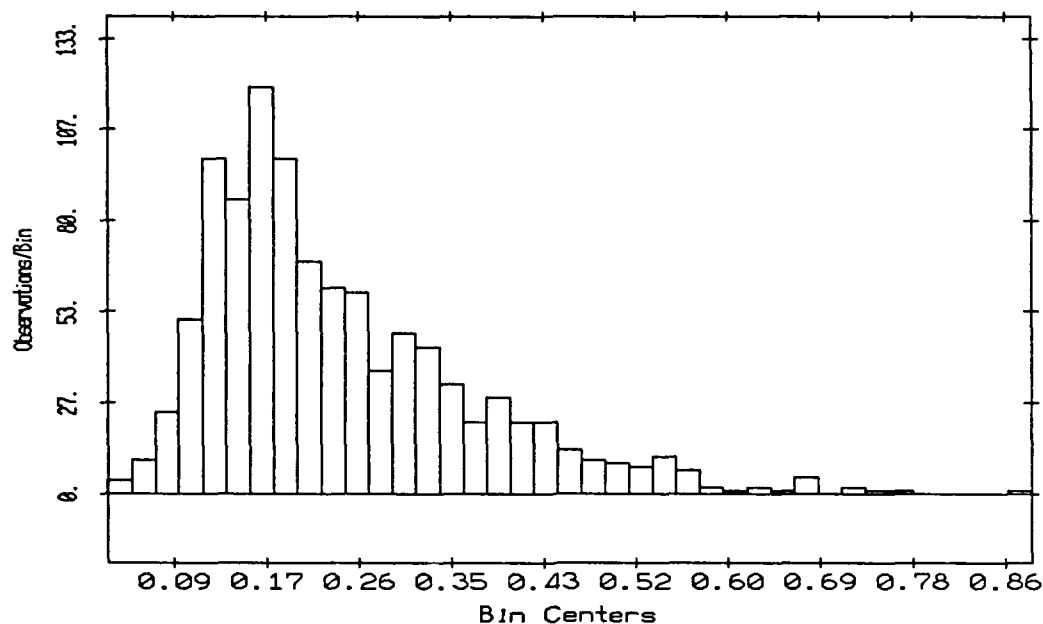


Fig. A.147 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated SMC at normalized frequency =  $1/32$

Histogram of Coherency at normalized freq. =  $2/32$ 

Mean=0.57273, Standard Deviation=0.10629

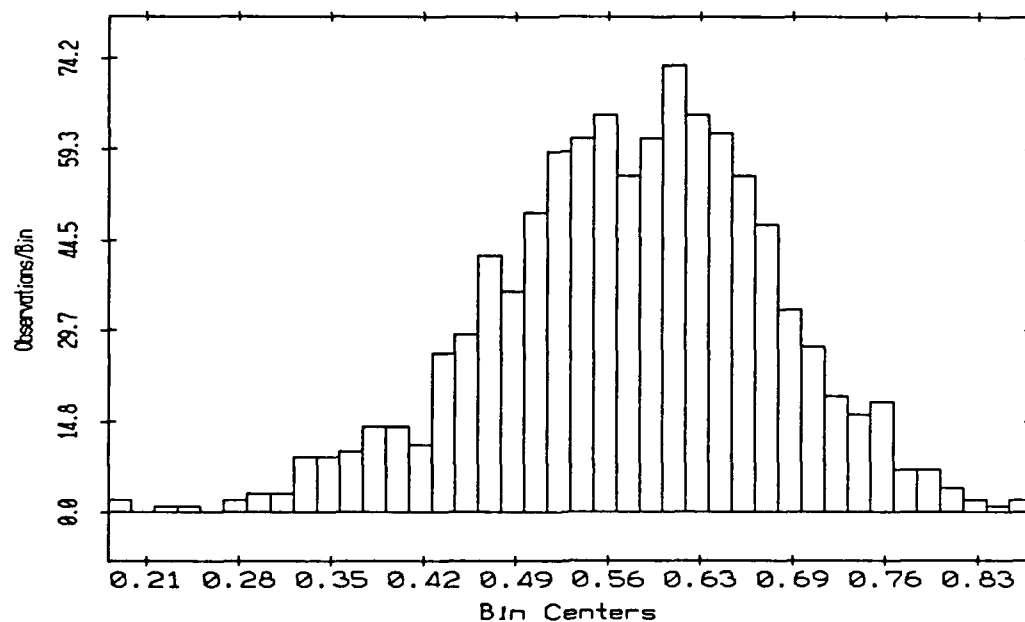


Fig. A.148 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated SMC at normalized frequency =  $2/32$

Histogram of Coherency at normalized freq. =  $3/32$   
 Mean=0.88558, Standard Deviation=0.72998e-01

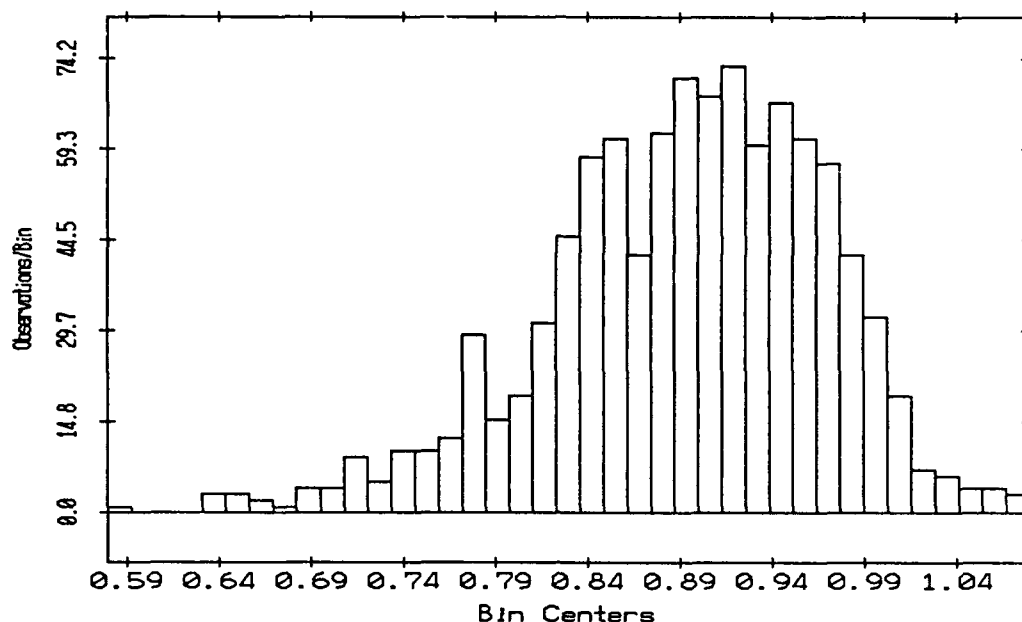


Fig. A.149 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $3/32$

Histogram of Coherency at normalized freq. =  $4/32$   
 Mean=0.99766, Standard Deviation=0.11664e-01

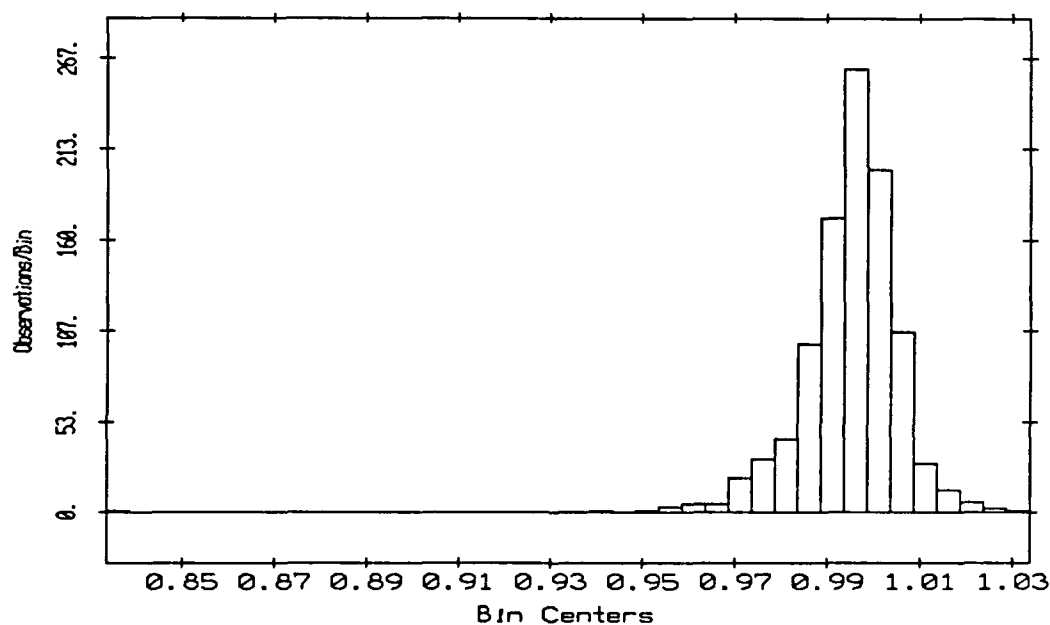


Fig. A.150 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $4/32$

Histogram of Coherency at normalized freq. =  $5/32$   
 Mean=0.95420, Standard Deviation=0.28563e-01

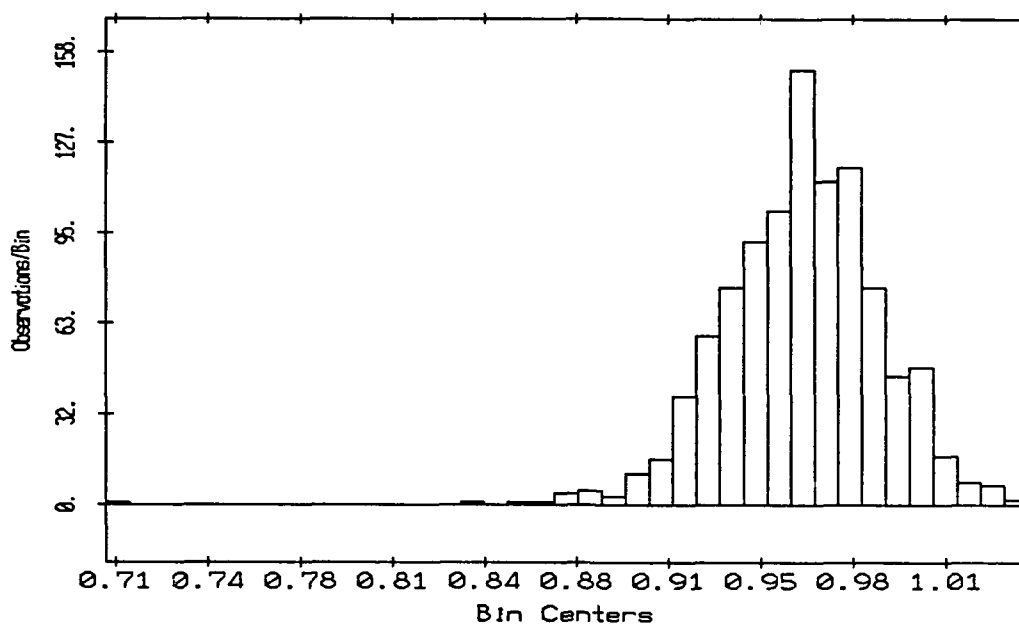


Fig. A.151 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $5/32$

Histogram of Coherency at normalized freq. =  $6/32$   
 Mean=0.84846, Standard Deviation=0.66012e-01

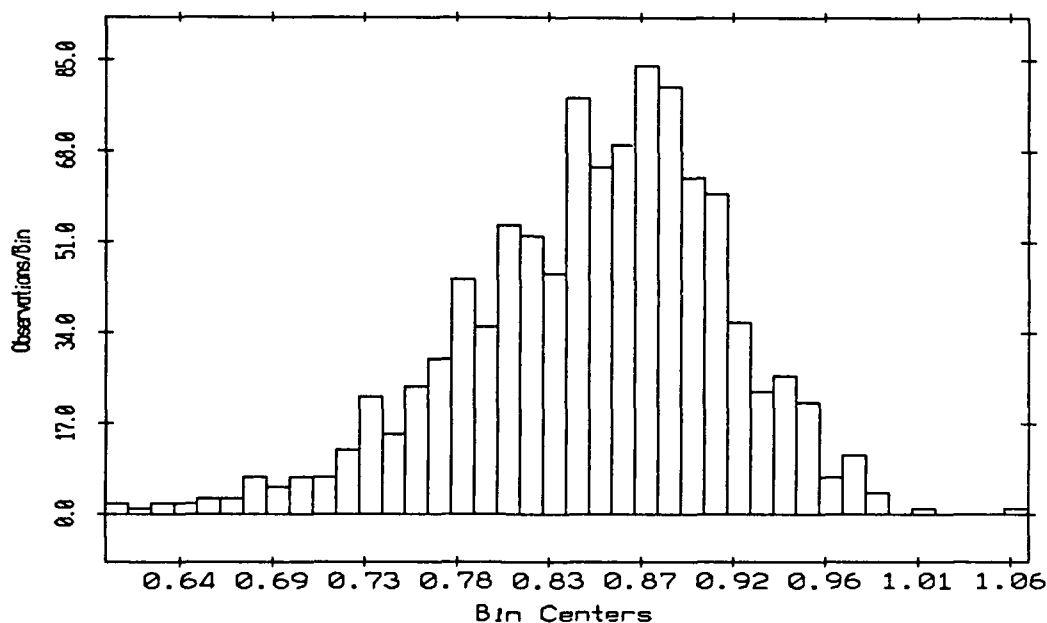


Fig. A.152 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $6/32$



Histogram of Coherency at normalized freq. =  $7/32$   
 Mean=0.72012, Standard Deviation=0.98322e-01

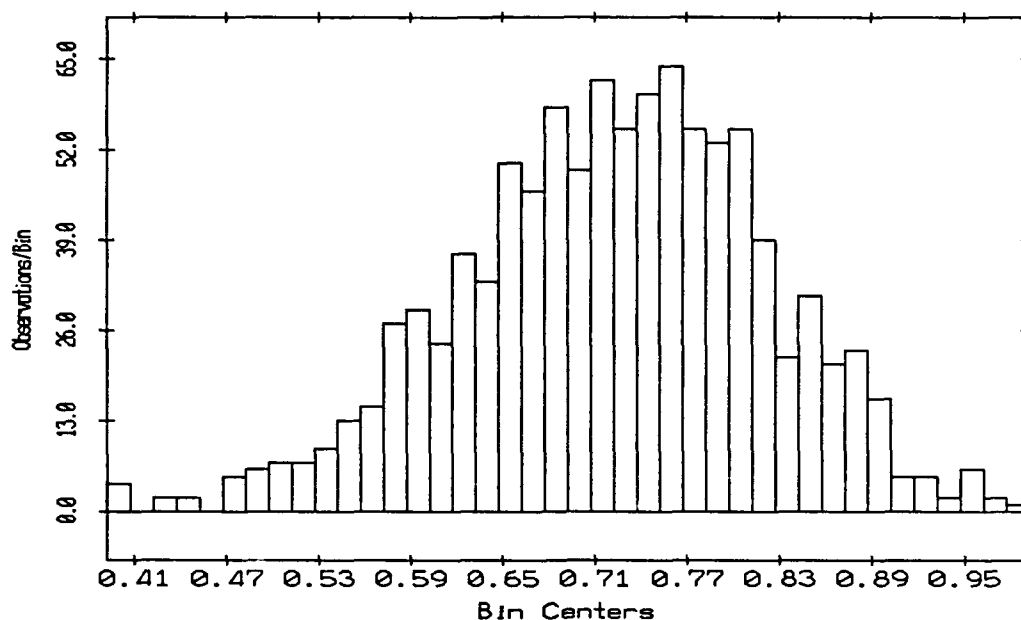


Fig. A.153 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $7/32$

Histogram of Coherency at normalized freq. =  $8/32$   
 Mean=0.57650, Standard Deviation=0.13149

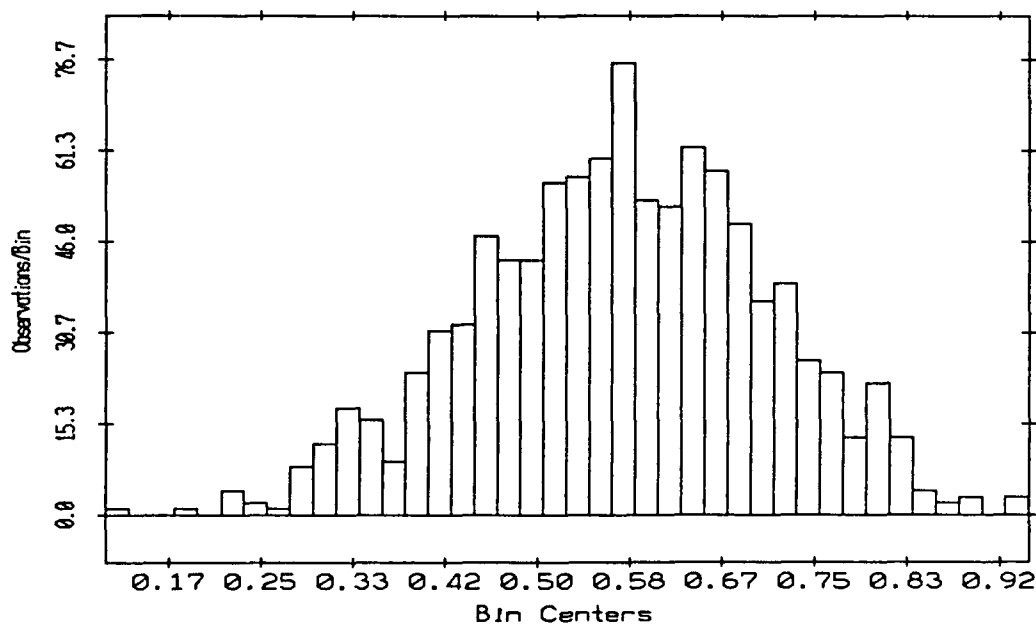


Fig. A.154 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $8/32$

Histogram of Coherency at normalized freq. =  $9/32$   
 Mean=0.43418, Standard Deviation=0.14736

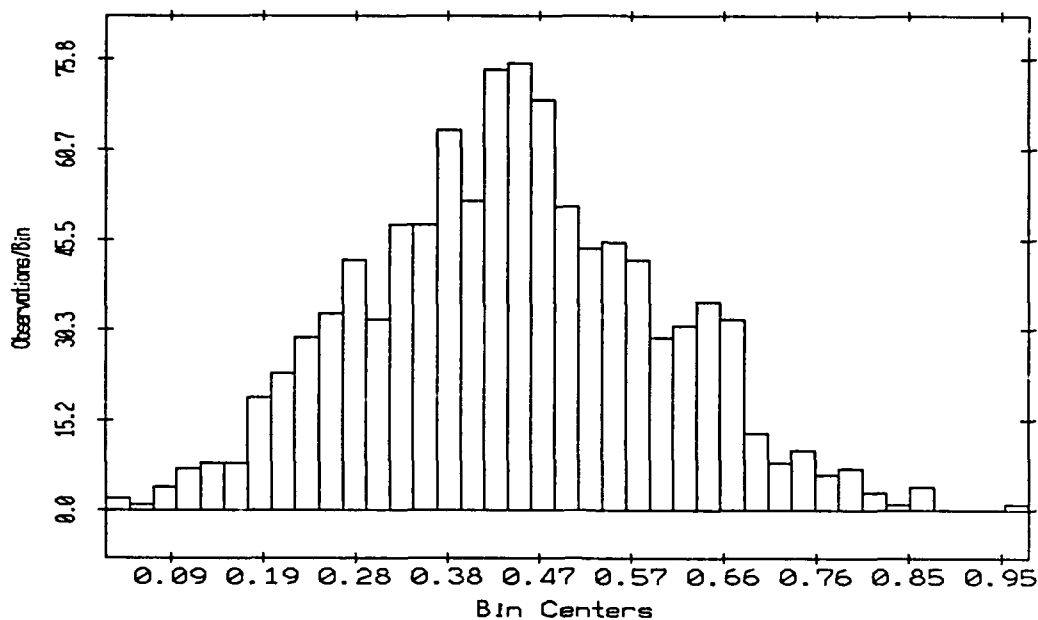


Fig. A.155 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $9/32$

Histogram of Coherency at normalized freq. =  $10/32$   
 Mean=0.33845, Standard Deviation=0.14408

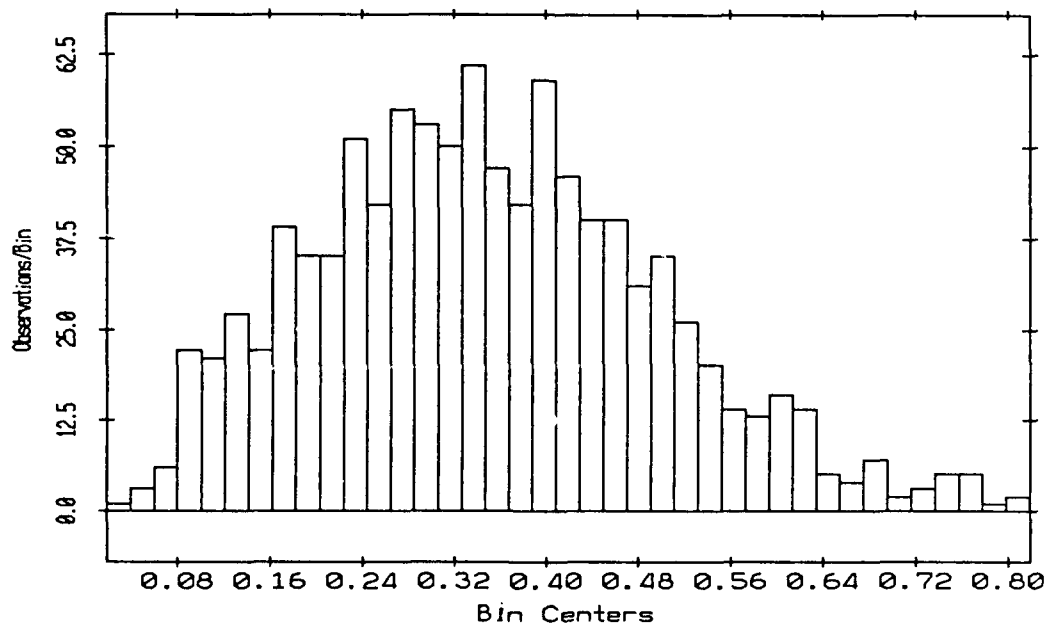


Fig. A.156 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $10/32$

Histogram of Coherency at normalized freq. =  $11/32$   
 Mean=0.27729, Standard Deviation=0.15153

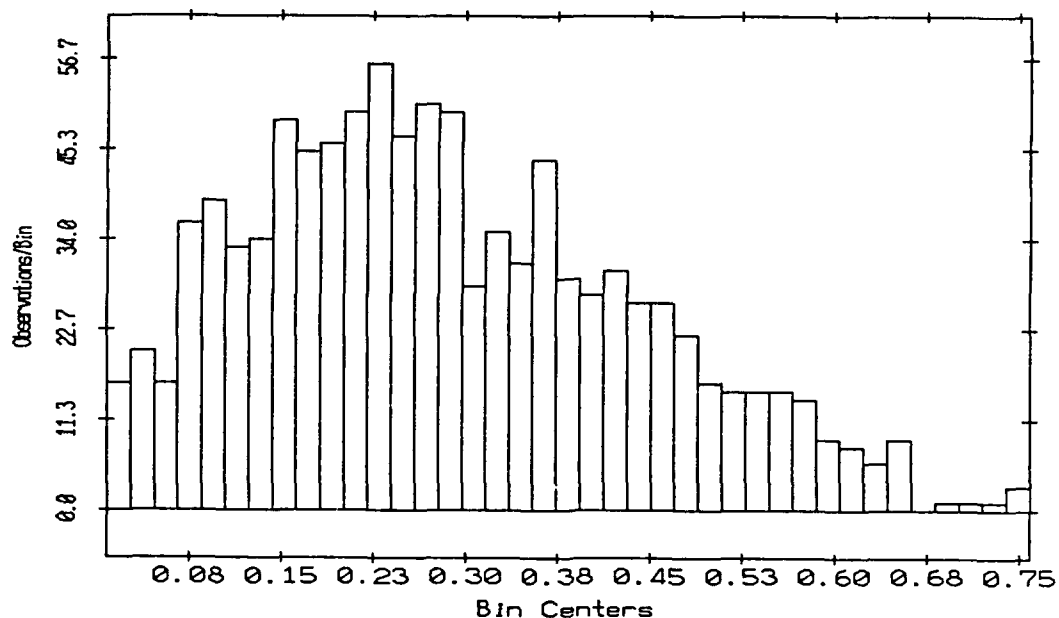


Fig. A.157 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $11/32$

Histogram of Coherency at normalized freq. =  $12/32$   
 Mean=0.22125, Standard Deviation=0.13793

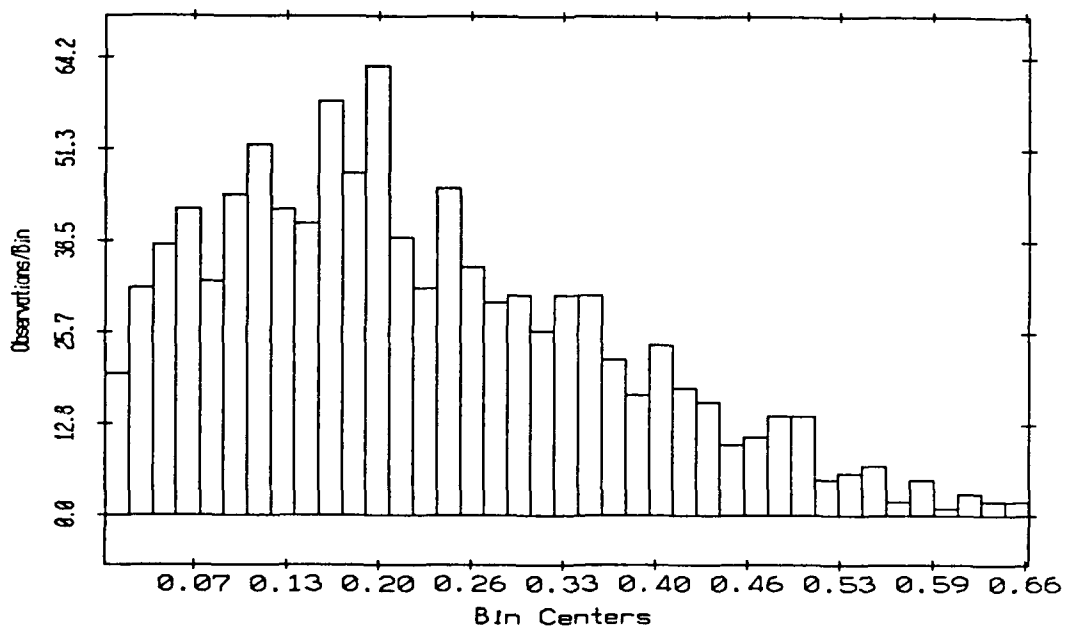


Fig. A.158 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $12/32$

Histogram of Coherency at normalized freq. =  $13/32$   
 Mean=0.17518, Standard Deviation=0.11943

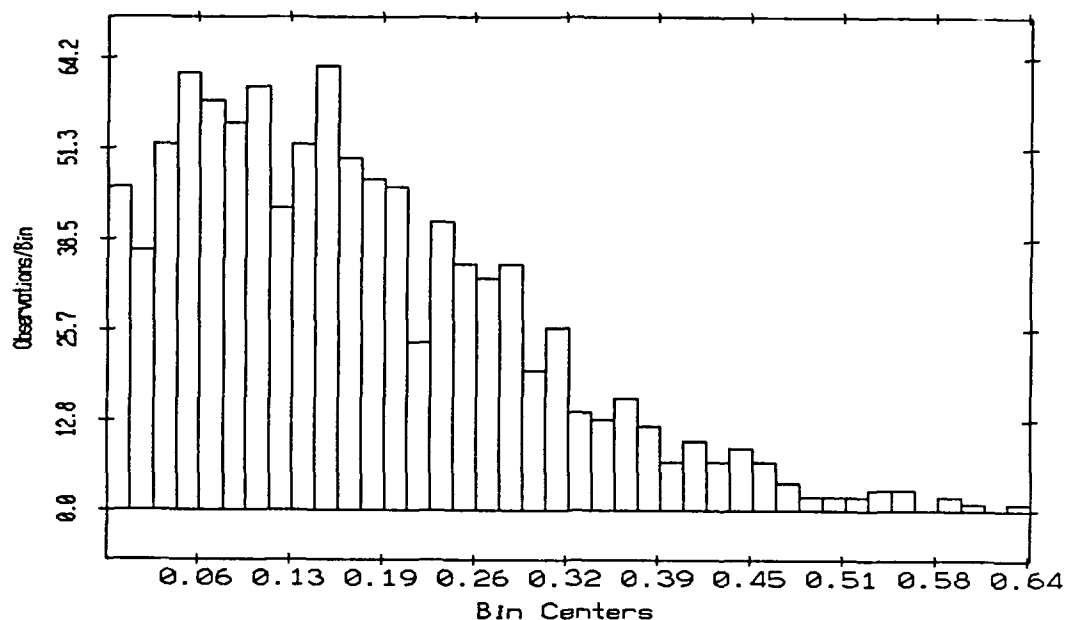


Fig. A.159 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $13/32$

Histogram of Coherency at normalized freq. =  $14/32$   
 Mean=0.13714, Standard Deviation=0.10886

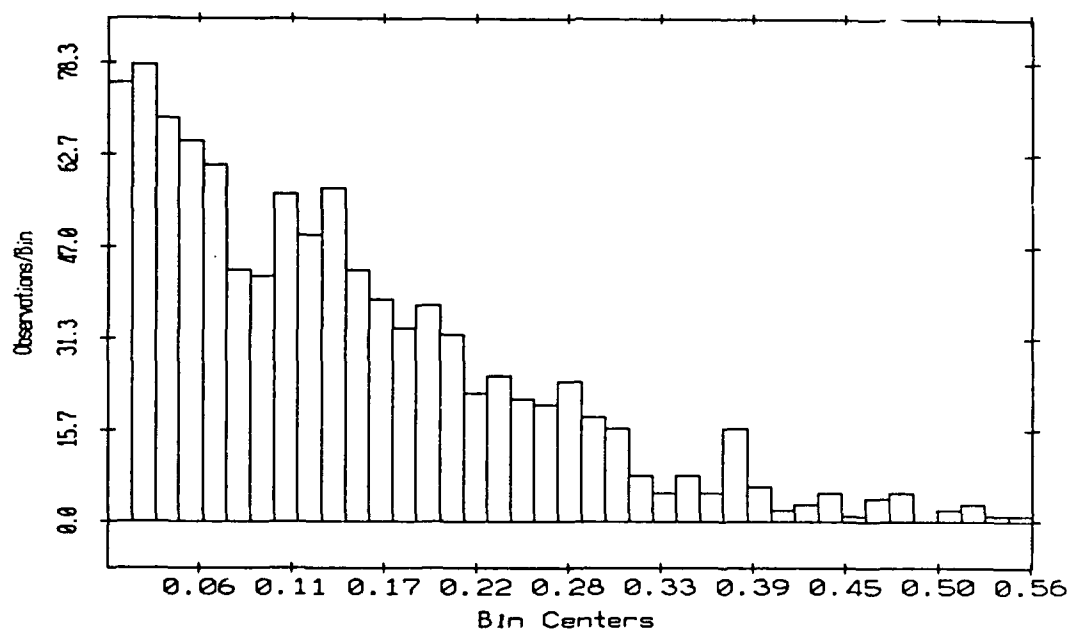


Fig. A.160 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $14/32$

Histogram of Coherency at normalized freq. =  $15/32$   
 Mean=0.11691, Standard Deviation=0.10475

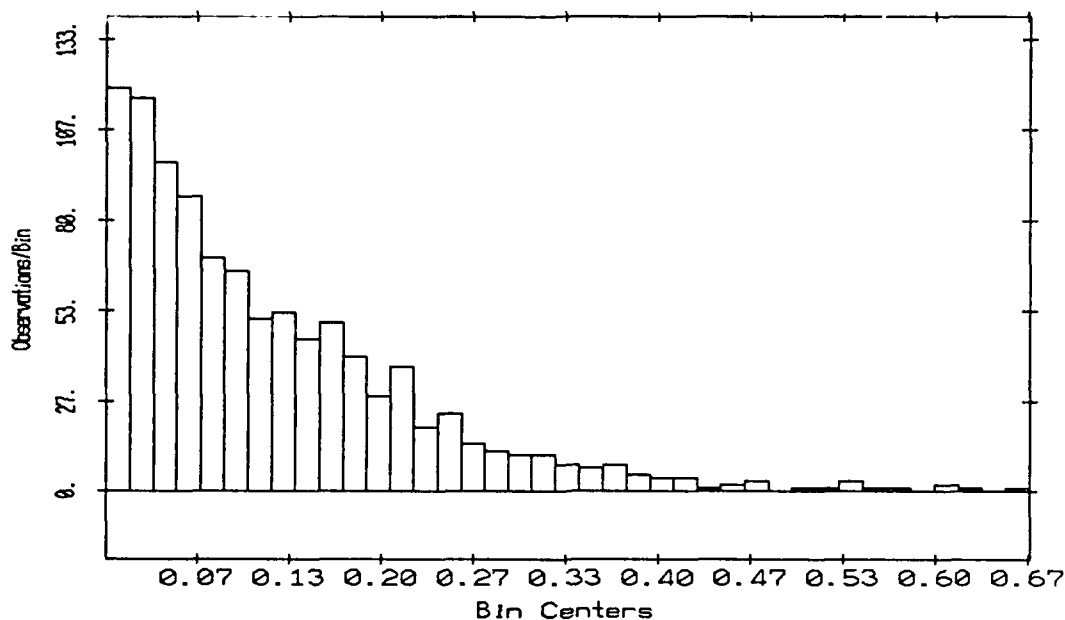


Fig. A.161 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $15/32$

Histogram of Coherency at normalized freq. =  $16/32$   
 Mean=0.11107, Standard Deviation=0.14282

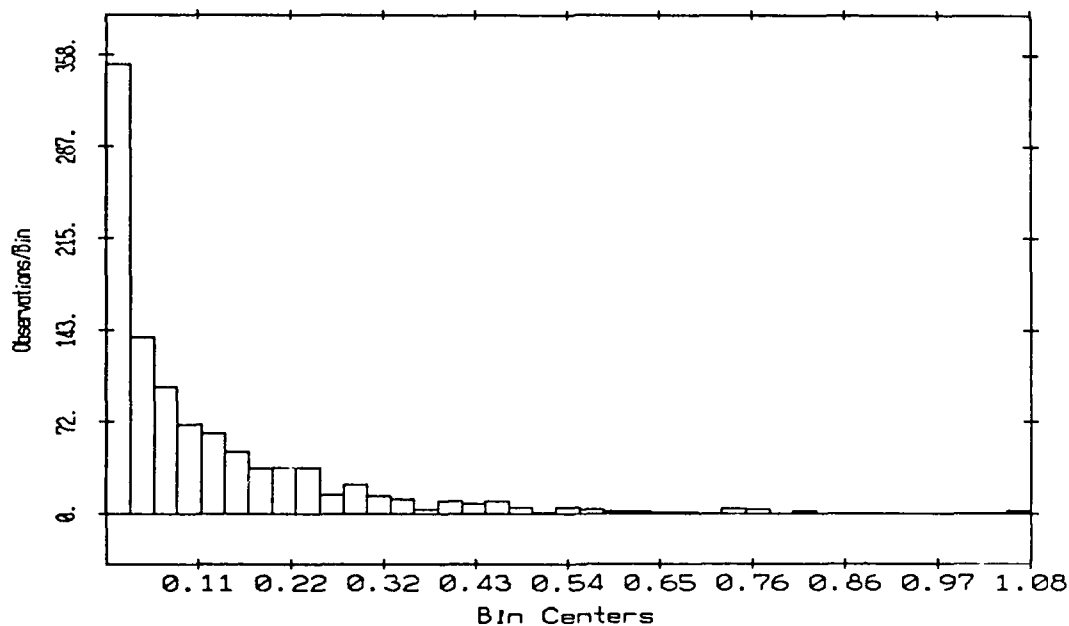


Fig. A.162 : SMC estimation, Nuttall's method, data length 128 points,  
 Histogram of estimated SMC at normalized frequency =  $16/32$

## Histogram of A1

Mean=-0.40961e-02, Standard Deviation=0.50835e-01

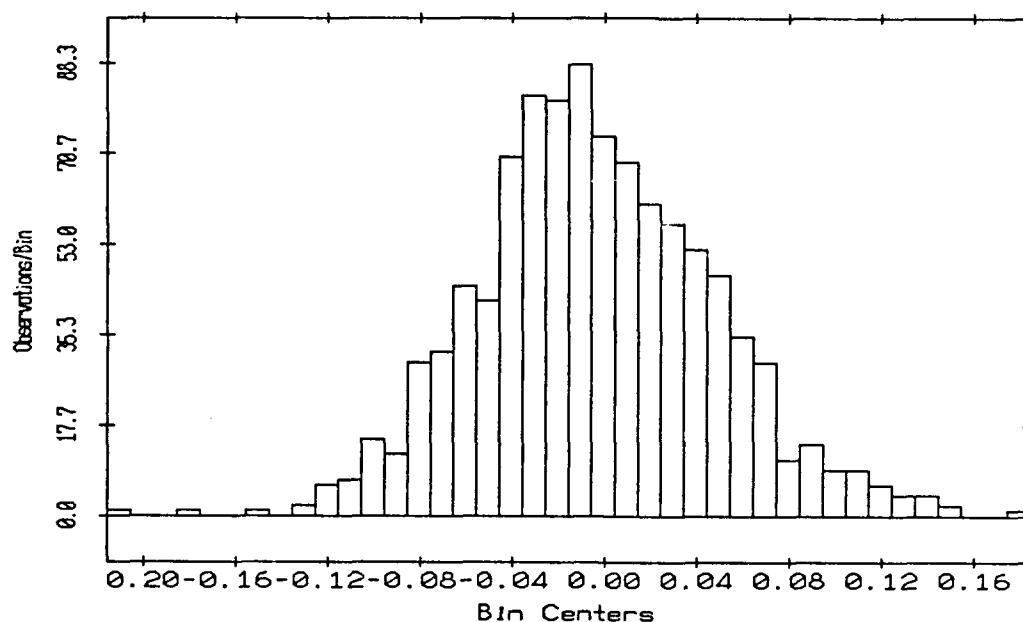


Fig. A.163 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A2'

## Histogram of A2

Mean=-0.55885e-02, Standard Deviation=0.60322e-01

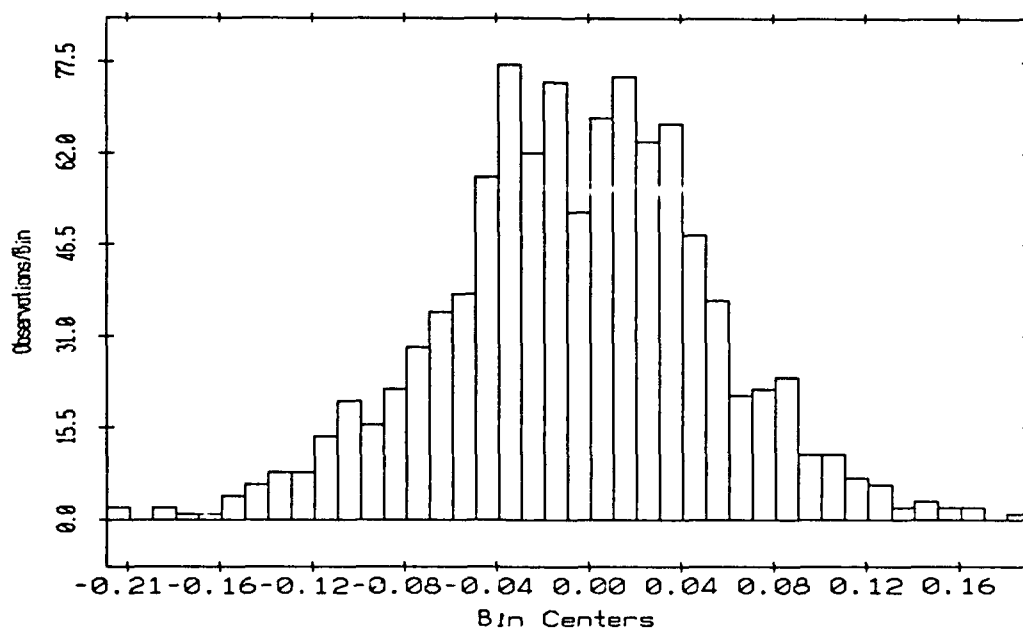


Fig. A.164 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A1'

## Histogram of A3

Mean=-0.16758e-01, Standard Deviation=0.63567e-01

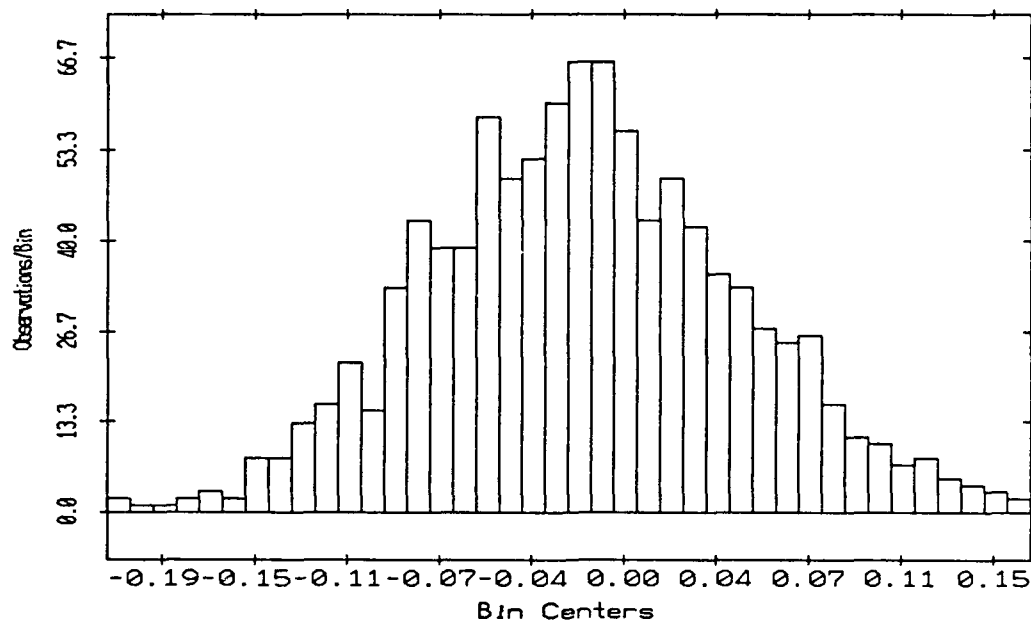


Fig. A.165 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A4'

## Histogram of A4

Mean=-0.49277e-01, Standard Deviation=0.67606e-01

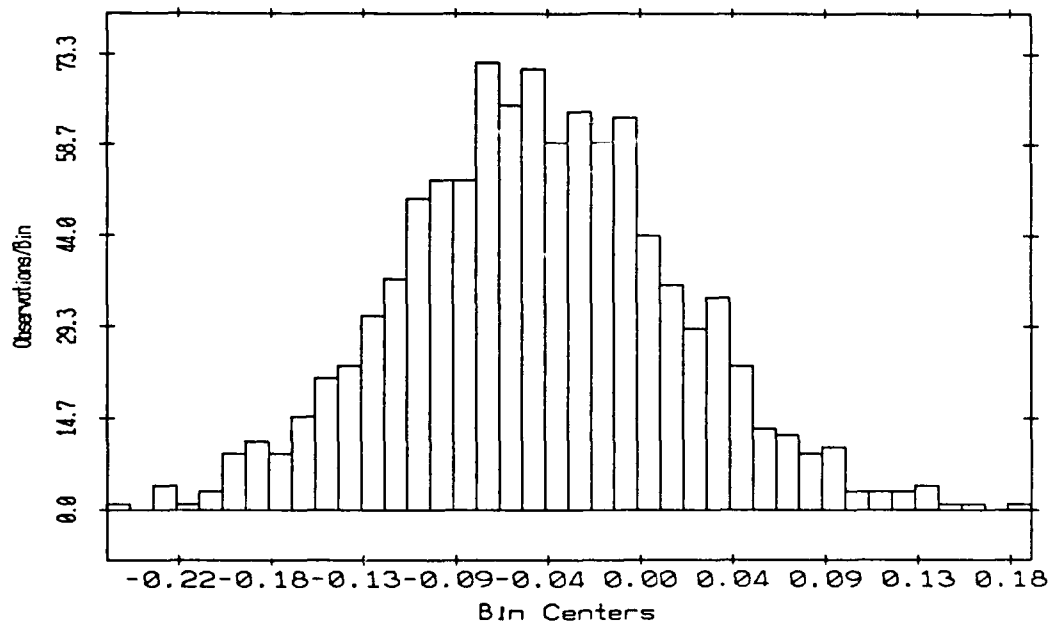


Fig. A.166 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A3'

## Histogram of A5

Mean=-0.13032, Standard Deviation=0.69405e-01

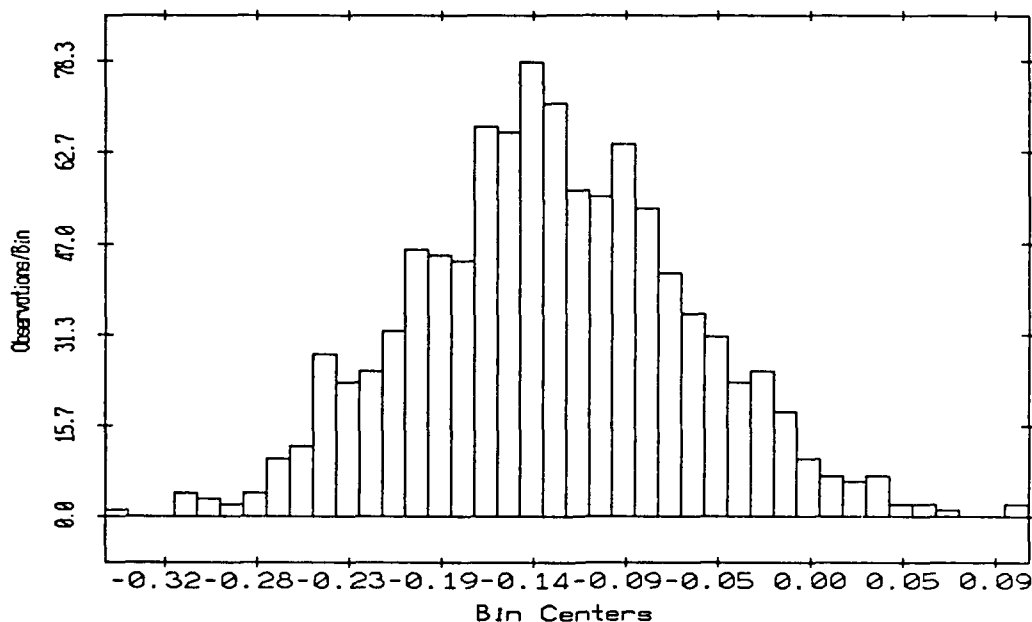


Fig. A.167 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A6'

## Histogram of A6

Mean=-0.38904, Standard Deviation=0.89156e-01

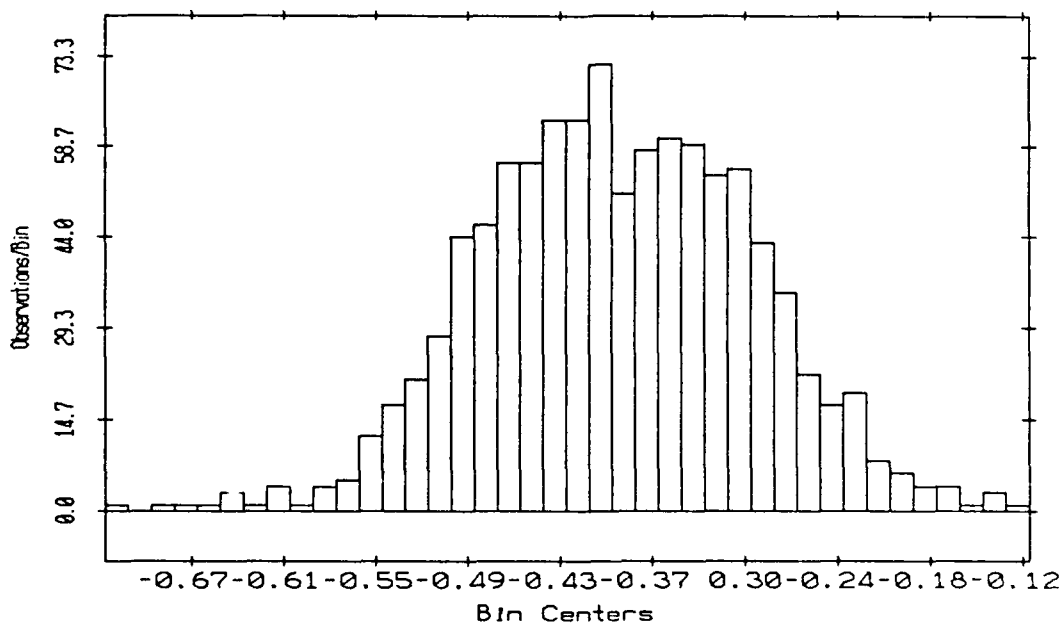


Fig. A.168 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A5'



## Histogram of A7

Mean=0.16658, Standard Deviation=0.95914e-01

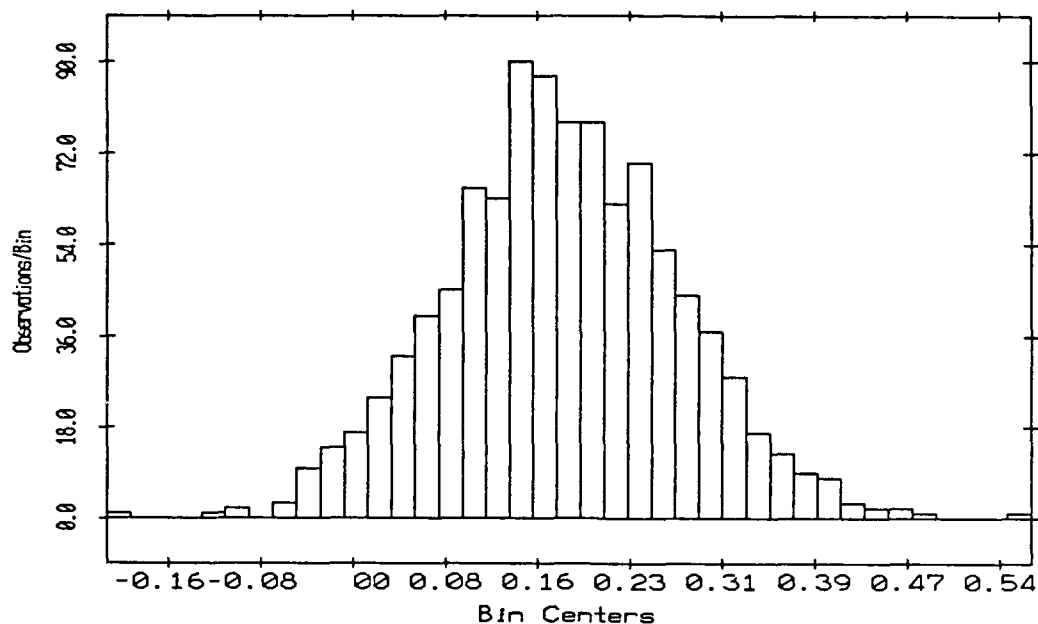


Fig. A.169 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A8'

## Histogram of A8

Mean=0.42730, Standard Deviation=0.91160e-01

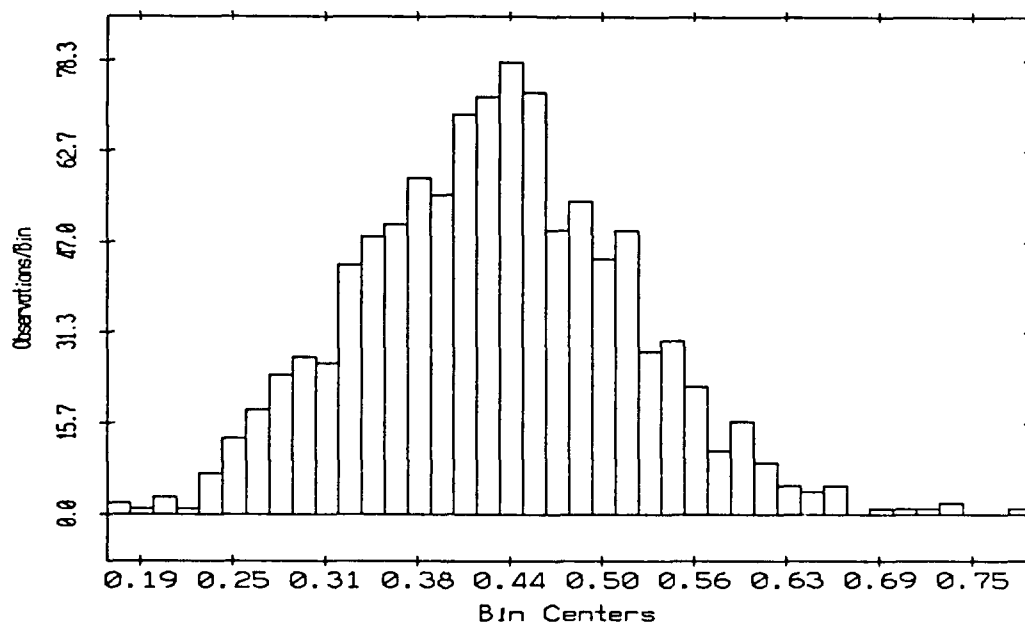


Fig. A.170 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A7'

## Histogram of A9

Mean=0.14539, Standard Deviation=0.70574e-01

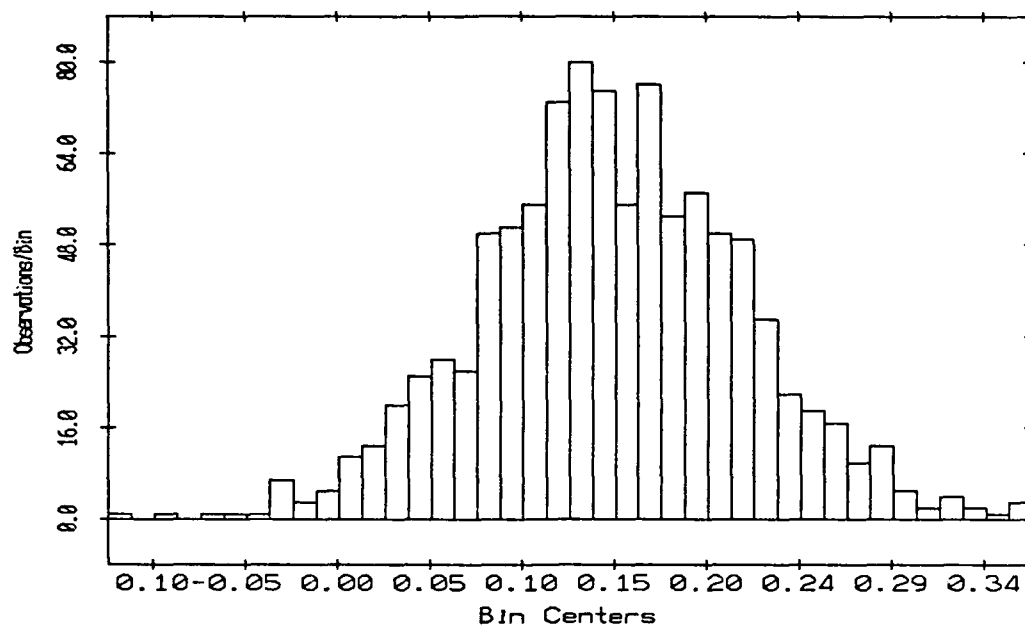


Fig. A.171 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A10'

## Histogram of A10

Mean=0.48110e-01, Standard Deviation=0.68515e-01

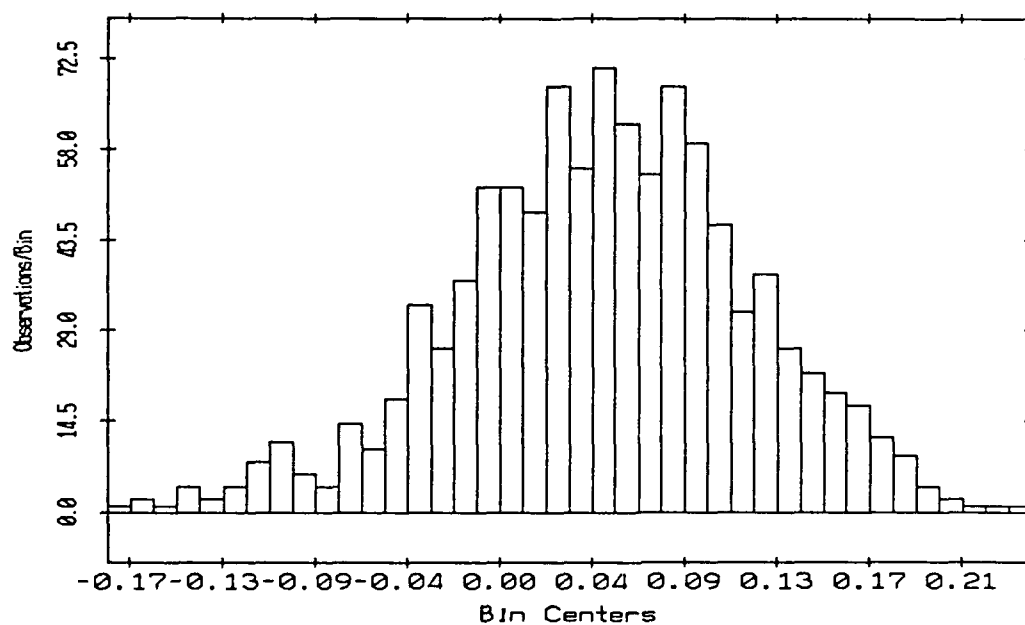


Fig. A.172 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A9'

## Histogram of A11

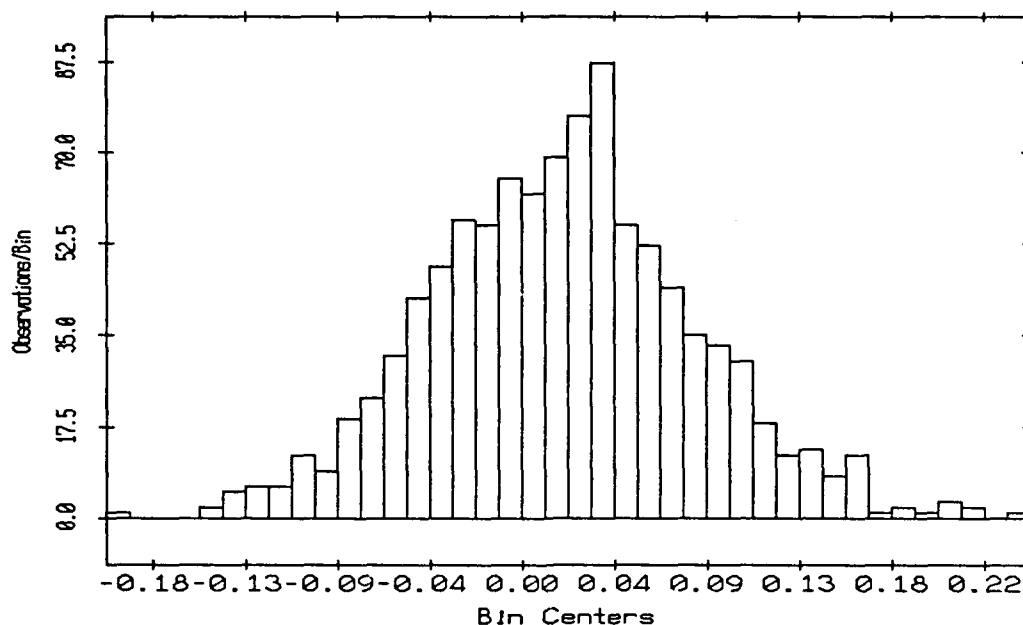
Mean= $0.19232e-01$ , Standard Deviation= $0.64131e-01$ 

Fig. A.173 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A12'

## Histogram of A12

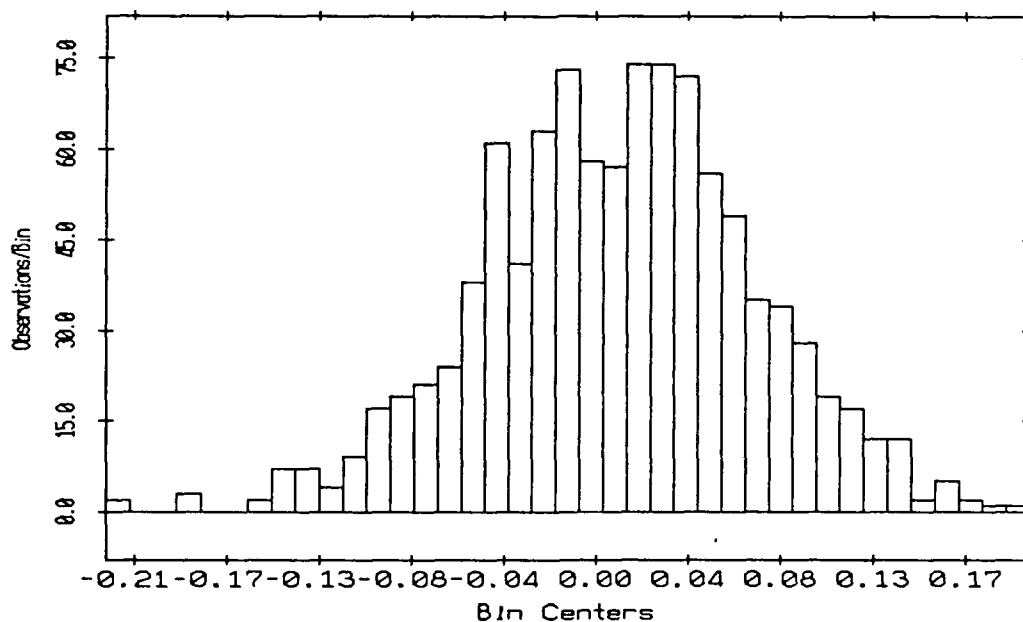
Mean= $0.43949e-02$ , Standard Deviation= $0.61129e-01$ 

Fig. A.174 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A11'

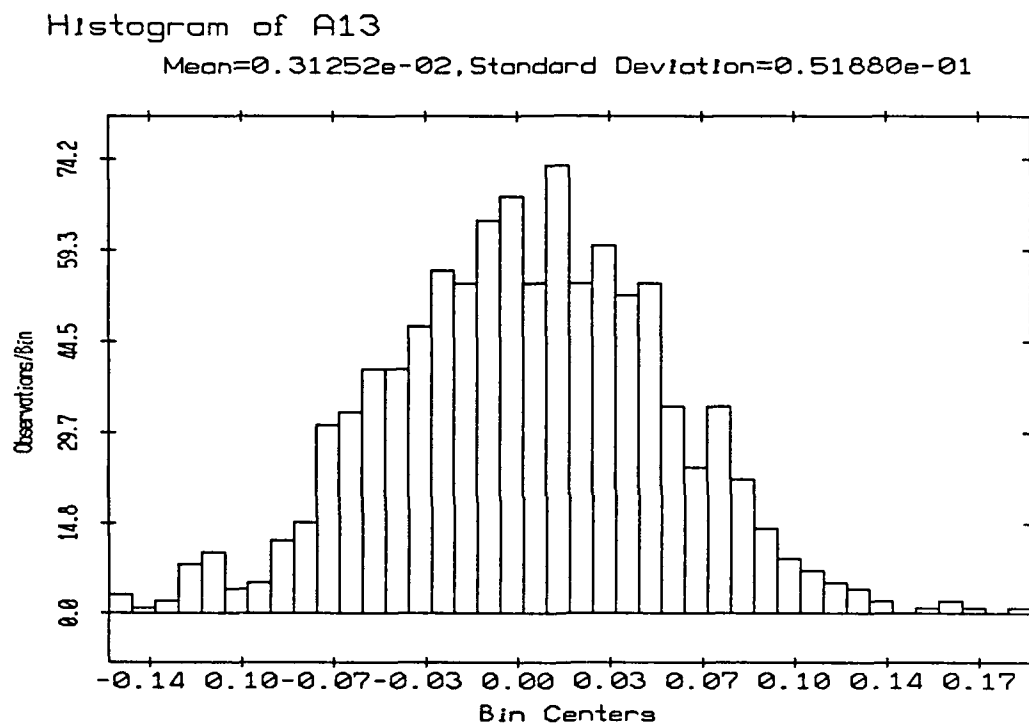


Fig. A.175 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'A13'

## Histogram of B1

Mean=0.17530e-01, standard deviation=0.57260e-01

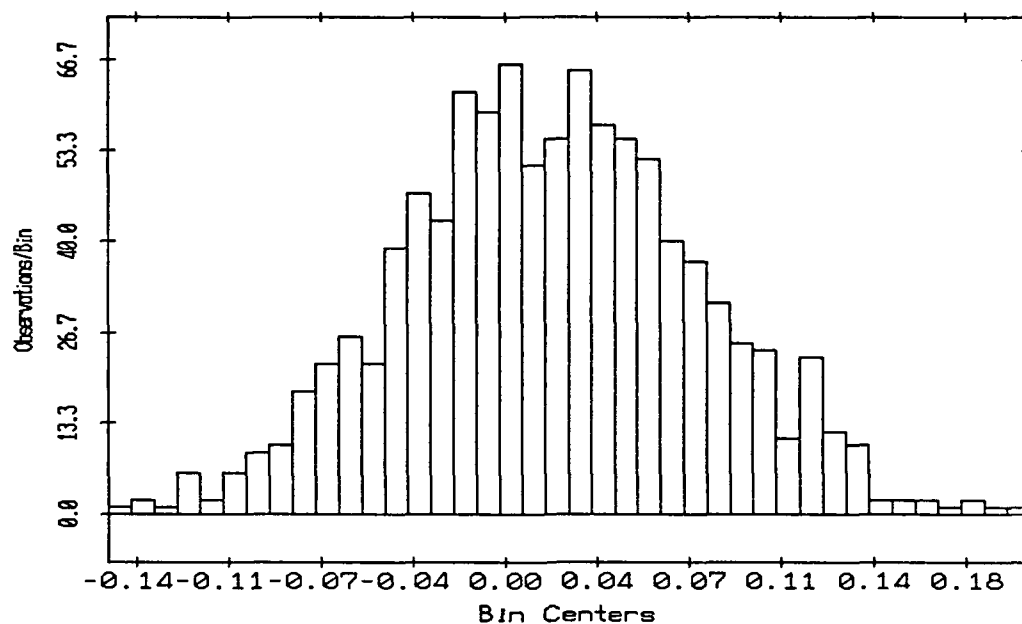


Fig. A.176 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'B2'

## Histogram of B2

Mean=0.20776e-01, standard deviation=0.56204e-01

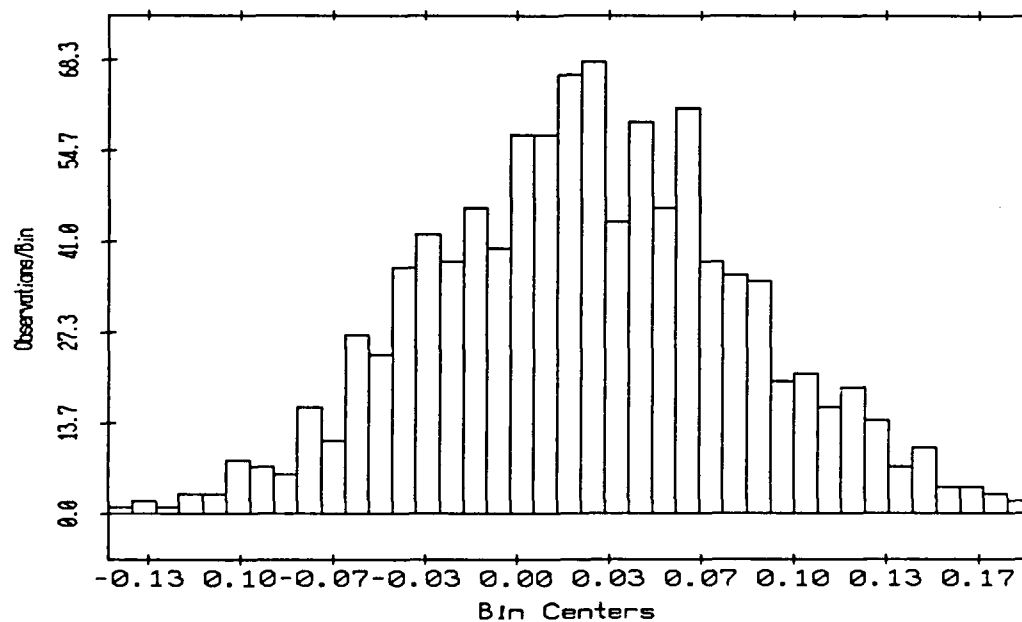


Fig. A.177 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'B1'

## Histogram of B3

Mean=0.47172e-01, standard deviation=0.58223e-01

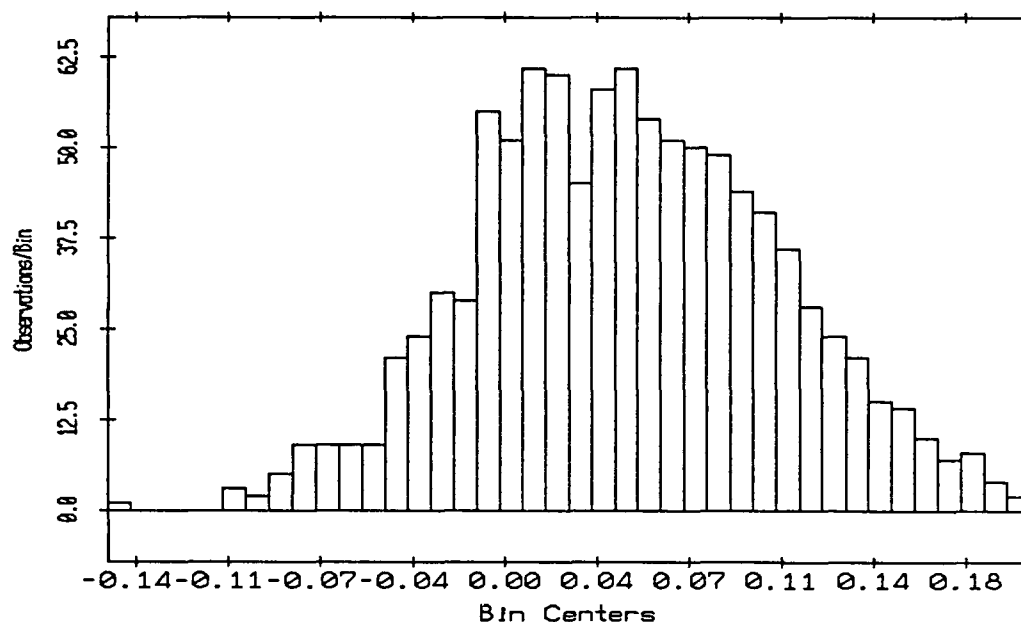


Fig. A.178 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'B4'

## Histogram of B4

Mean=0.10284, standard deviation=0.63936e-01

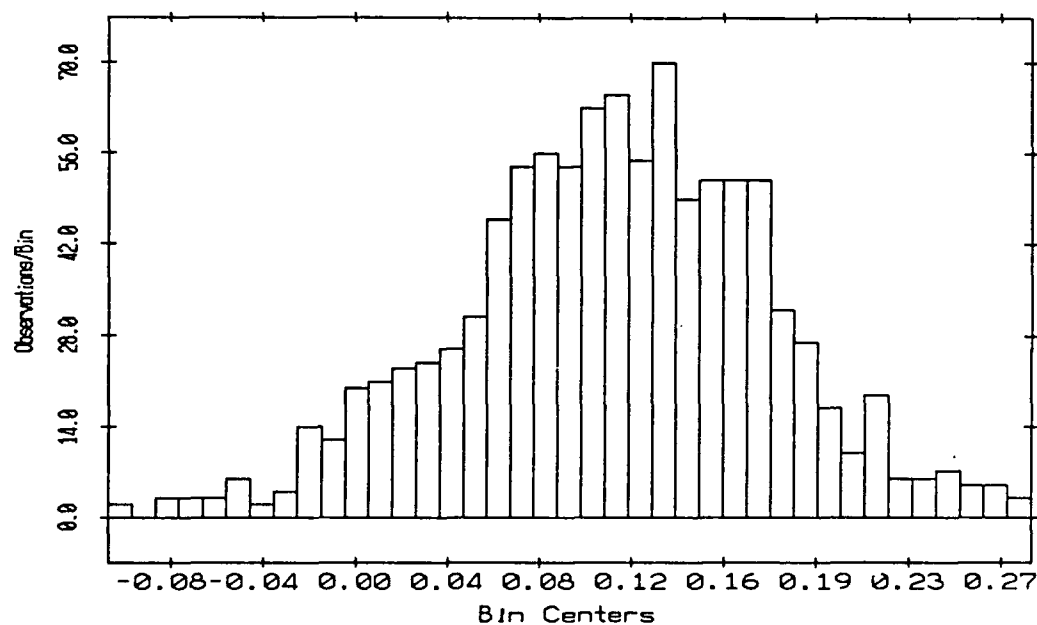


Fig. A.179 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'B3'

## Histogram of B5

Mean=0.21980, standard deviation=0.66878e-01

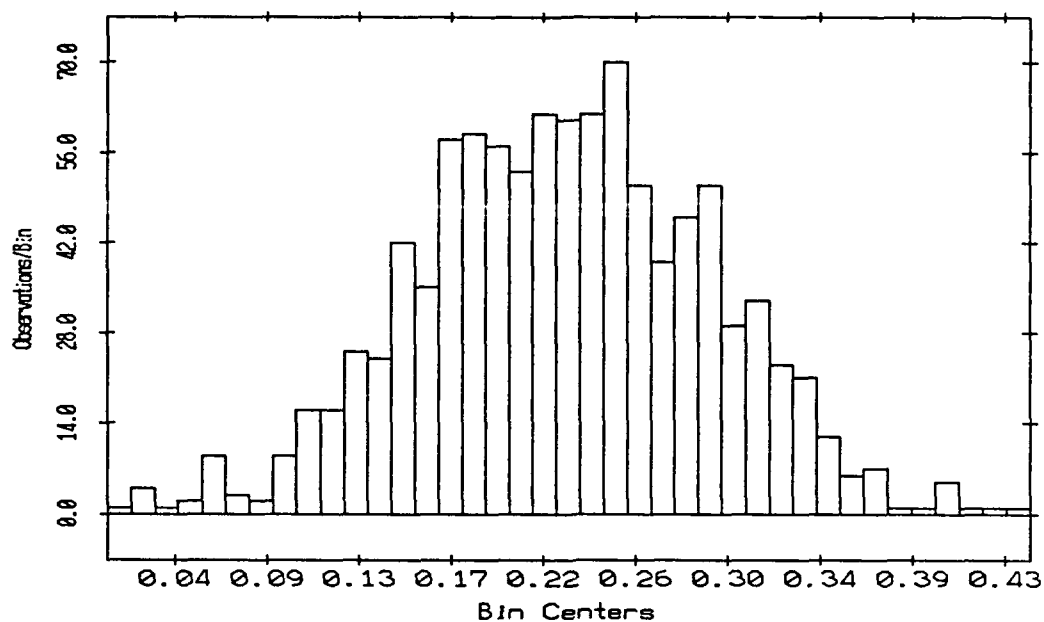


Fig. A.180 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'B6'

## Histogram of B6

Mean=0.46183, standard deviation=0.84502e-01

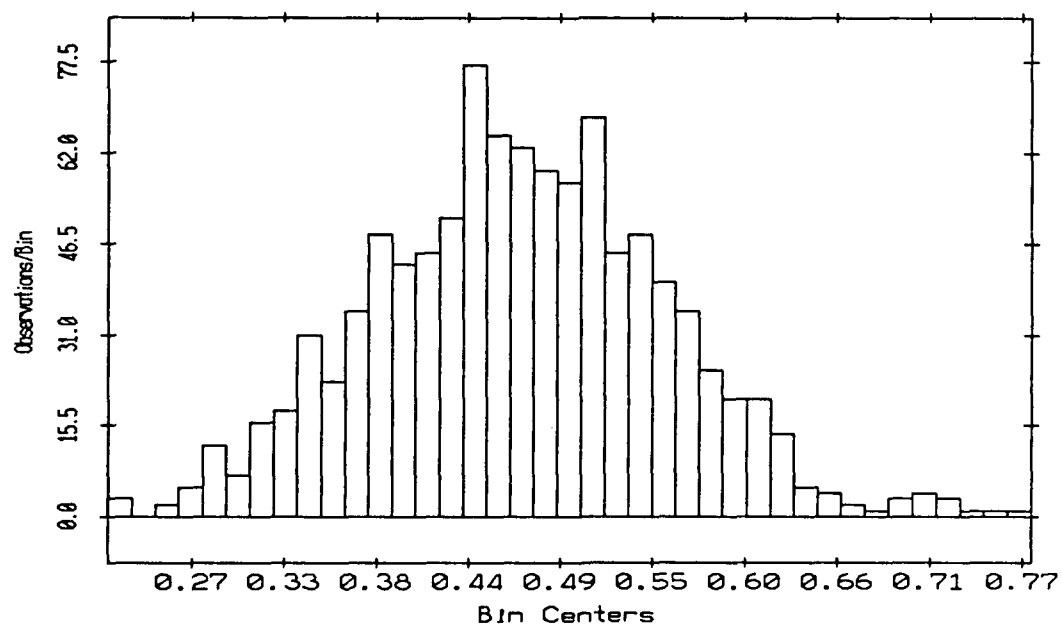


Fig. A.181 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'B5'

## Histogram of B7

Mean=0.18008, standard deviation=0.84607e-01

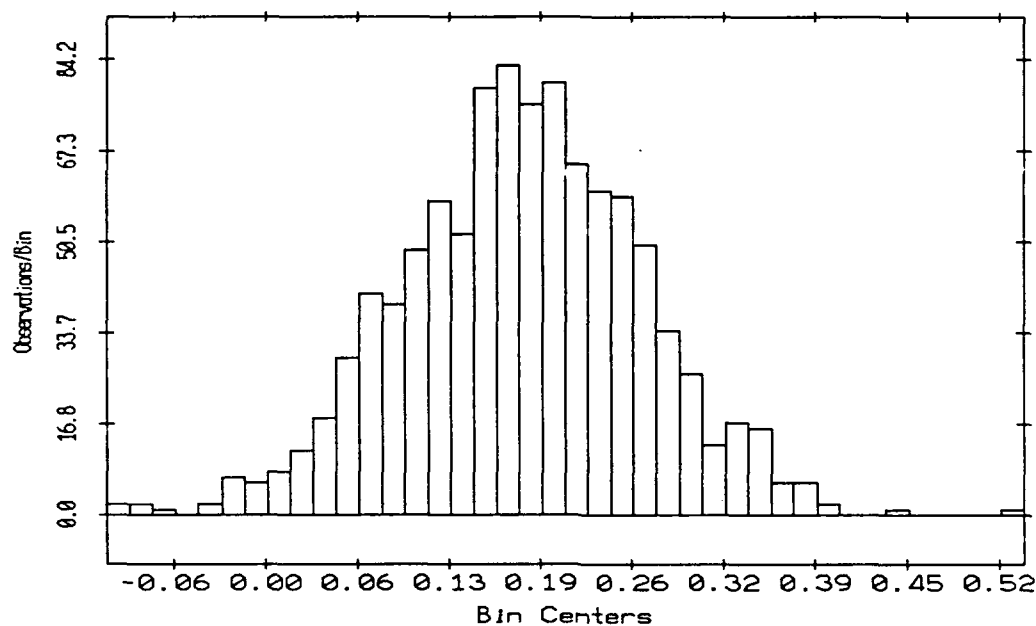


Fig. A.182 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'B8'

## Histogram of B8

Mean=-0.36415, standard deviation=0.85241e-01

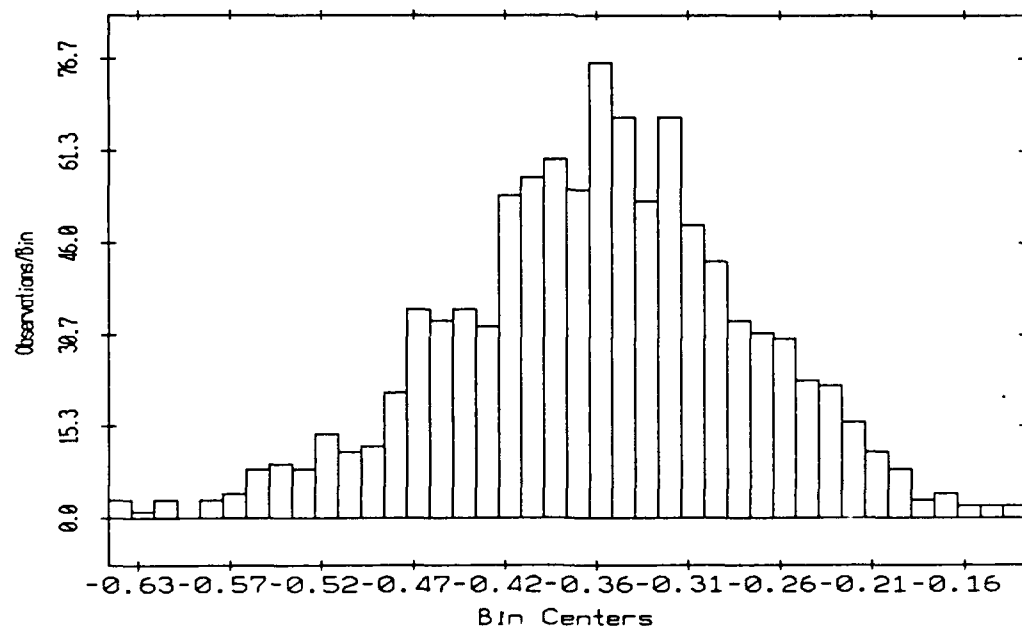


Fig. A.183 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'B7'



## Histogram of B9

Mean=-0.17360, standard deviation=0.68976e-01

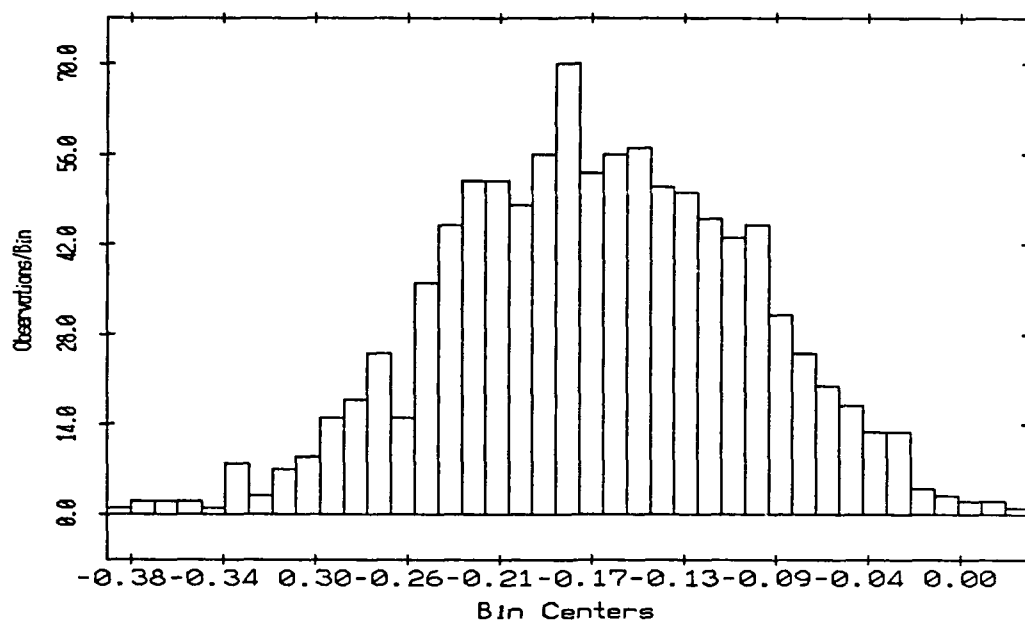


Fig. A.184 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'B10'

## Histogram of B10

Mean=-0.84265e-01, standard deviation=0.66344e-01

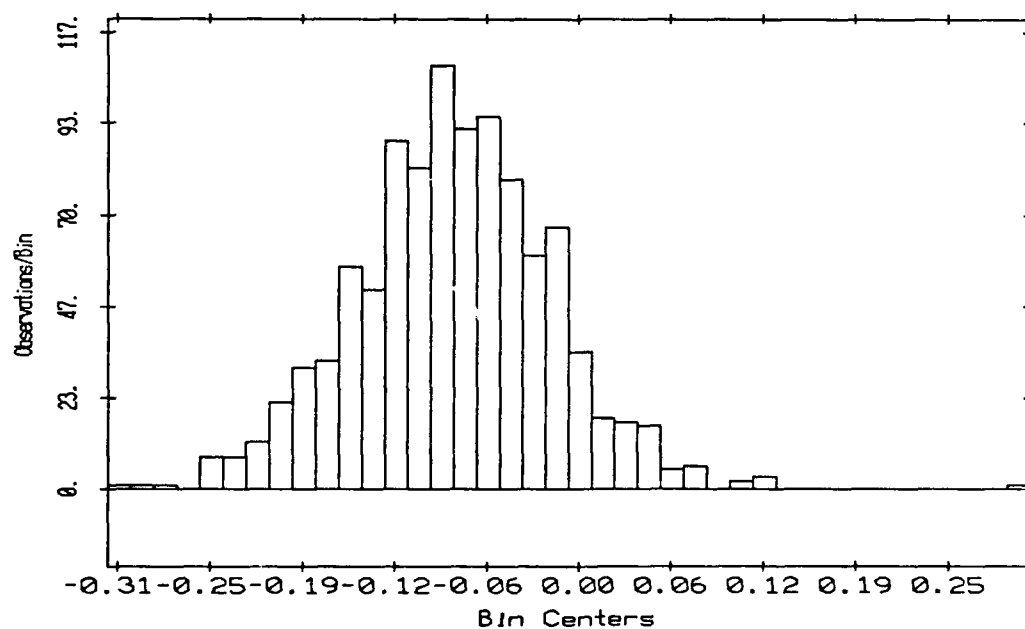


Fig. A.185 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'B9'

## Histogram of B11

Mean=-0.38480e-01, standard deviation=0.57751e-01

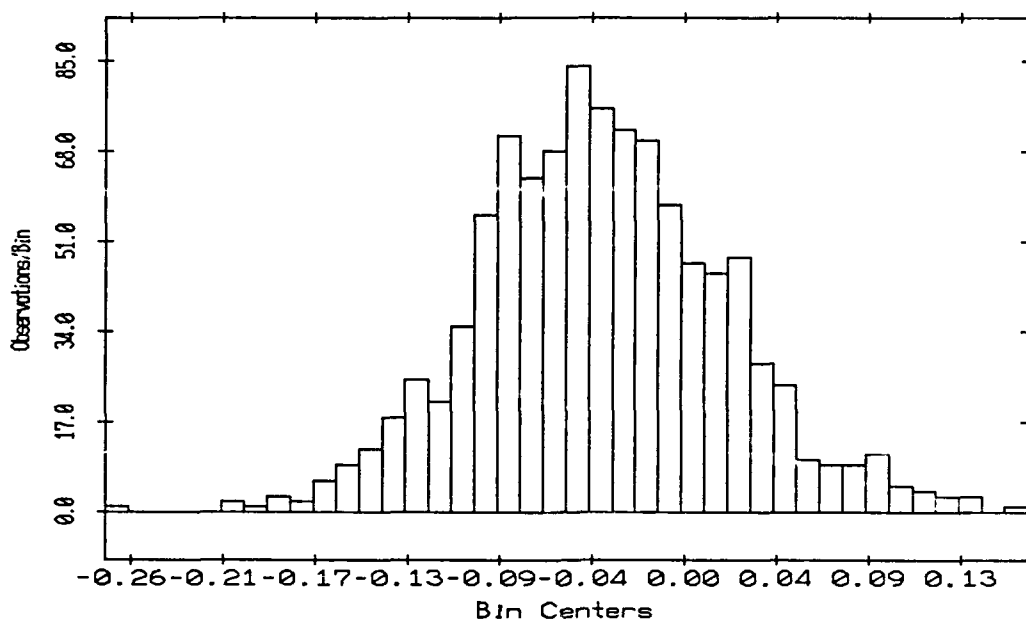


Fig. A.186 : SMC estimation, Nuttall's method, data length 128 points, Histogram of estimated model coefficient 'B12'

## Histogram of B12

Mean=-0.15019e-01, standard deviation=0.54865e-01

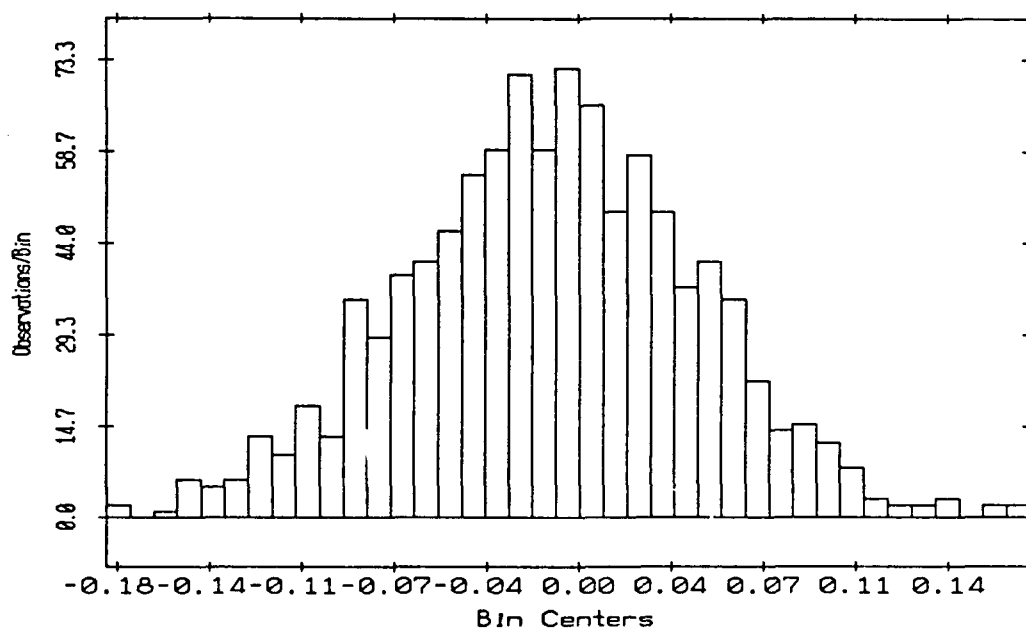


Fig. A.187 : SMC estimation, Nuttall's method, data length 128 points, Histogram of estimated model coefficient 'B11'

## Histogram of B13

Mean=-0.15845e-01, standard deviation=0.56226e-01

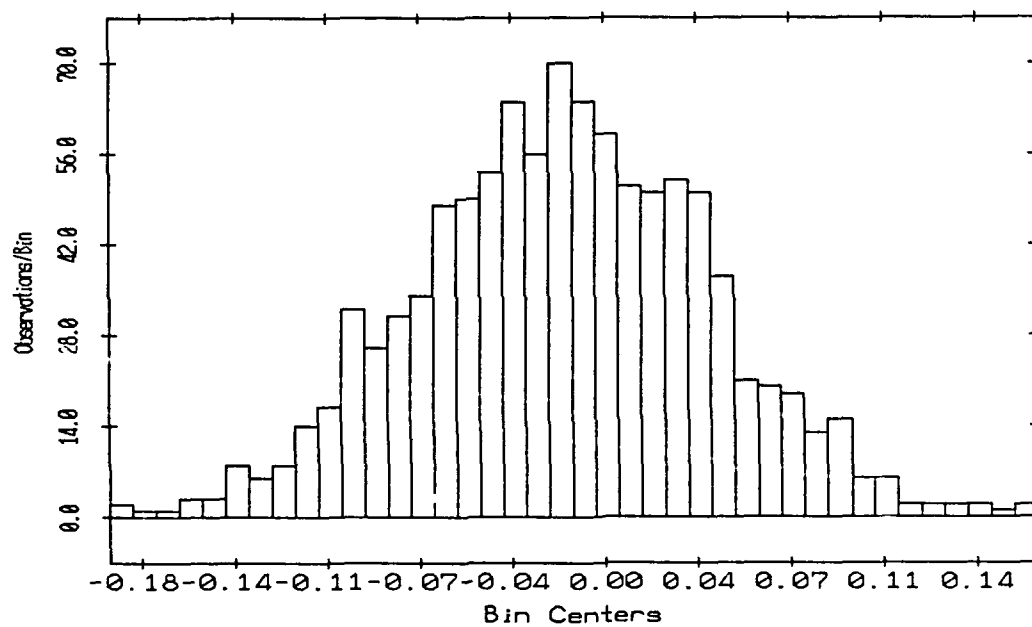


Fig. A.188 : SMC estimation, Nuttall's method, data length 128 points,  
Histogram of estimated model coefficient 'B13'

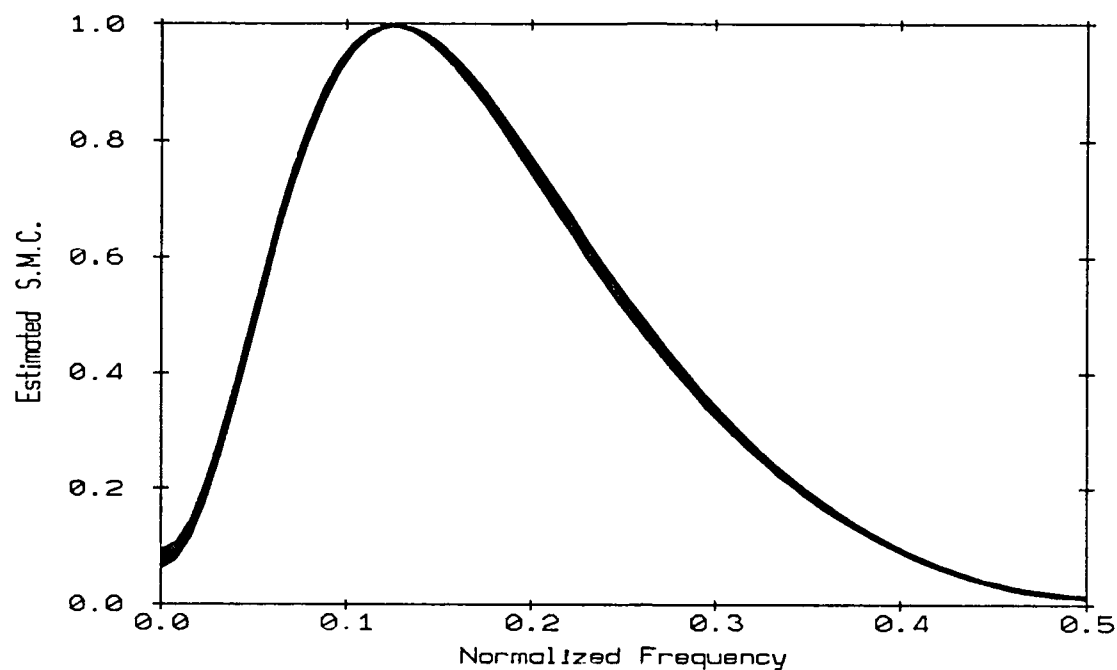


Fig. A.189 : SMC estimation, assumed model order = 1,  
data length 1024 points

SMC estimation, general method, data length 1024 points

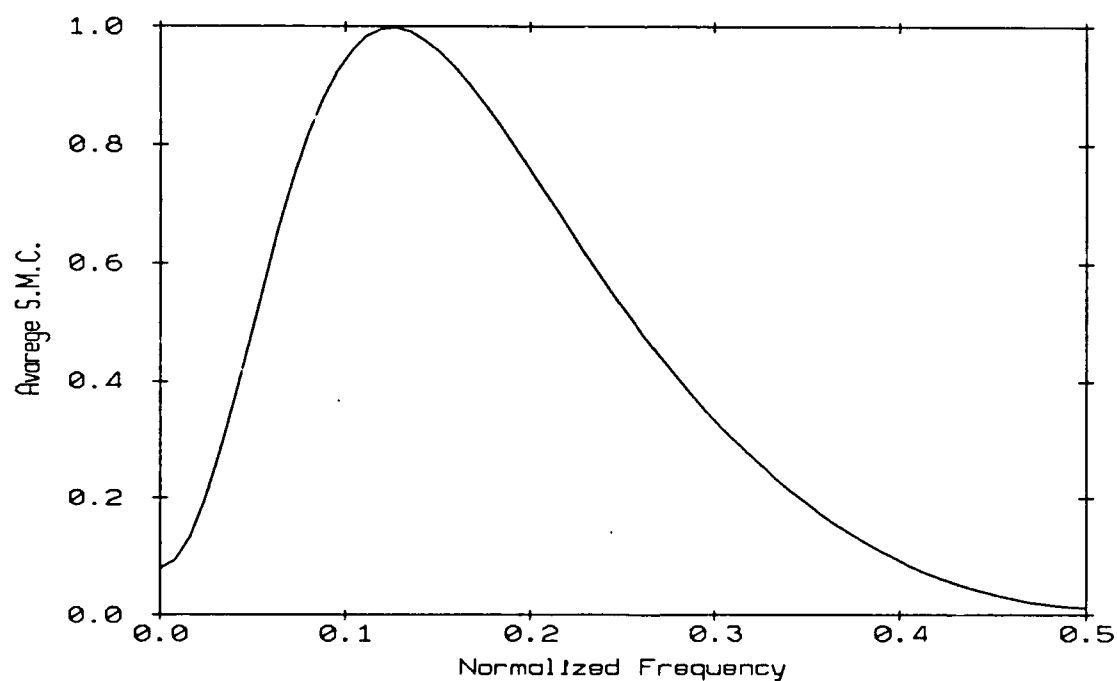


Fig. A.189.1 : SMC estimation, assumed model order = 1,  
data length 1024 points, Average estimated SMC over 100 runs

SMC estimation, general method, data length 1024 points

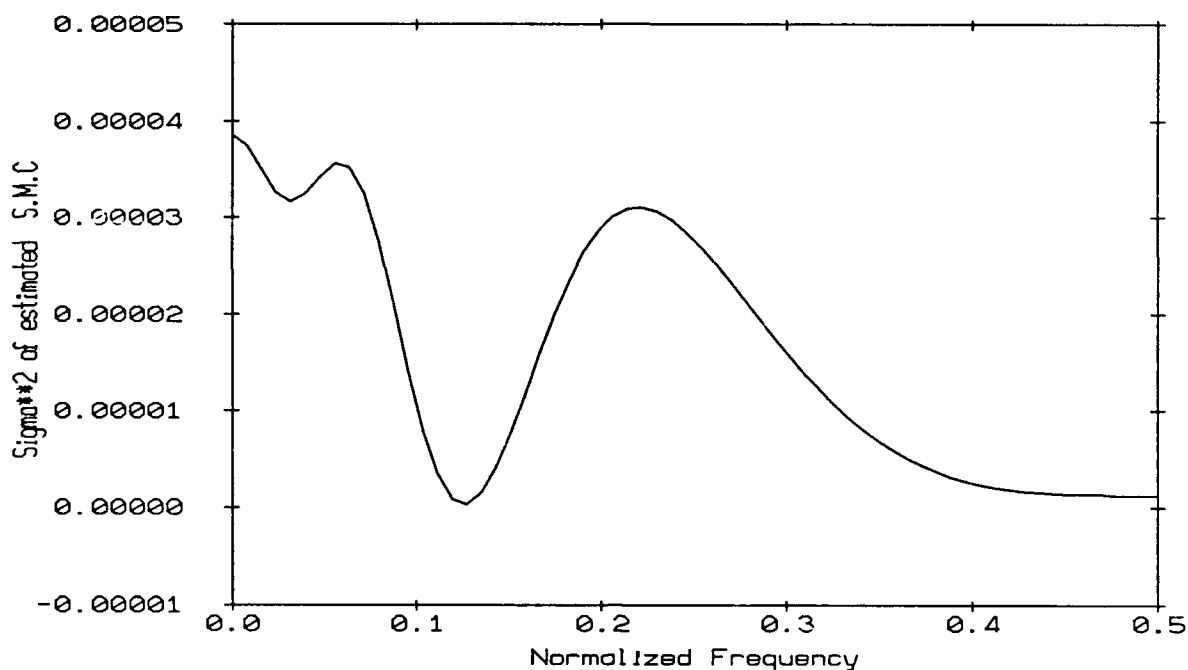


Fig. A.190 : Squared Standard Deviation, of estimated SMC,  
data length 1024 points, 100 runs

SMC estimation, general method, data length 1024 points

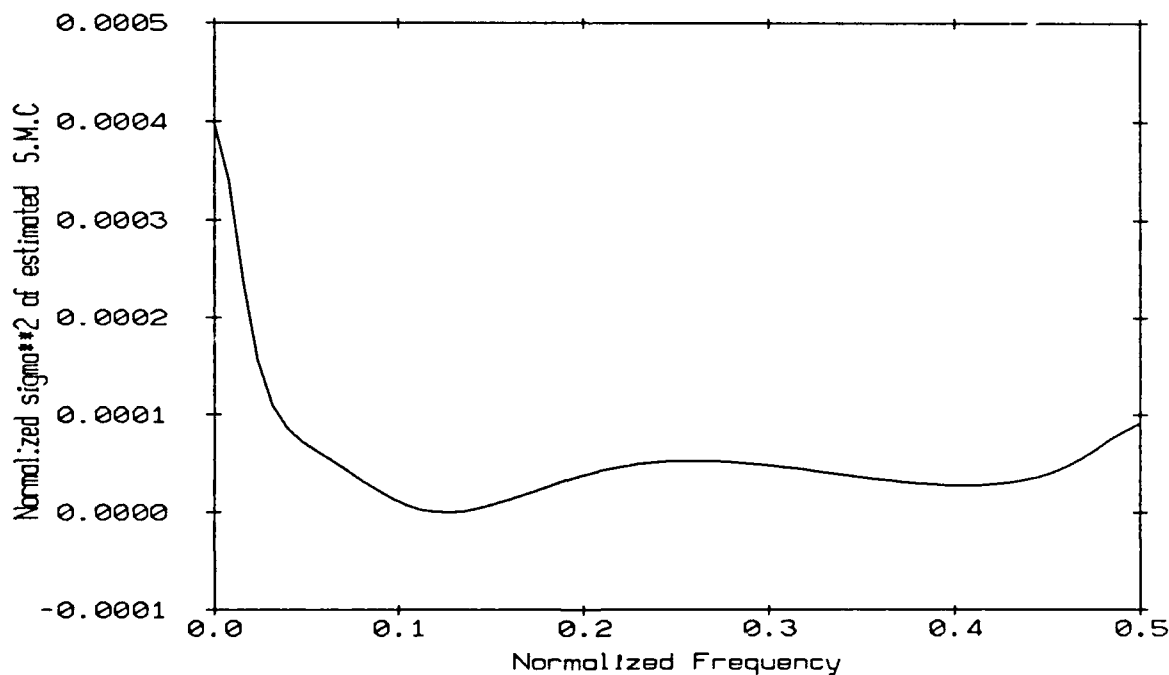


Fig. A.190.1: Normalized Squared Standard Deviation of estimated SMC,  
data length 1024 points, 100 runs

Histogram of Coherency at normalized frequency =  $1/32$

Mean= $0.19527$ , Var= $0.32664e-04$ , assumed model order = 1

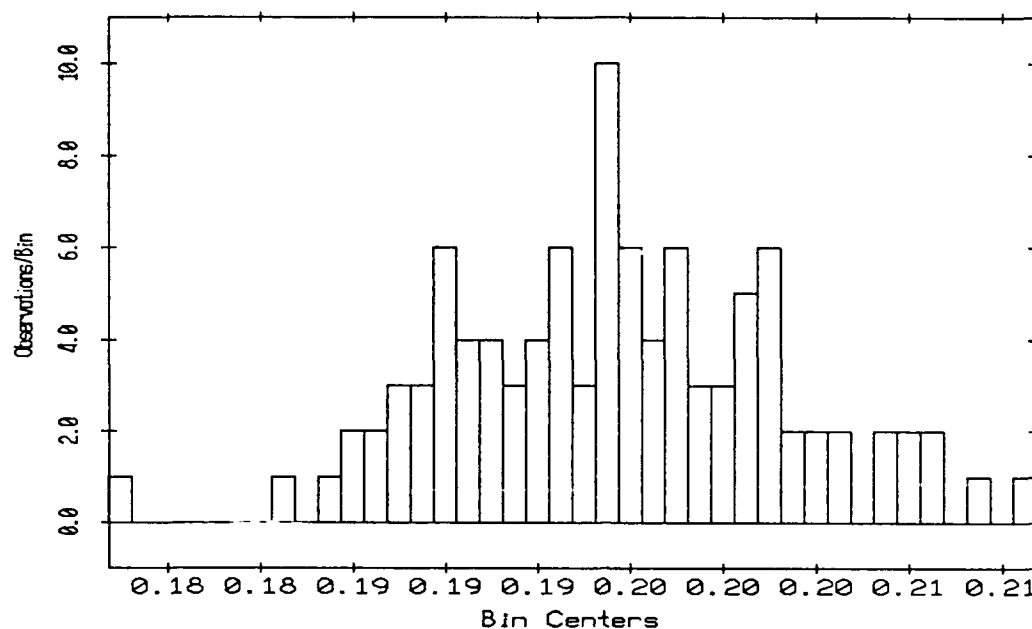


Fig. A.191 : SMC estimation, assumed model order = 1 (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency =  $1/32$

Histogram of Coherency at normalized frequency =  $2/32$

Mean= $0.56245$ , Var= $0.35717e-04$ , assumed model order = 1

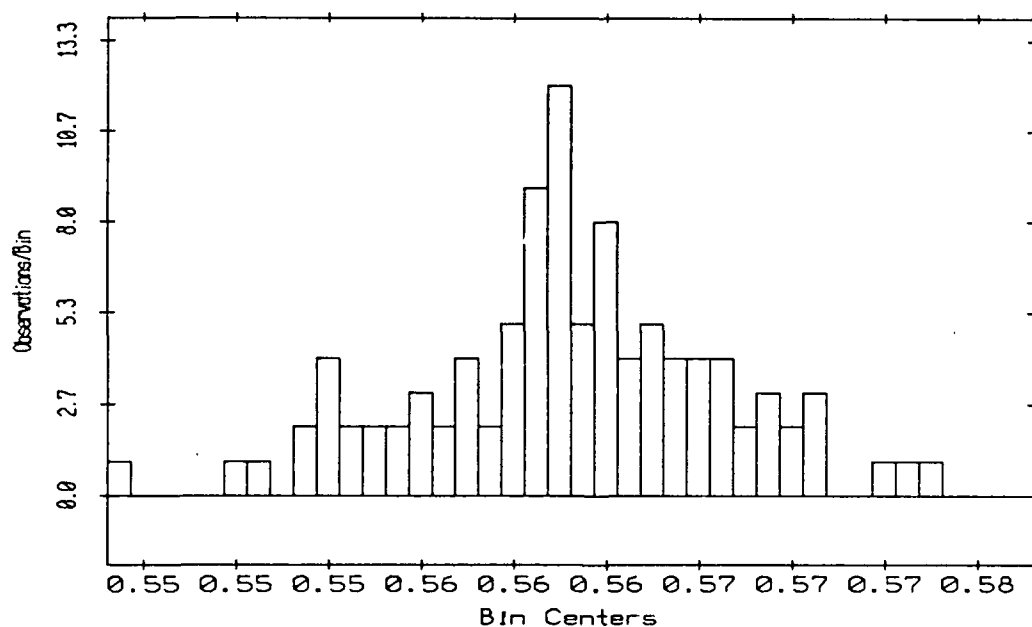


Fig. A.192 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency =  $2/32$

Histogram of Coherency at normalized frequency = 3/32

Mean=0.87603, Var=0.20951e-04, assumed model order = 1

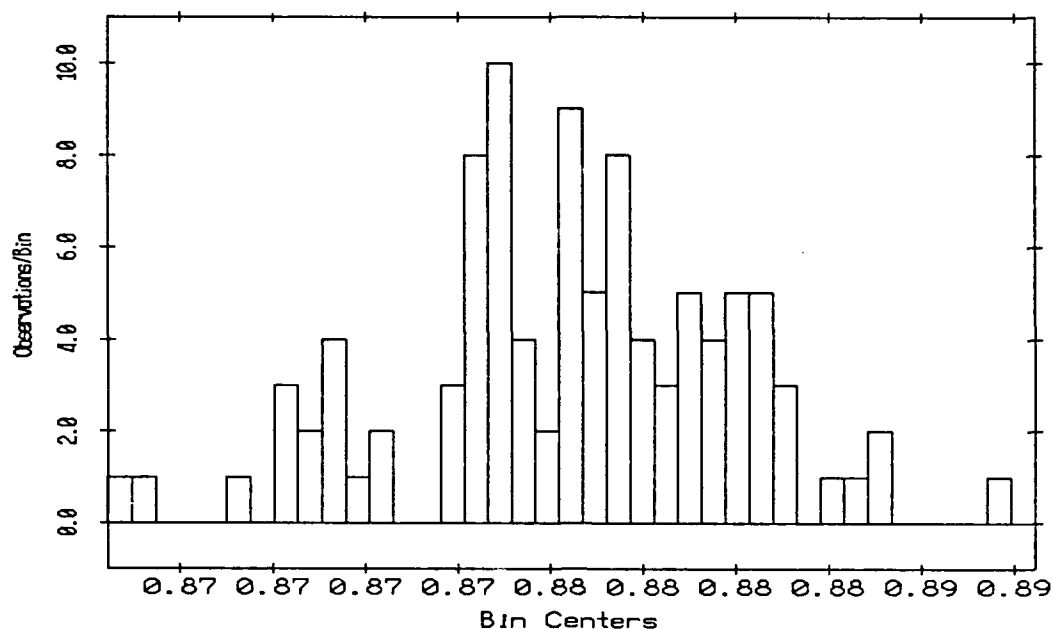


Fig. A.193 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 3/32

Histogram of Coherency at normalized frequency = 4/32

Mean=0.99616, Var=0.89094e-06, assumed model order = 1

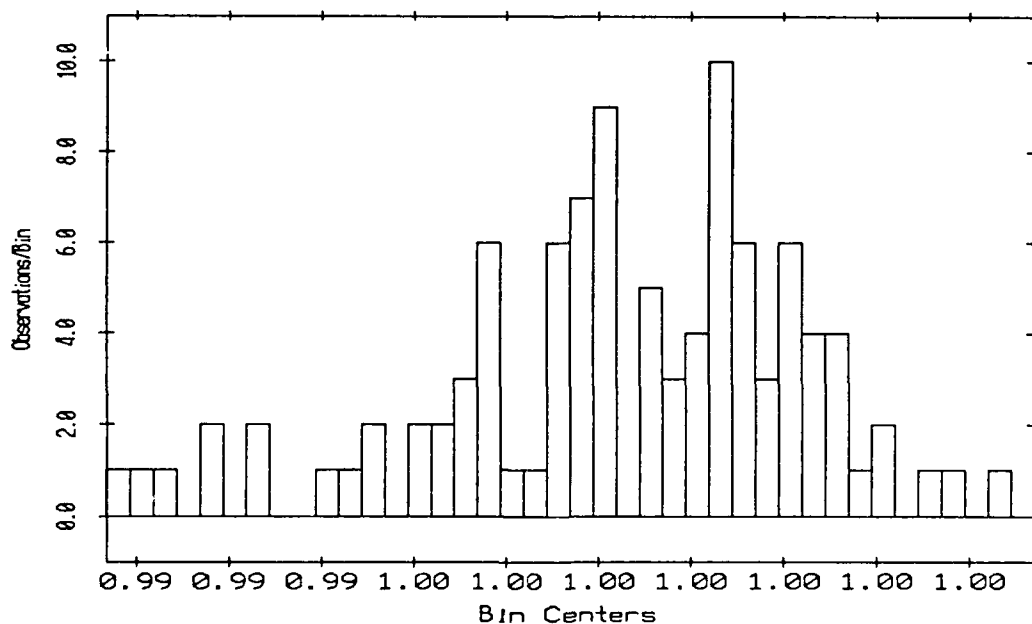


Fig. A.194 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 4/32

Histogram of Coherency at normalized frequency =  $5/32$

Mean=0.95822, Var=0.77998e-05, assumed model order = 1

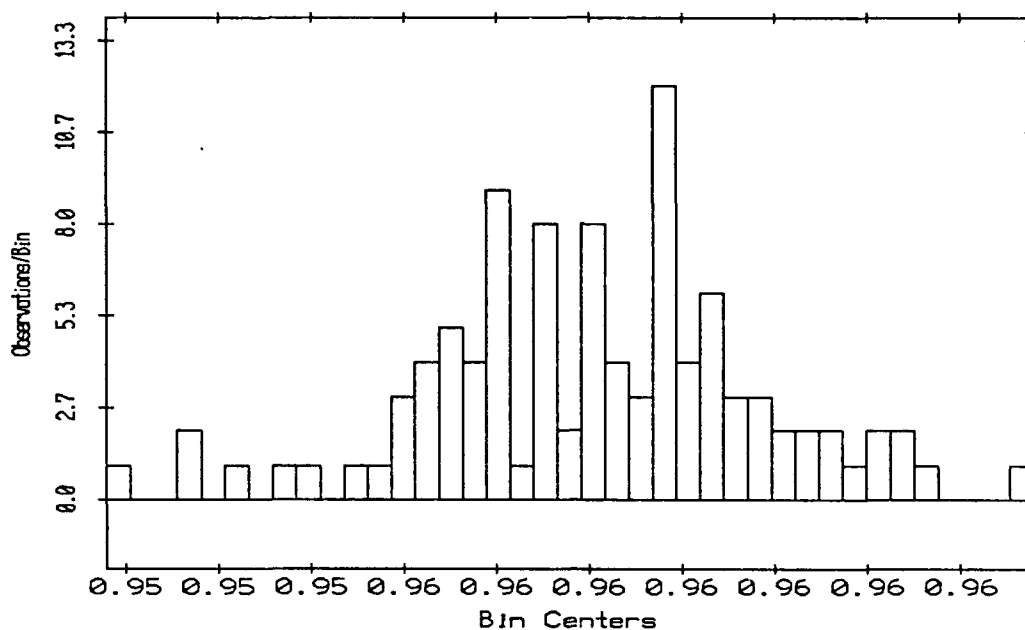


Fig. A.195 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency =  $5/32$

Histogram of Coherency at normalized frequency =  $6/32$

Mean=0.83740, Var=0.23607e-04, assumed model order = 1

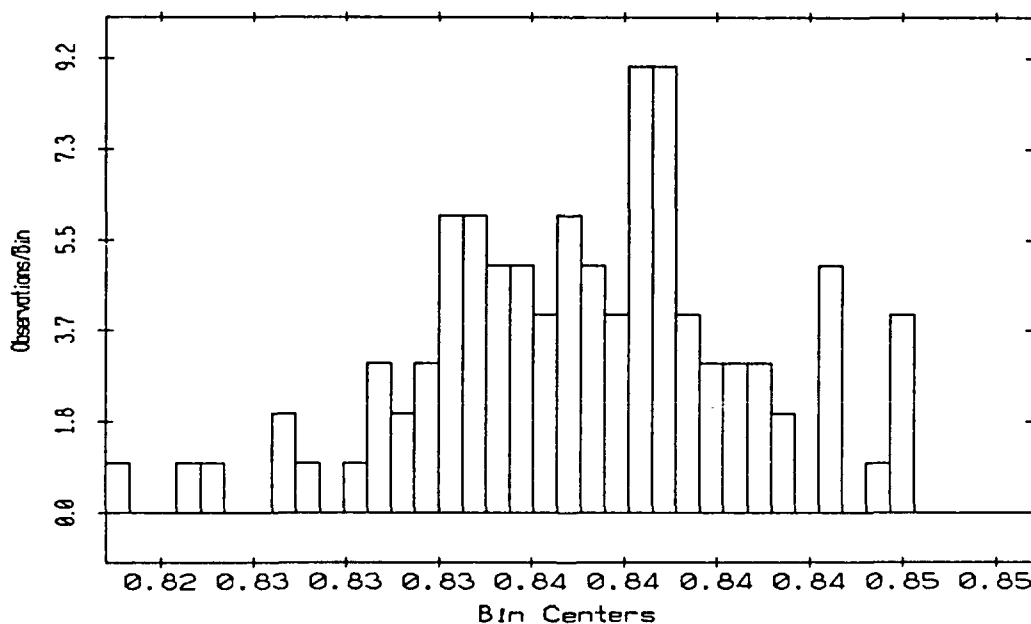


Fig. A.196 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency =  $6/32$



## Histogram of Coherency at normalized frequency = 7/32

Mean=0.68890, Var=0.30908e-04, assumed model order = 1

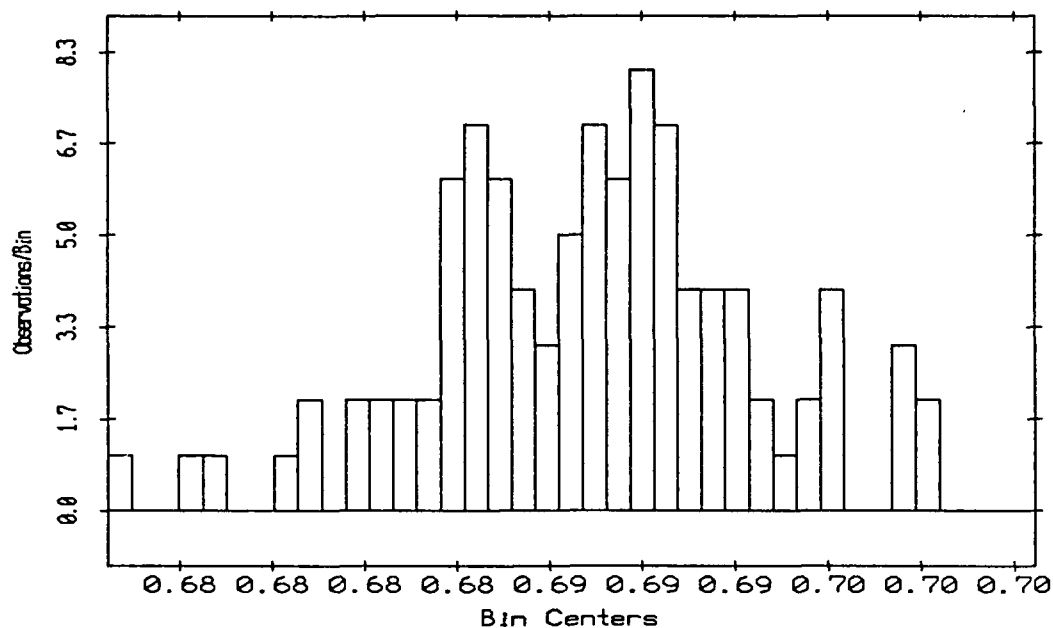


Fig. A.197 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 7/32

## Histogram of Coherency at normalized frequency = 8/32

Mean=0.54264, Var=0.28481e-04, assumed model order = 1

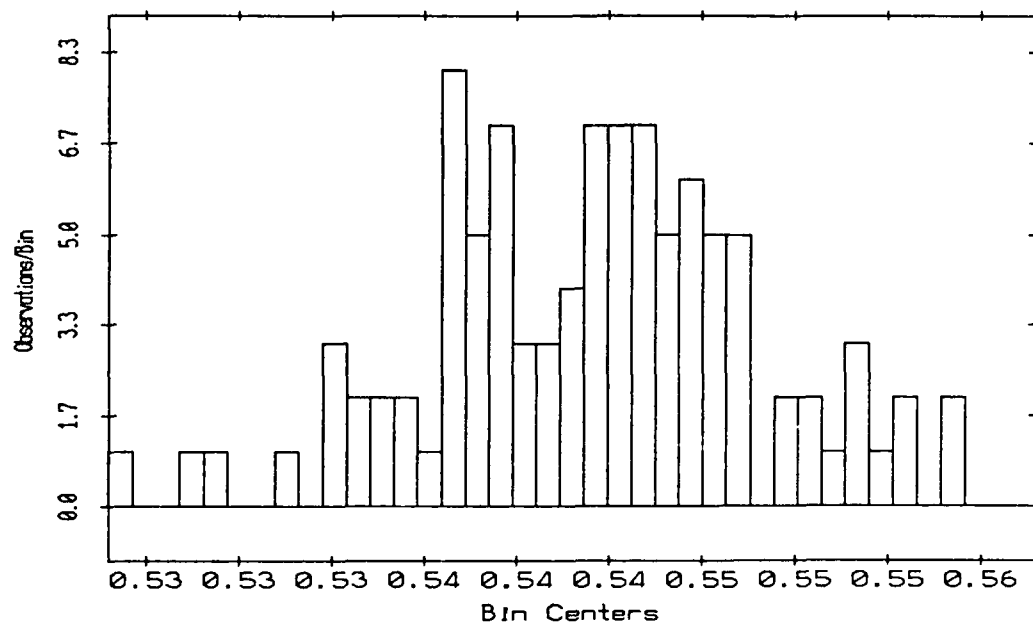


Fig. A.198 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 8/32

Histogram of Coherency at normalized frequency = 9/32

Mean=0.41192, Var=0.21366e-04, assumed model order = 1

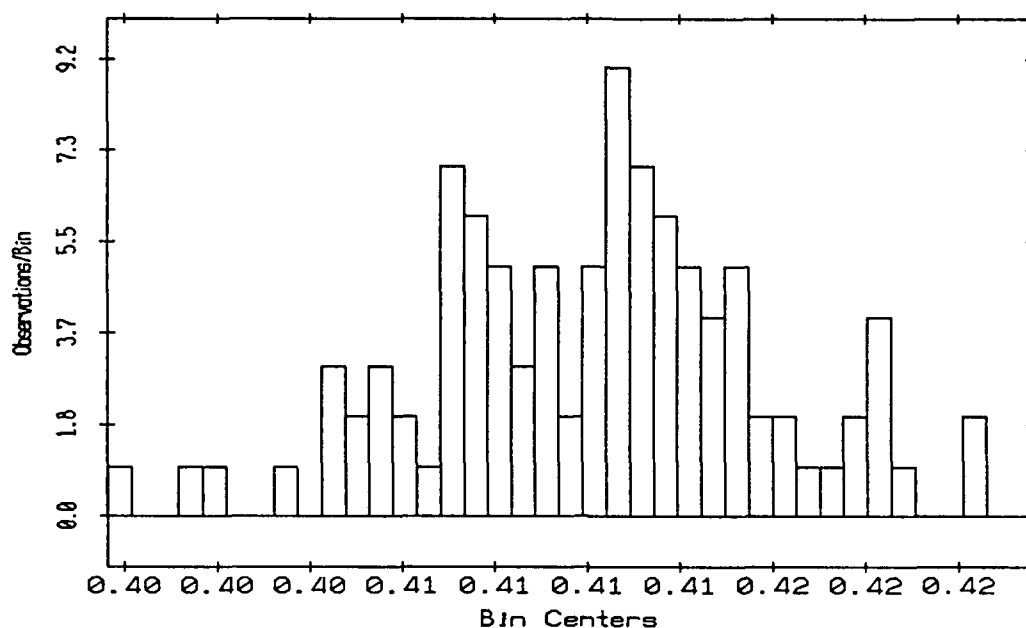


Fig. A.199 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 9/32

Histogram of Coherency at normalized frequency = 10/32

Mean=0.30117, Var=0.13861e-04, assumed model order = 1

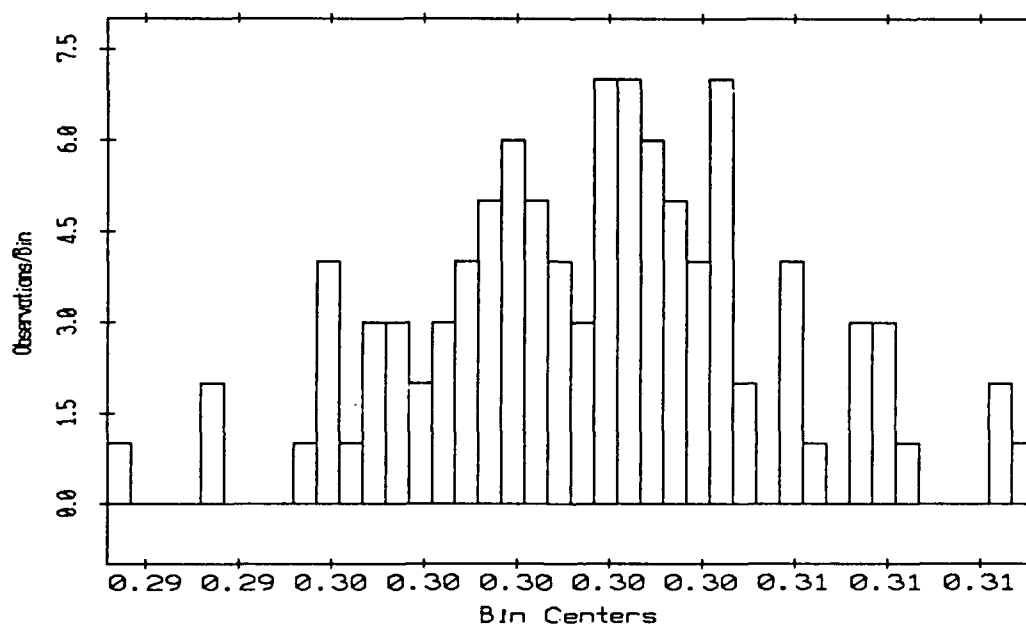


Fig. A.200 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 10/32

## Histogram of Coherency at normalized frequency = 11/32

Mean=0.21074, Var=0.80343e-05, assumed model order = 1

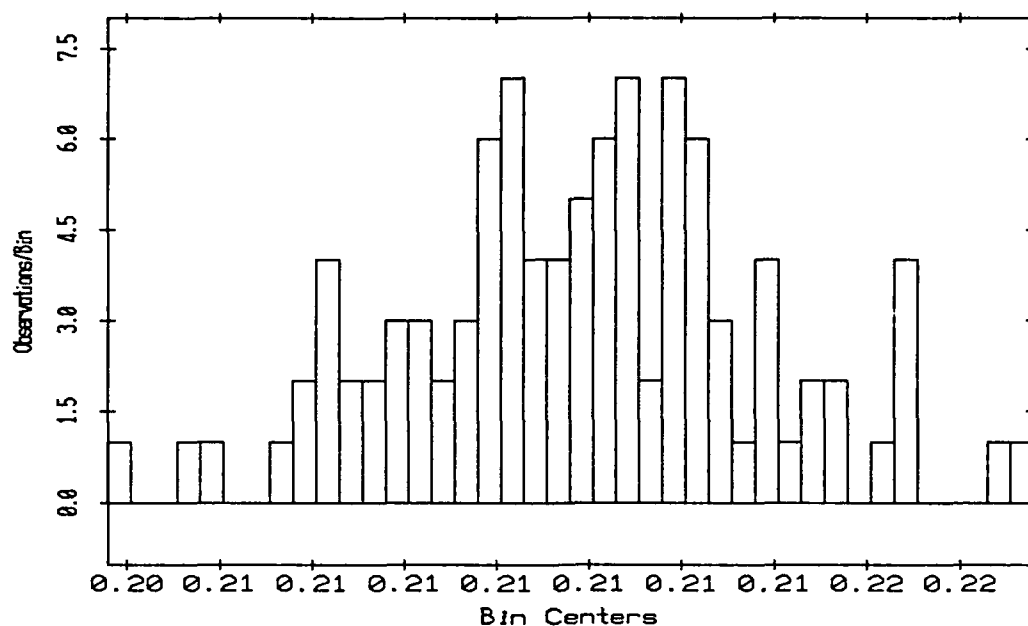


Fig. A.201 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 11/32

## Histogram of Coherency at normalized frequency = 12/32

Mean=0.13935, Var=0.43223e-05, assumed model order = 1

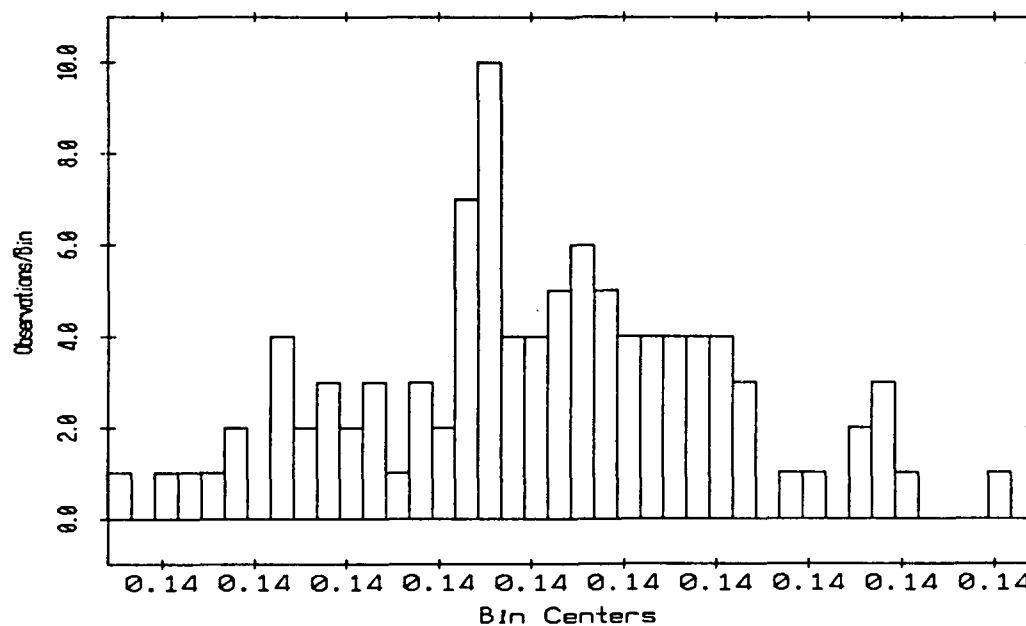


Fig. A.202 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 12/32

## Histogram of Coherency at normalized frequency = 13/32

Mean=0.85290e-01, Var=0.23633e-05, assumed model order = 1

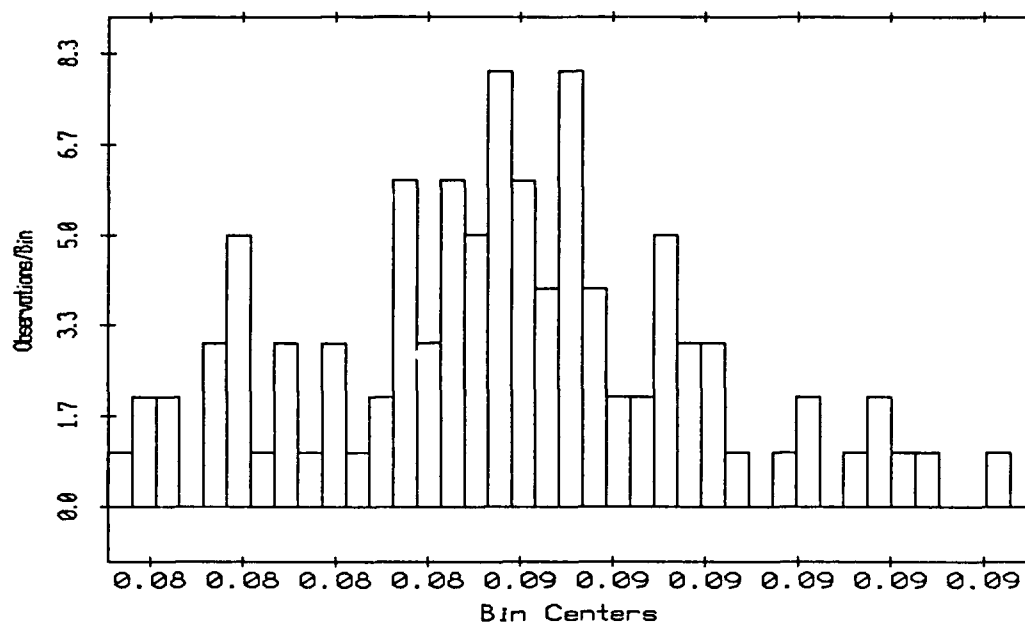


Fig. A.203 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 13/32

## Histogram of Coherency at normalized frequency = 14/32

Mean=0.46992e-01, Var=0.15520e-05, assumed model order = 1

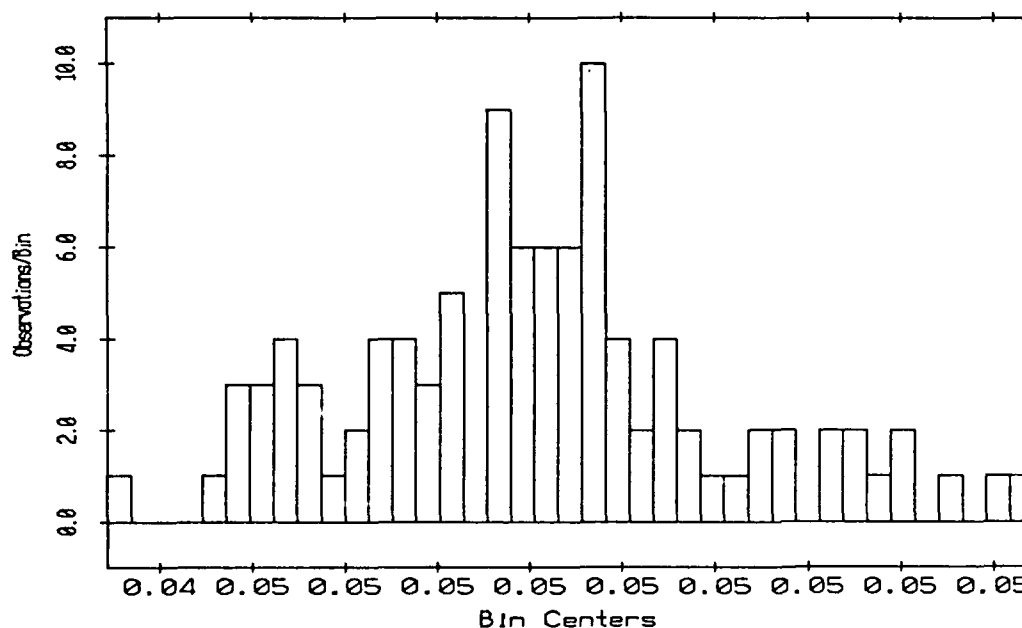


Fig. A.204 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 14/32

## Histogram of Coherency at normalized frequency = 15/32

Mean=0.23222e-01, Var=0.13285e-05, assumed model order = 1

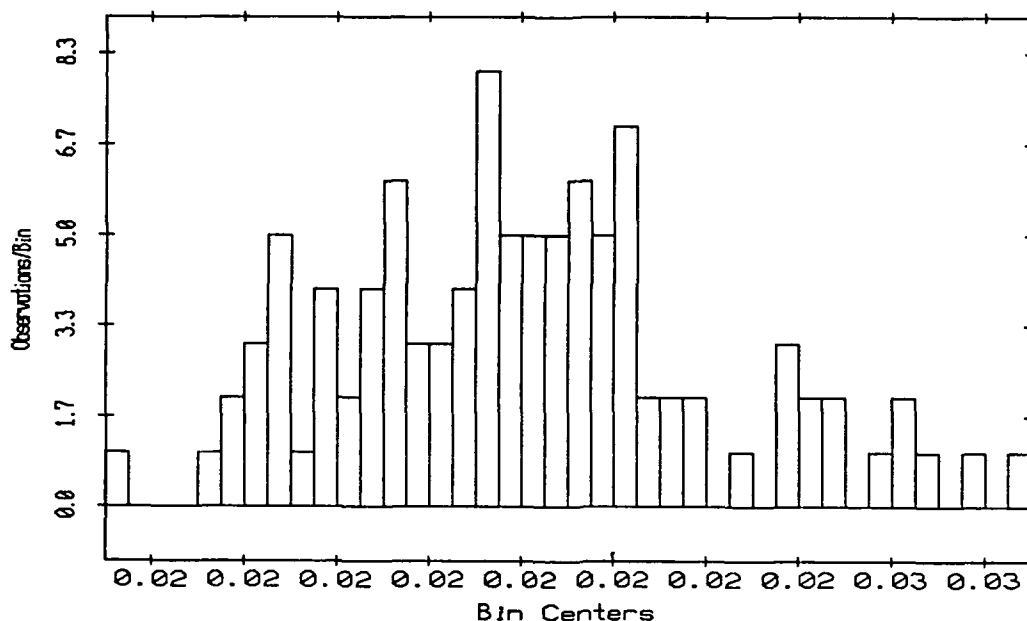


Fig. A.205 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 15/32

## Histogram of Coherency at normalized frequency = 16/32

Mean=0.13179e-01, Var=0.13042e-05, assumed model order = 1

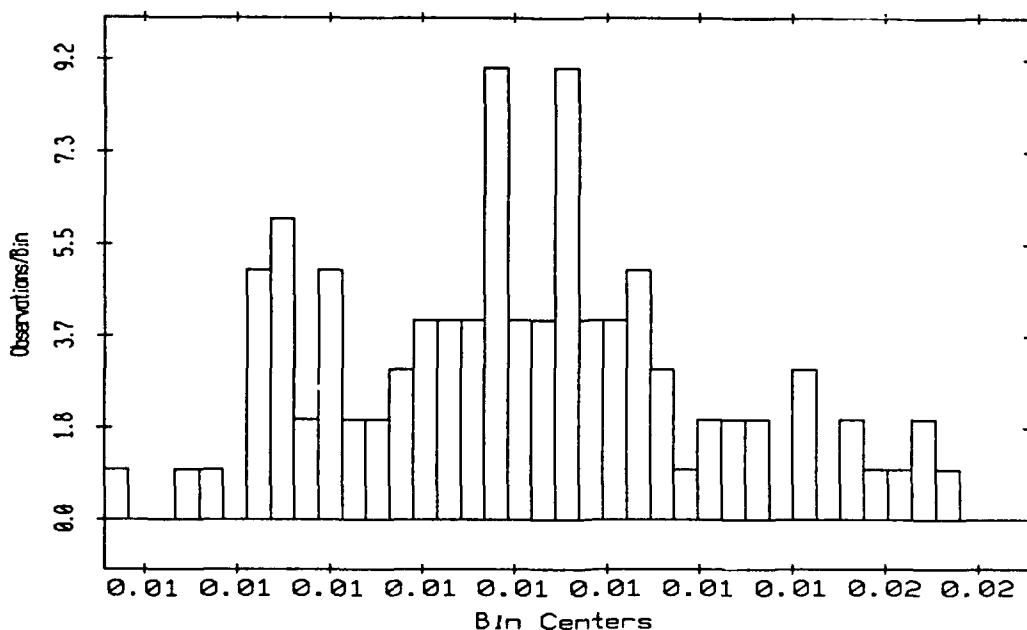


Fig. A.206 : SMC estimation, assumed model order = 1, (model 1), data length 1024 points  
Histogram of estimated SMC at normalized frequency = 16/32

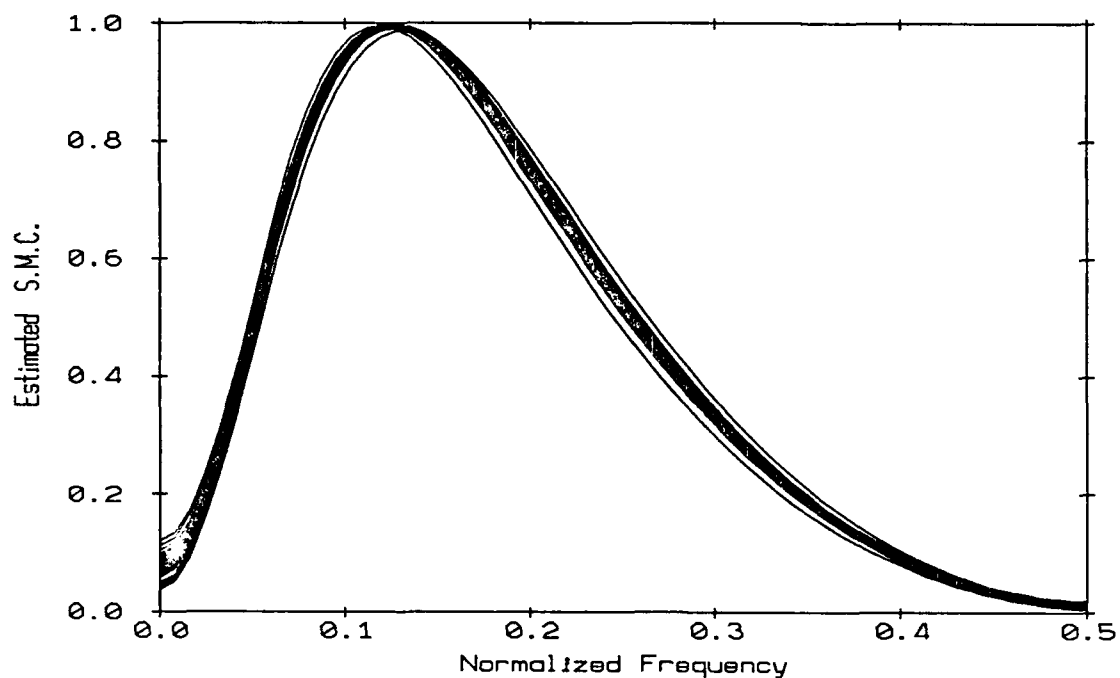


Fig. A.207 : SMC estimation, assumed model order = 1,  
data length 256 points

SMC estimation, general method, data length 256 points

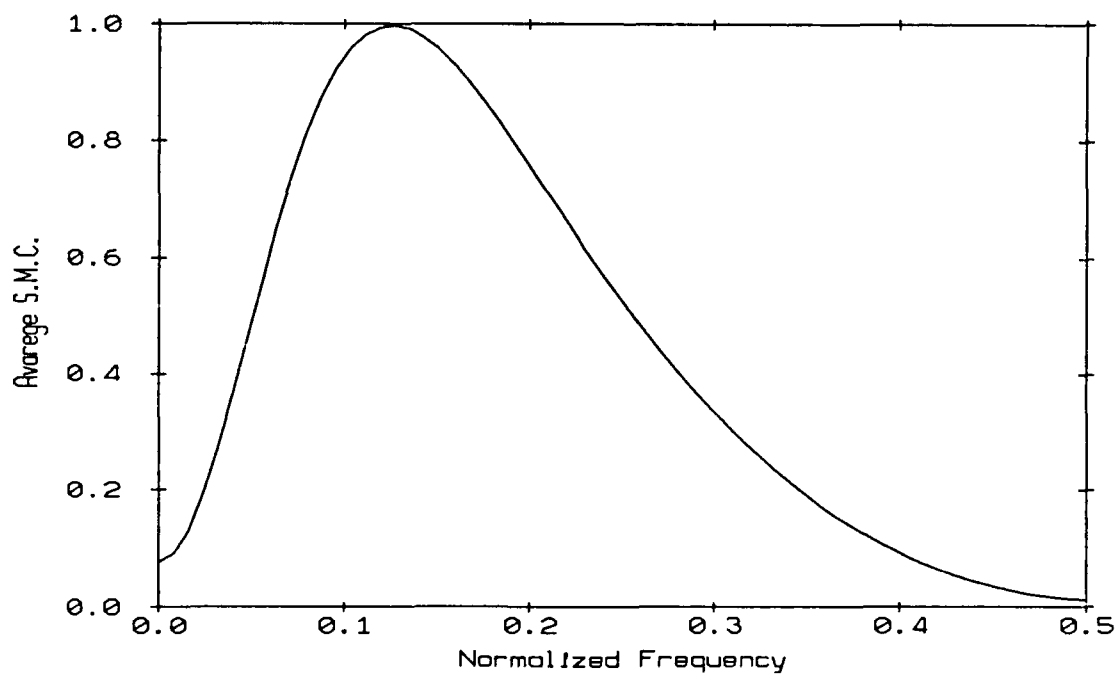


Fig. A.207.1 : SMC estimation, assumed model order = 1,  
data length 256 points, Average estimated SMC over 100 runs

SMC estimation, general method, data length 256 points

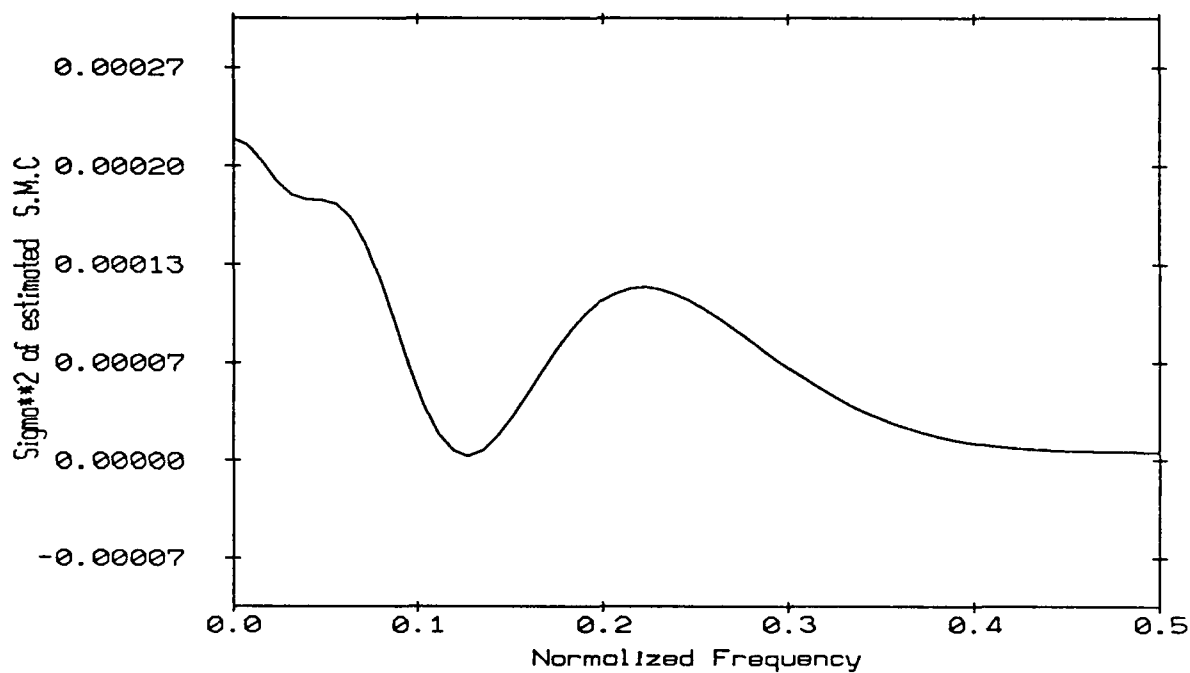


Fig. A.208 : Squared Standard Deviation, of estimated SMC,  
data length 256 points, 100 runs

SMC estimation, general method, data length 256 points

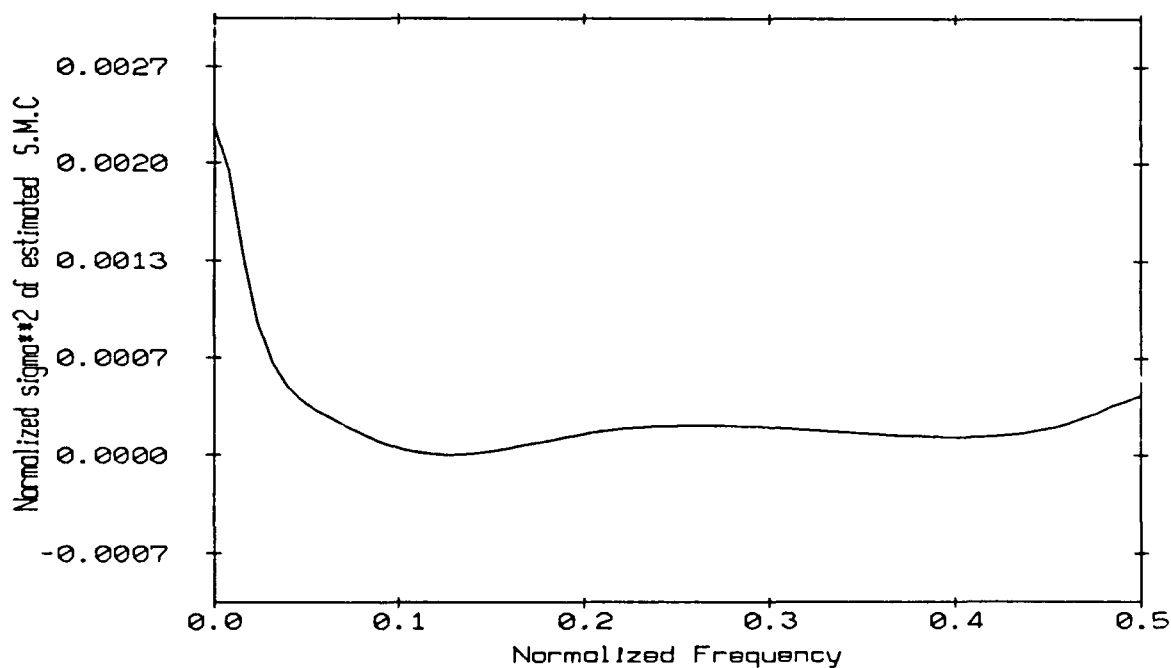


Fig. A.208.1: Normalized Squared Standard Deviation of estimated SMC,  
data length 256 points, 100 runs

Histogram of Coherency at normalized frequency =  $1/32$ 

Mean=0.19260, Var=0.18989e-03, assumed model order = 1

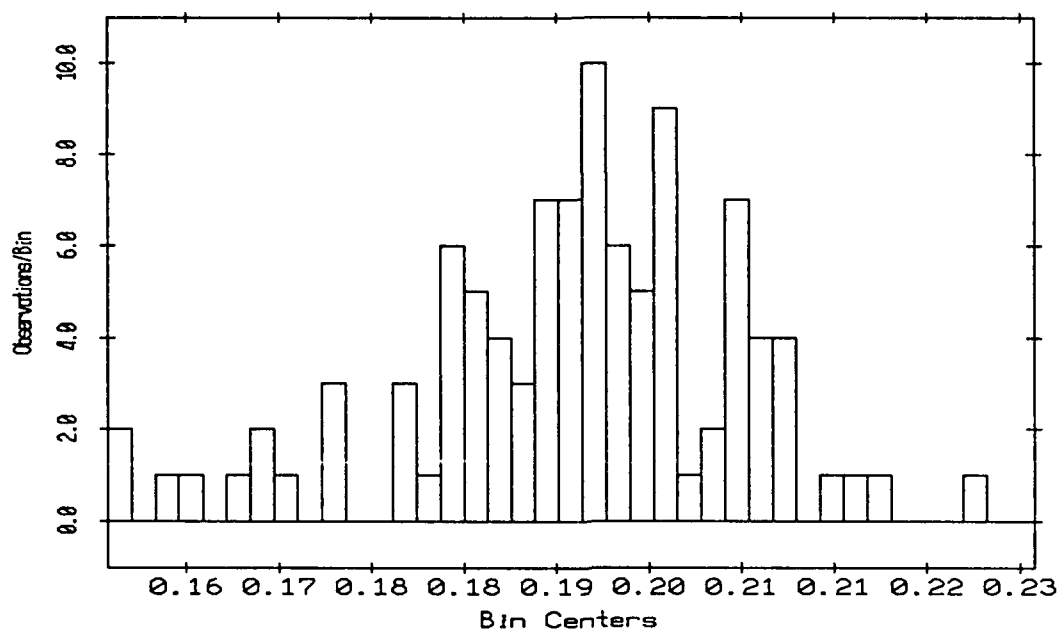


Fig. A.209 : SMC estimation, assumed model order = 1, (model 1), data length 256 points,  
Histogram of estimated SMC at normalized frequency =  $1/32$

Histogram of Coherency at normalized frequency =  $2/32$ 

Mean=0.56000, Var=0.17464e-03, assumed model order = 1

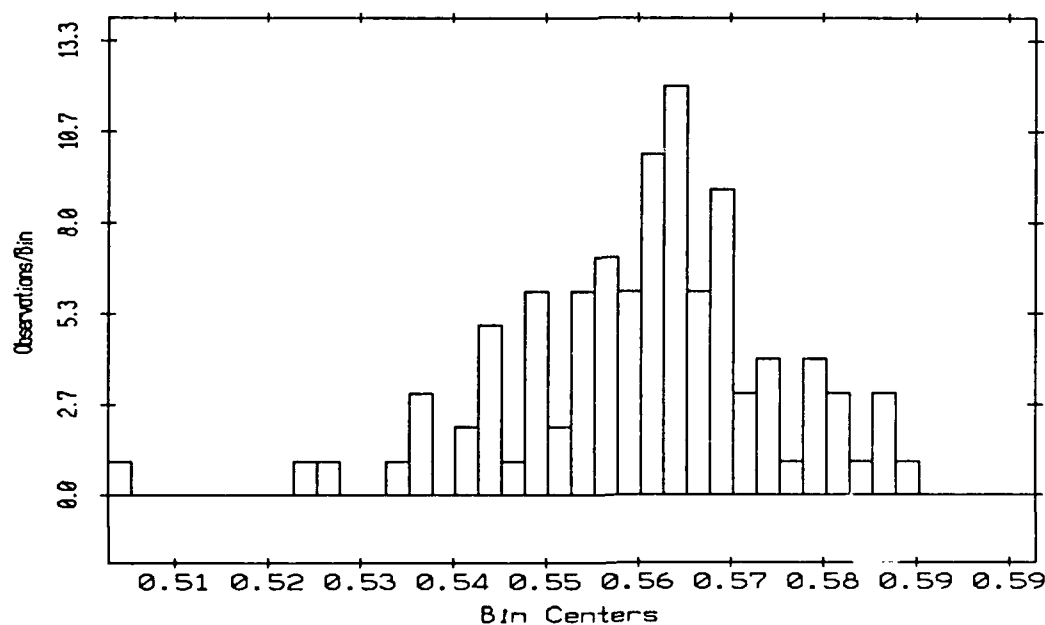


Fig. A.210 : SMC estimation, assumed model order = 1, (model 1), data length 256 points,  
Histogram of estimated SMC at normalized frequency =  $2/32$



## Histogram of Coherency at normalized frequency = 3/32

Mean=0.87412, Var=0.93503e-04, assumed model order = 1

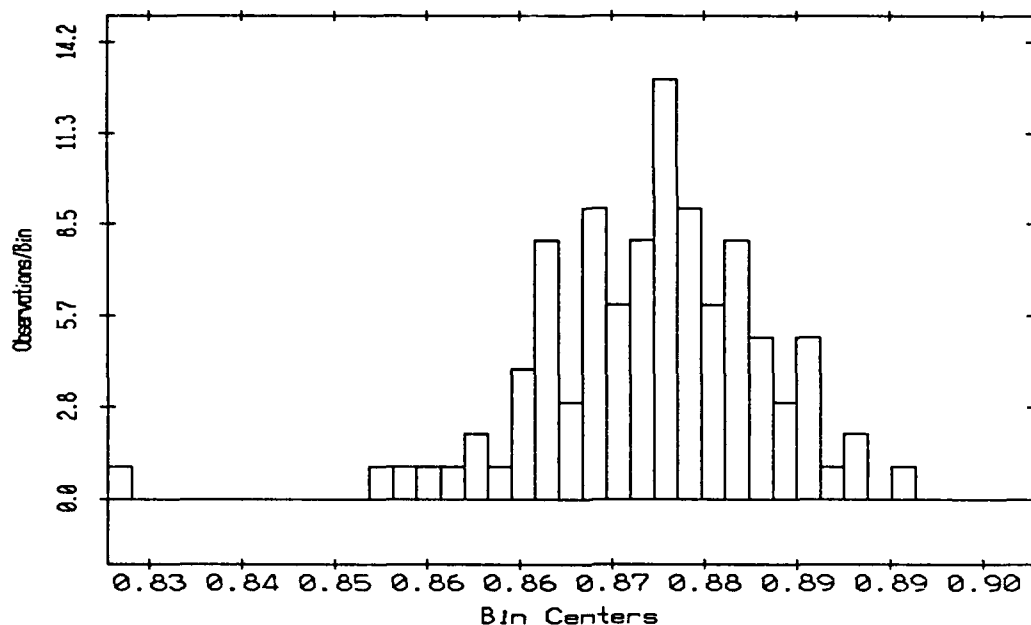


Fig. A.211 : SMC estimation, assumed model order = 1, (model 1), data length 256 points,  
Histogram of estimated SMC at normalized frequency = 3/32

## Histogram of Coherency at normalized frequency = 4/32

Mean=0.99501, Var=0.68641e-05, assumed model order = 1

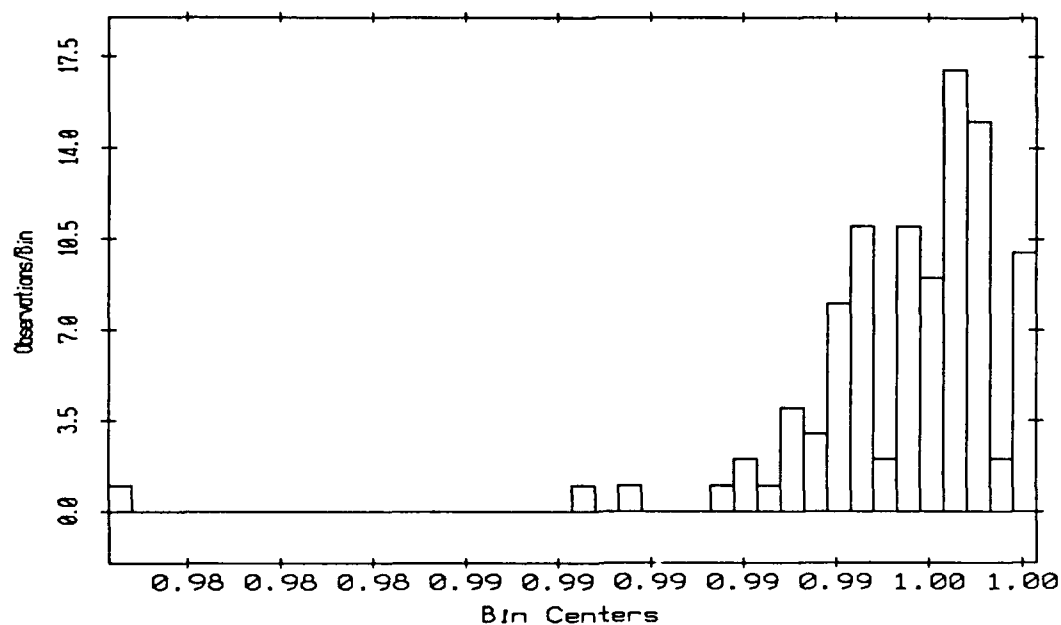


Fig. A.212 : SMC estimation, assumed model order = 1, (model 1), data length 256 points,  
Histogram of estimated SMC at normalized frequency = 4/32

## Histogram of Coherency at normalized frequency = 5/32

Mean=0.95776, Var=0.29676e-04, assumed model order = 1

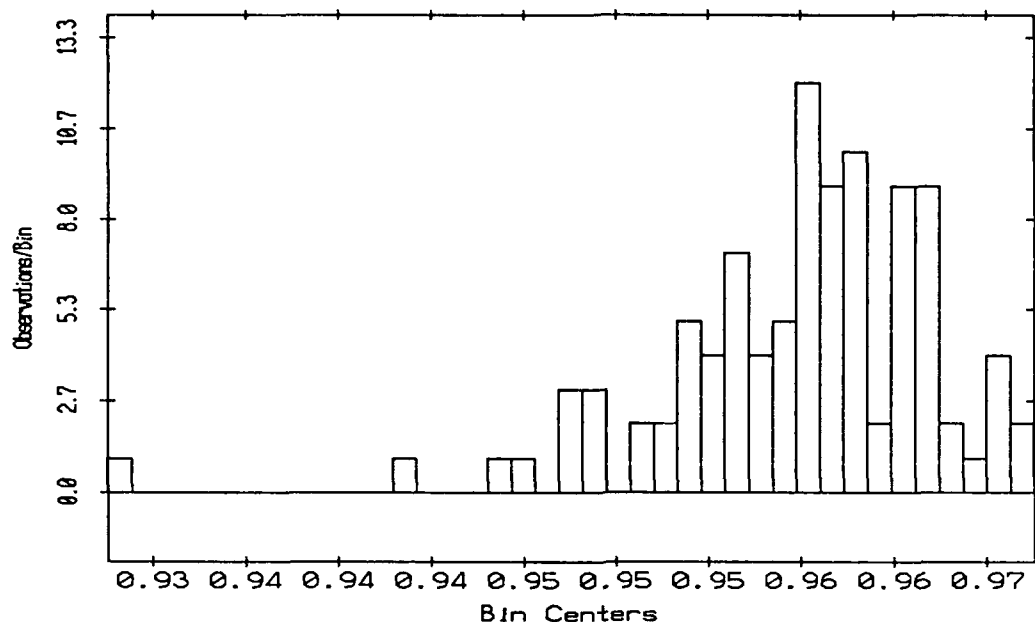


Fig. A.213 : SMC estimation, assumed model order = 1, (model 1), data length 256 points,  
Histogram of estimated SMC at normalized frequency = 5/32

## Histogram of Coherency at normalized frequency = 6/32

Mean=0.83742, Var=0.88927e-04, assumed model order = 1

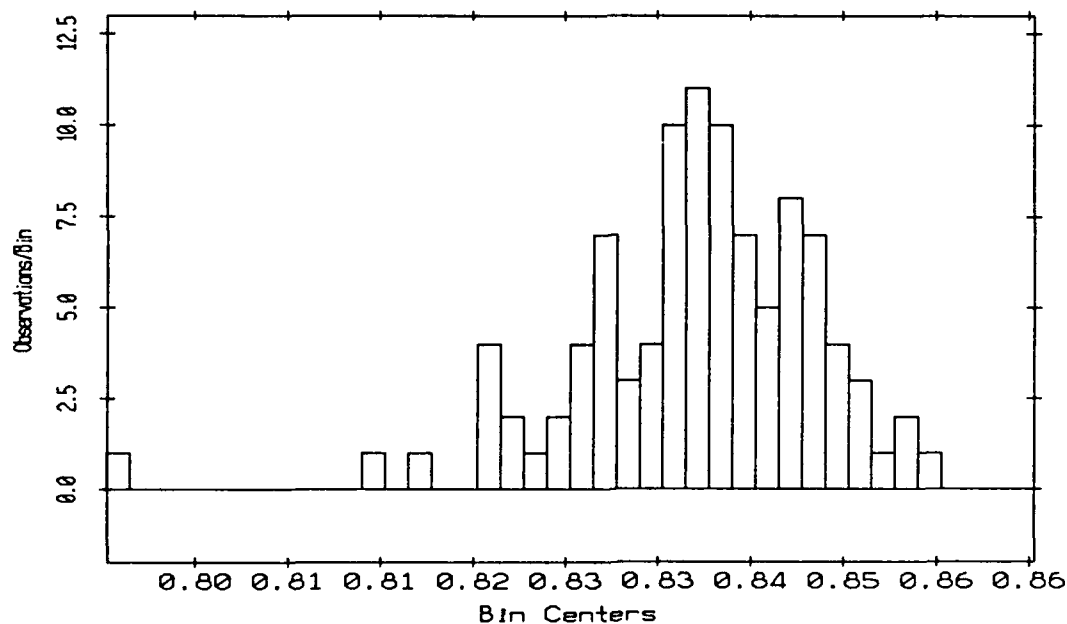


Fig. A.214 : SMC estimation, assumed model order = 1, (model 1), data length 256 points,  
Histogram of estimated SMC at normalized frequency = 6/32

# Histogram of Coherency at normalized frequency = 7/32

Mean=0.68919, Var=0.11725e-03, assumed model order = 1

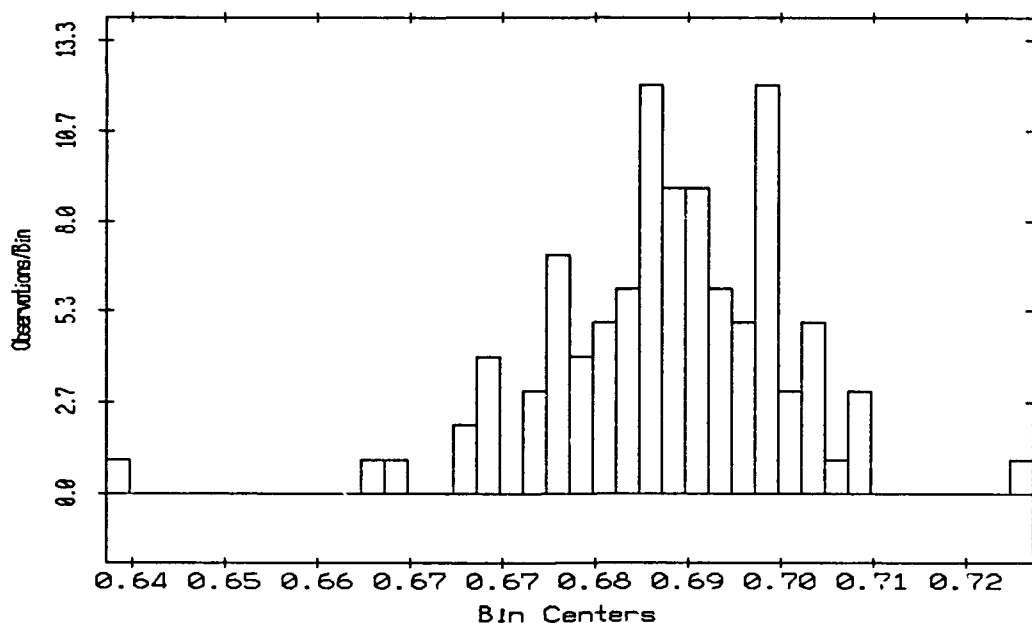


Fig. A.215 : SMC estimation, assumed model order = 1, (model 1), data length 256 points, Histogram of estimated SMC at normalized frequency = 7/32

# Histogram of Coherency at normalized frequency = 8/32

Mean=0.54305, Var=0.10916e-03, assumed model order = 1

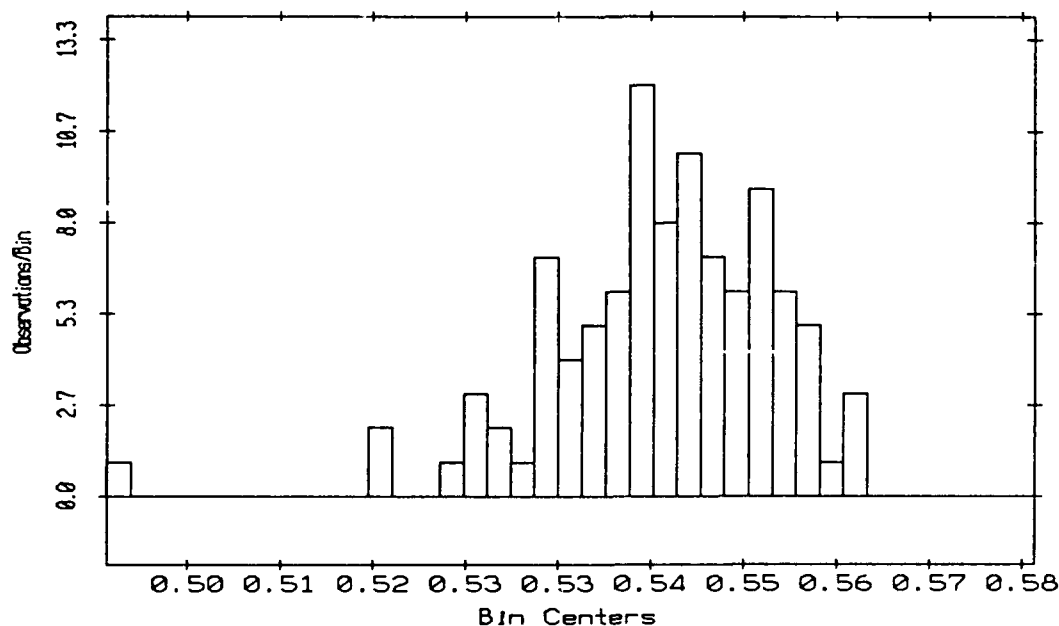


Fig. A.216 : SMC estimation, assumed model order = 1, (model 1), data length 256 points, Histogram of estimated SMC at normalized frequency = 8/32

Histogram of Coherency at normalized frequency = 9/32

Mean=0.41233, Var=0.83099e-04, assumed model order = 1

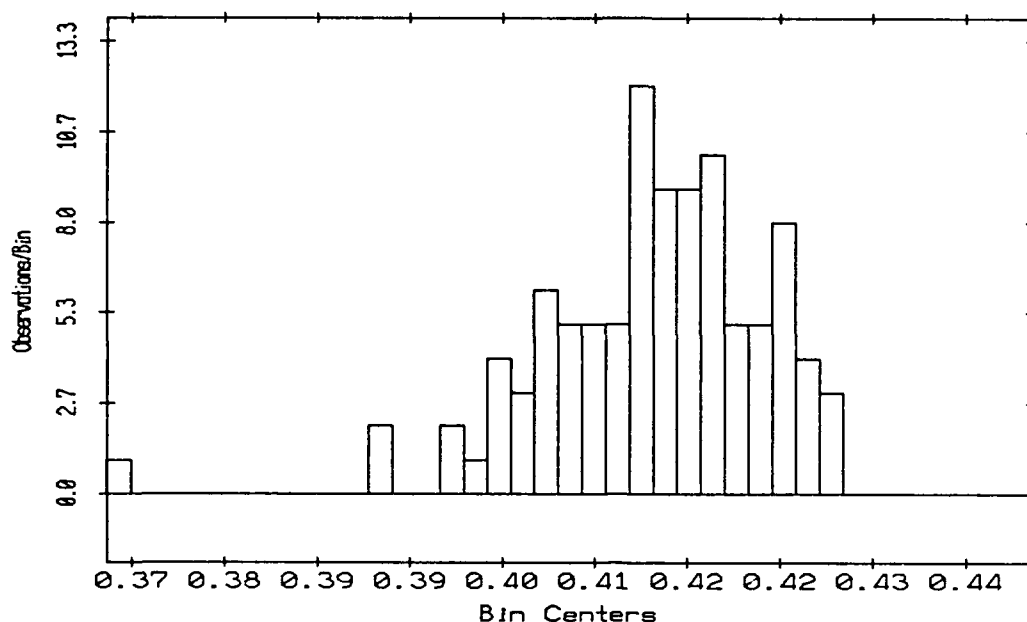


Fig. A.217 : SMC estimation, assumed model order = 1, (model 1), data length 256 points, Histogram of estimated SMC at normalized frequency = 9/32

Histogram of Coherency at normalized frequency = 10/32

Mean=0.30153, Var=0.55094e-04, assumed model order = 1

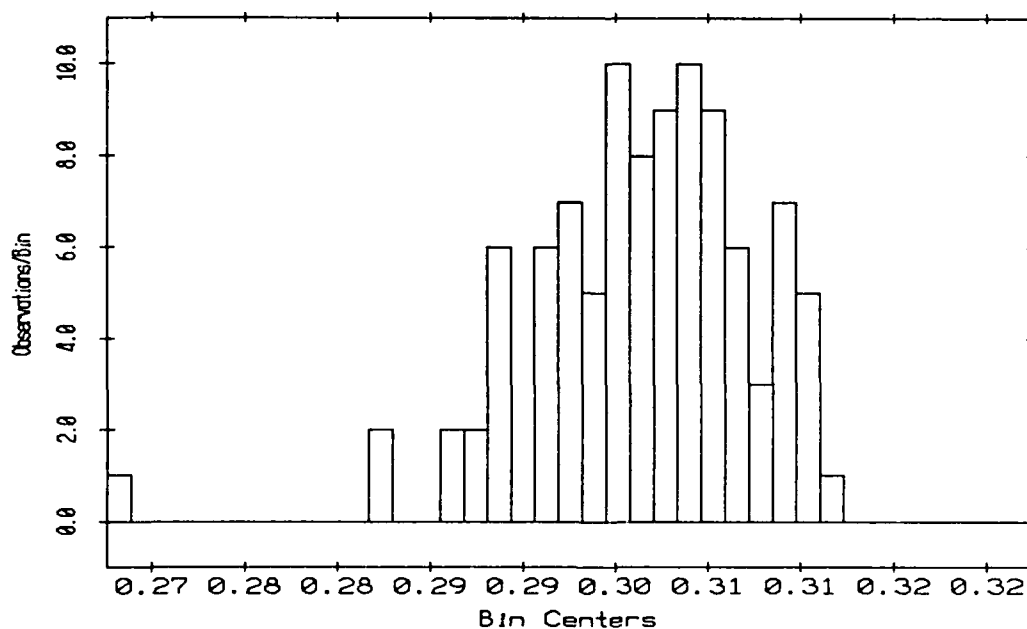


Fig. A.218 : SMC estimation, assumed model order = 1, (model 1), data length 256 points, Histogram of estimated SMC at normalized frequency = 10/32

## Histogram of Coherency at normalized frequency = 11/32

Mean=0.21102, Var=0.32998e-04, assumed model order = 1

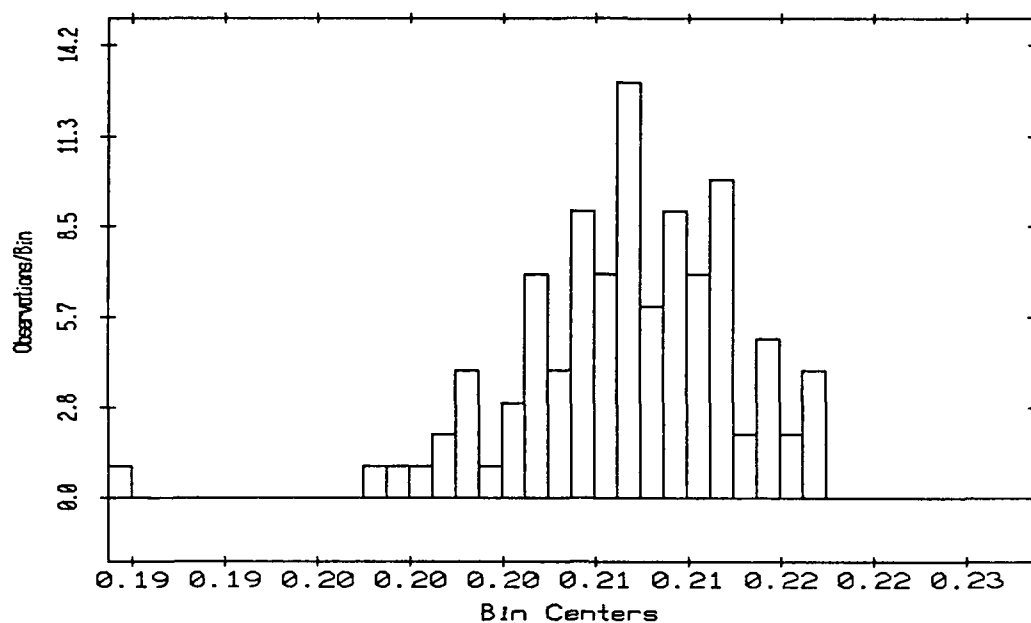


Fig. A.219 : SMC estimation, assumed model order = 1, (model 1), data length 256 points,  
Histogram of estimated SMC at normalized frequency = 11/32

## Histogram of Coherency at normalized frequency = 12/32

Mean=0.13954, Var=0.18611e-04, assumed model order = 1

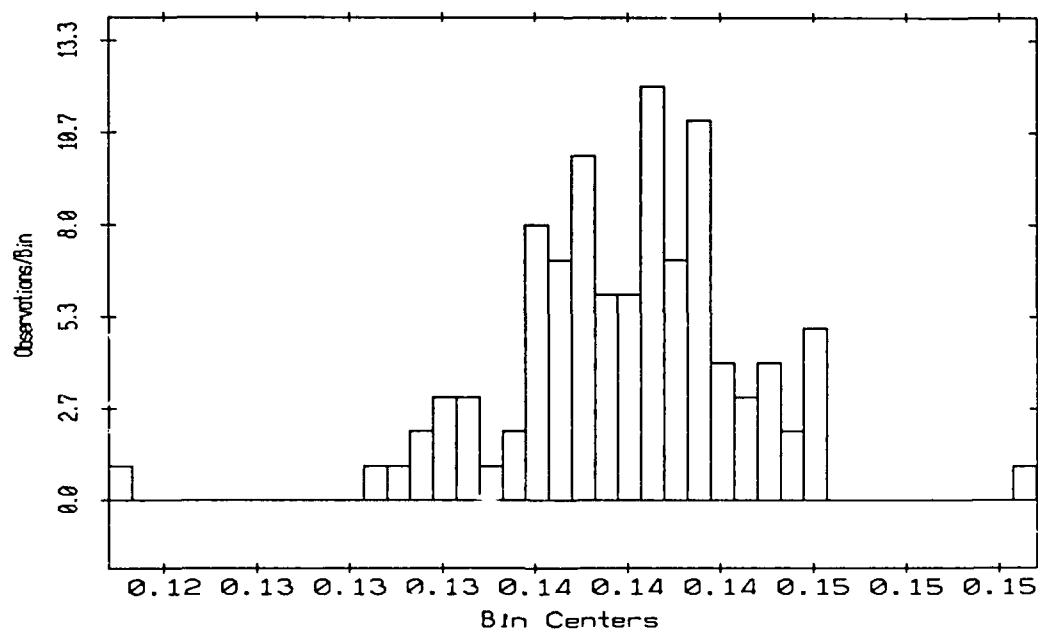


Fig. A.220 : SMC estimation, assumed model order = 1, (model 1), data length 256 points,  
Histogram of estimated SMC at normalized frequency = 12/32

## Histogram of Coherency at normalized frequency = 13/32

Mean=0.85412e-01, Var=0.10732e-04, assumed model order = 1

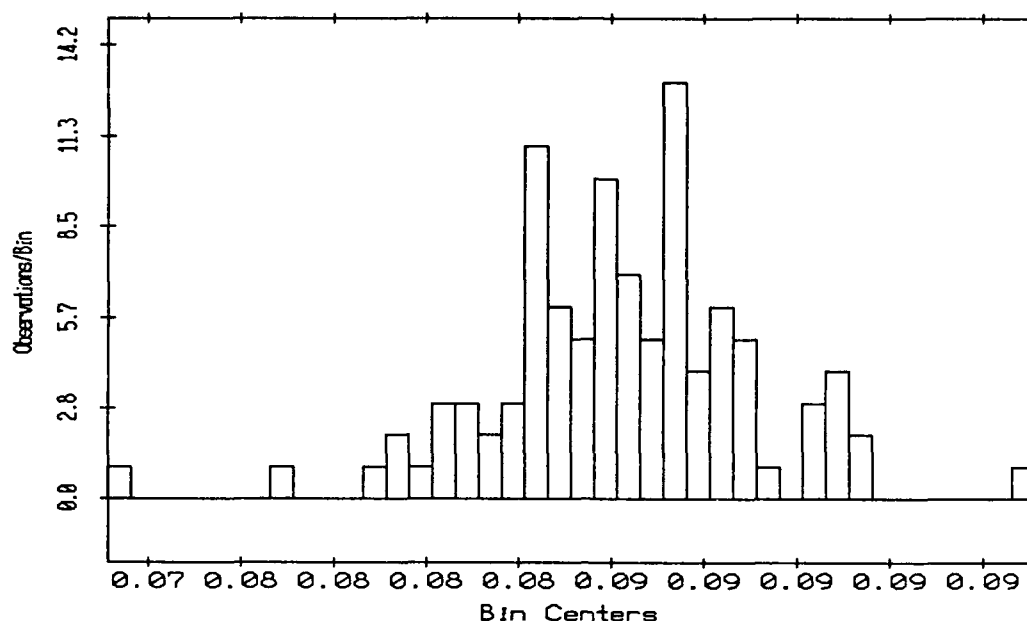


Fig. A.221 : SMC estimation, assumed model order = 1, (model 1), data length 256 points,  
Histogram of estimated SMC at normalized frequency = 13/32

## Histogram of Coherency at normalized frequency = 14/32

Mean=0.47057e-01, Var=0.72120e-05, assumed model order = 1

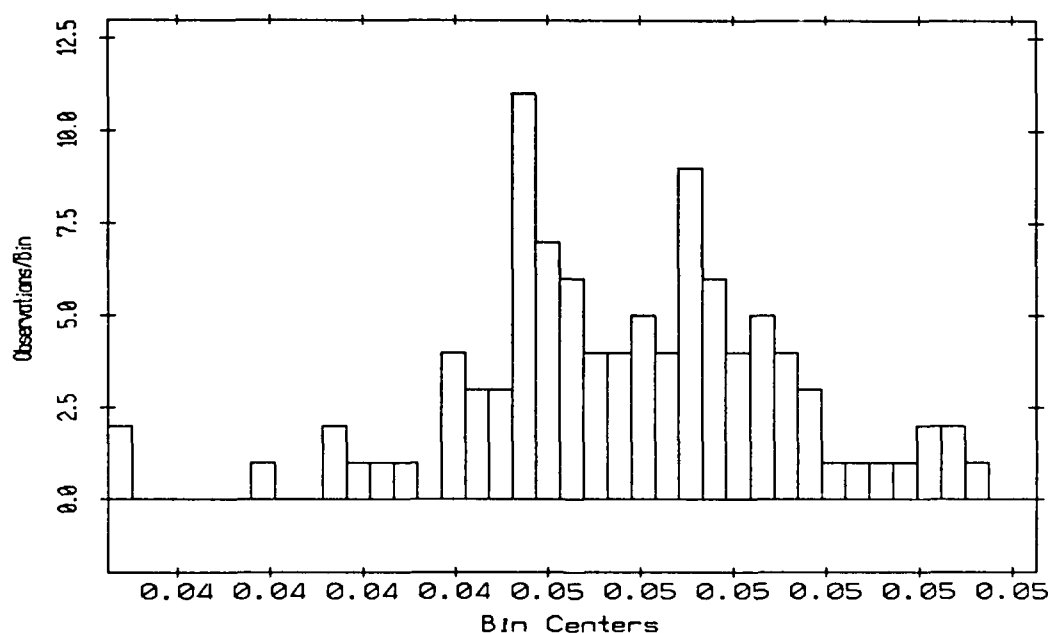


Fig. A.222 : SMC estimation, assumed model order = 1, (model 1), data length 256 points,  
Histogram of estimated SMC at normalized frequency = 14/32

# Histogram of Coherency at normalized frequency = 15/32

Mean=0.23250e-01, Var=0.60335e-05, assumed model order = 1

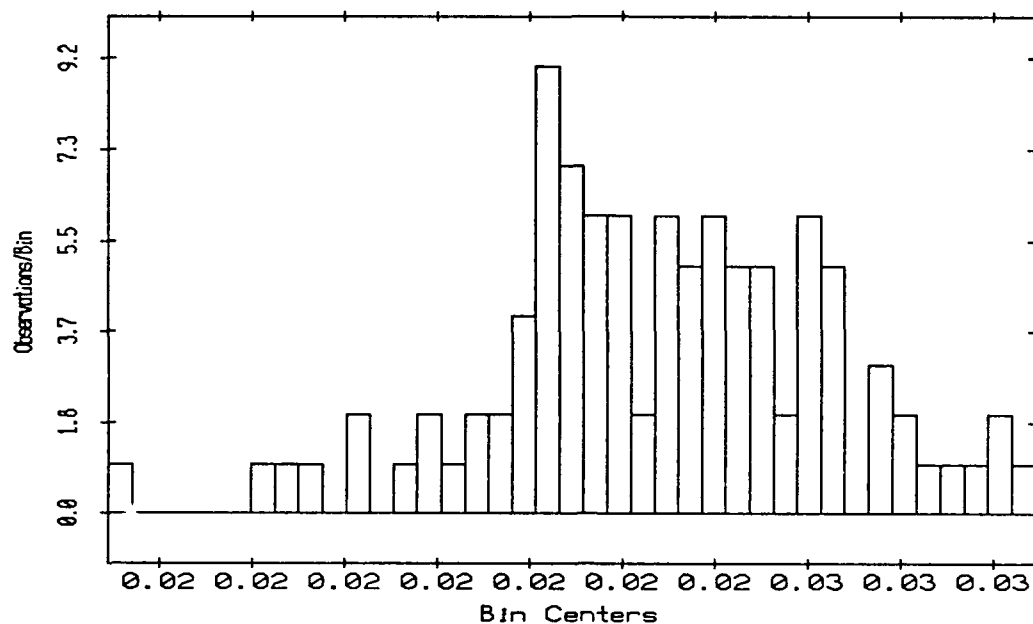


Fig. A.223 : SMC estimation, assumed model order = 1, (model 1), data length 256 points, Histogram of estimated SMC at normalized frequency = 15/32

# Histogram of Coherency at normalized frequency = 16/32

Mean=0.13190e-01, Var=0.57883e-05, assumed model order = 1

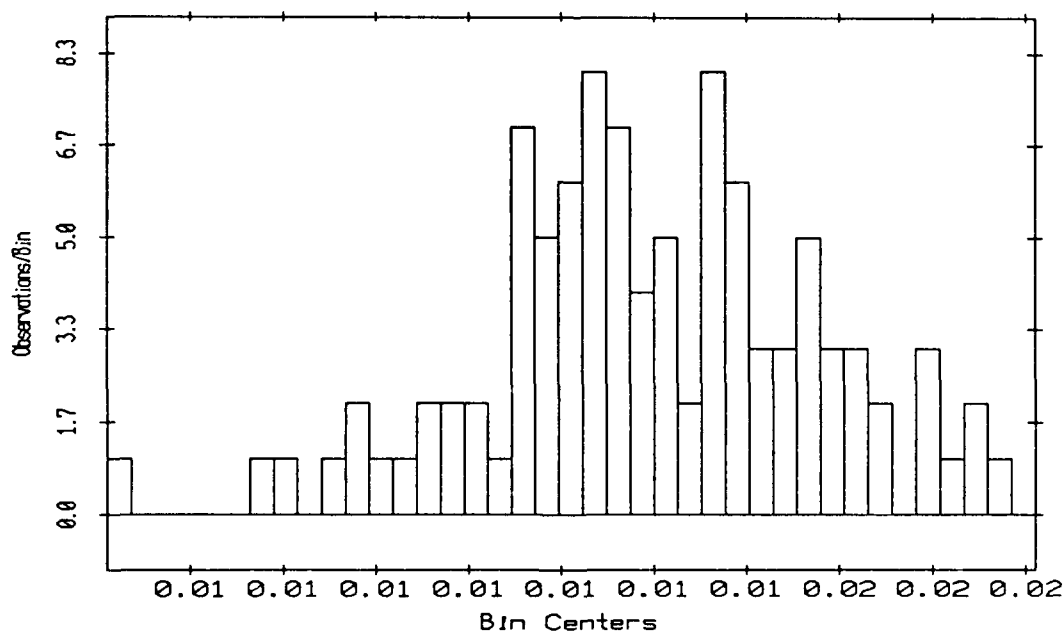


Fig. A.224 : SMC estimation, assumed model order = 1, (model 1), data length 256 points, Histogram of estimated SMC at normalized frequency = 16/32

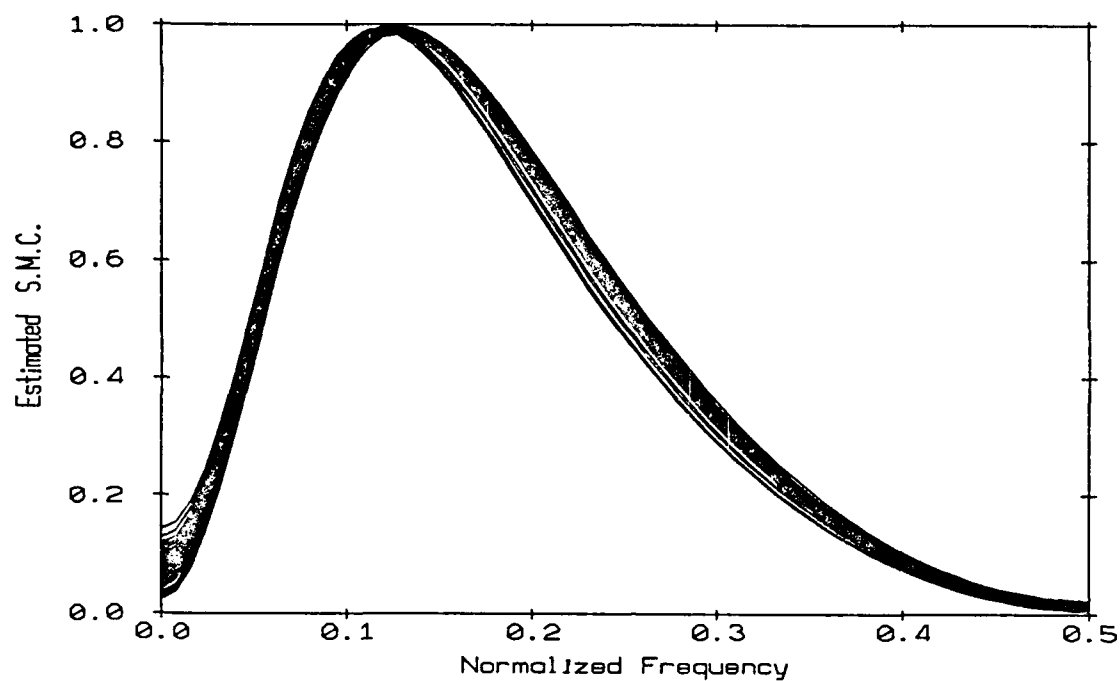


Fig. A.225 : SMC estimation, assumed model order = 1,  
data length 128 points

SMC estimation, general method, data length 128 points

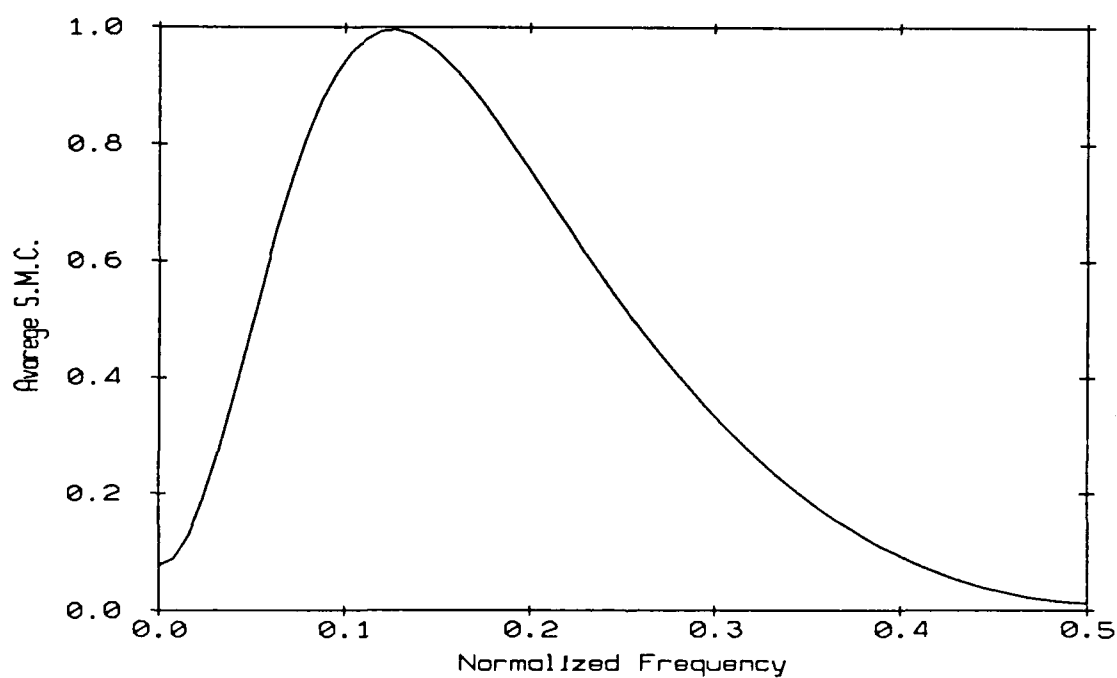


Fig. A.225.1 : SMC estimation, assumed model order = 1,  
data length 128 points, Average estimated SMC over 100 runs



SMC estimation, general method, data length 128 points

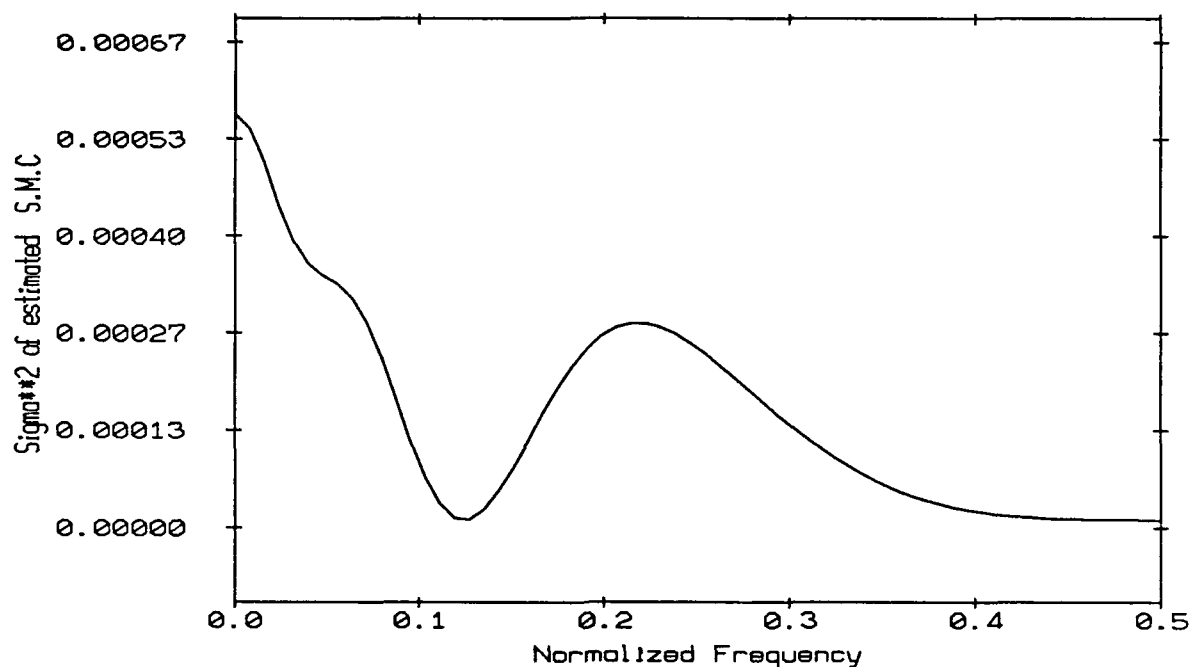


Fig. A.226 : Squared Standard Deviation, of estimated SMC,  
data length 128 points, 100 runs

SMC estimation, general method, data length 128 points

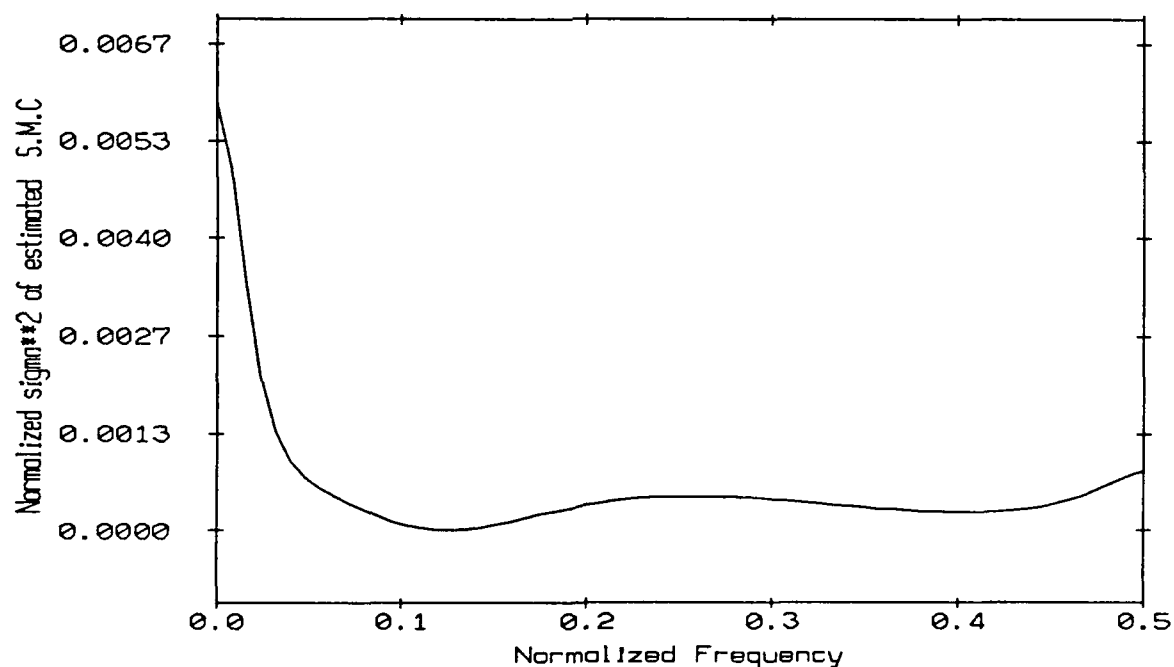


Fig. A.226.1: Normalized Squared Standard Deviation of estimated SMC,  
data length 128 points, 100 runs

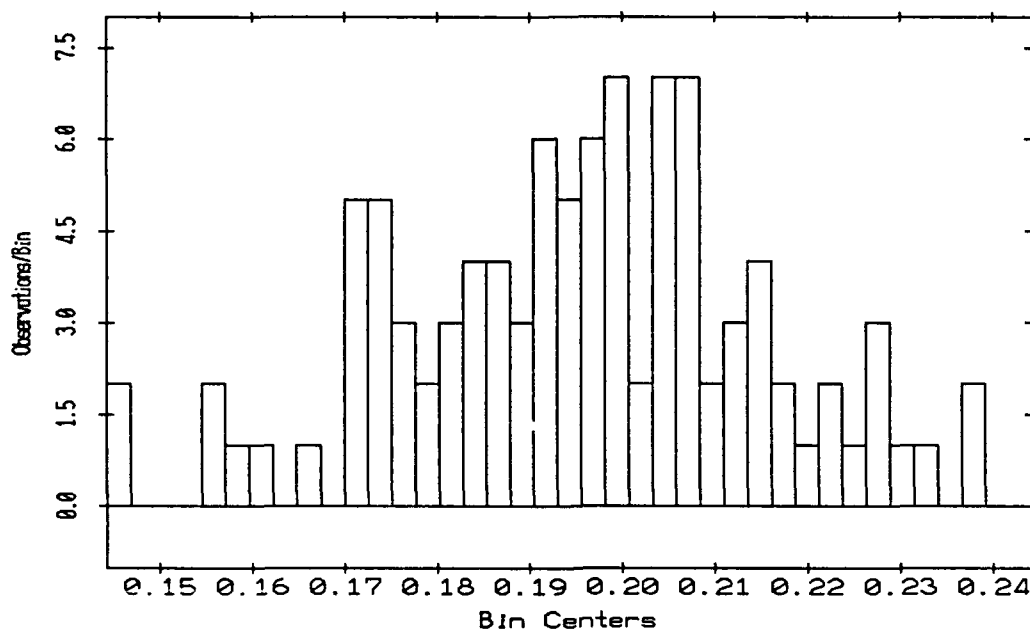
Histogram of Coherency at normalized frequency =  $1/32$ Mean= $0.19187$ , Var= $0.44390e-03$ , assumed model order = 1

Fig. A.227 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency =  $1/32$

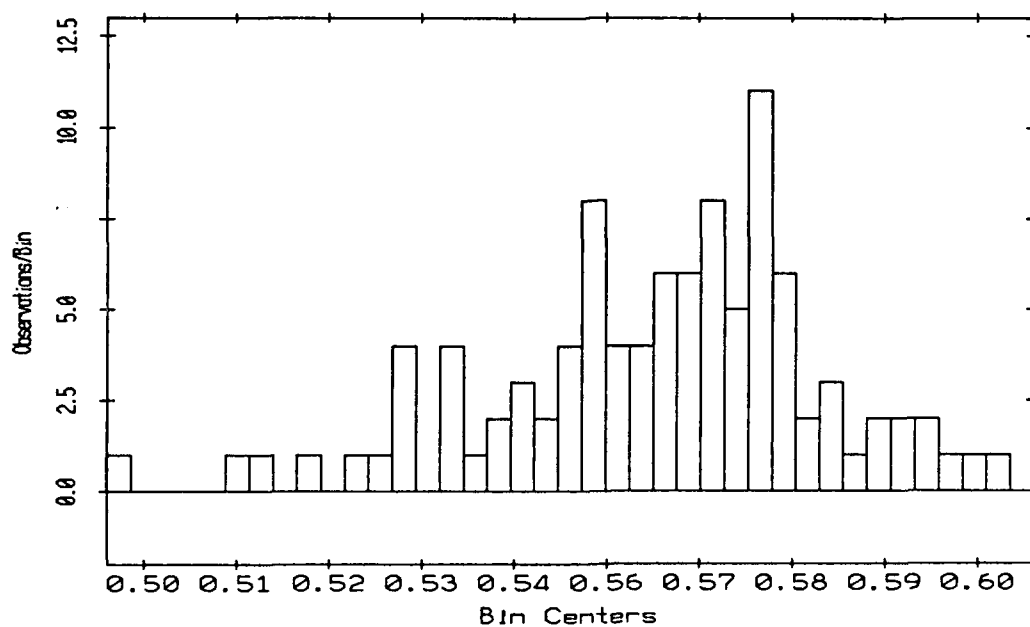
Histogram of Coherency at normalized frequency =  $2/32$ Mean= $0.55994$ , Var= $0.33466e-03$ , assumed model order = 1

Fig. A.228 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency =  $2/32$

## Histogram of Coherency at normalized frequency = 3/32

Mean=0.87378, Var=0.17577e-03, assumed model order = 1

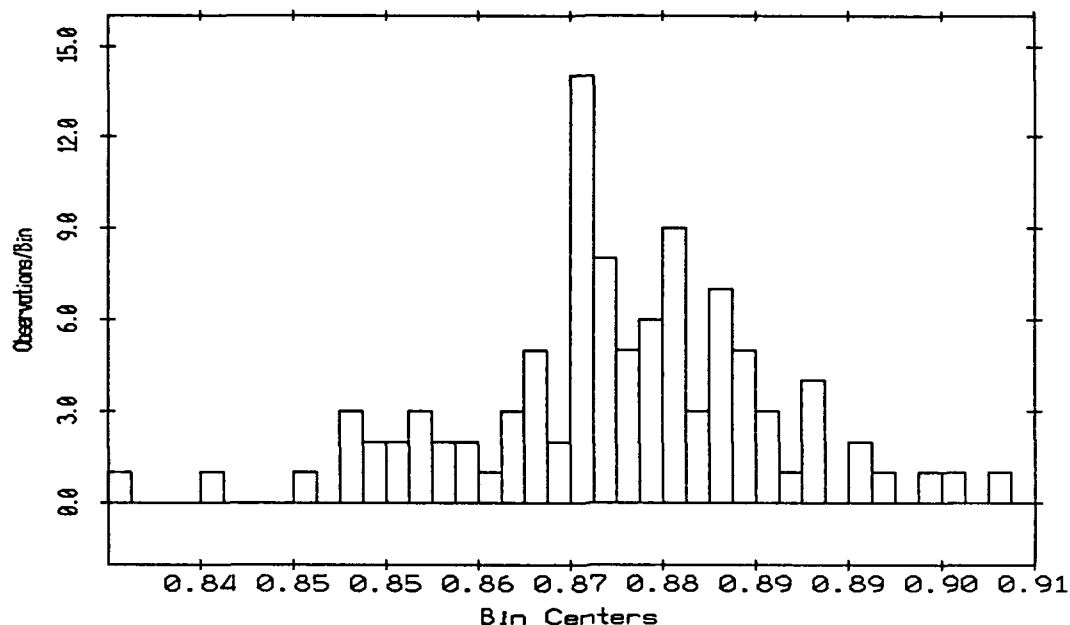


Fig. A.229 : SMC estimation, assumed model order = 1, (model 1), data length 128 points.  
Histogram of estimated SMC at normalized frequency = 3/32

## Histogram of Coherency at normalized frequency = 4/32

Mean=0.99387, Var=0.13844e-04, assumed model order = 1

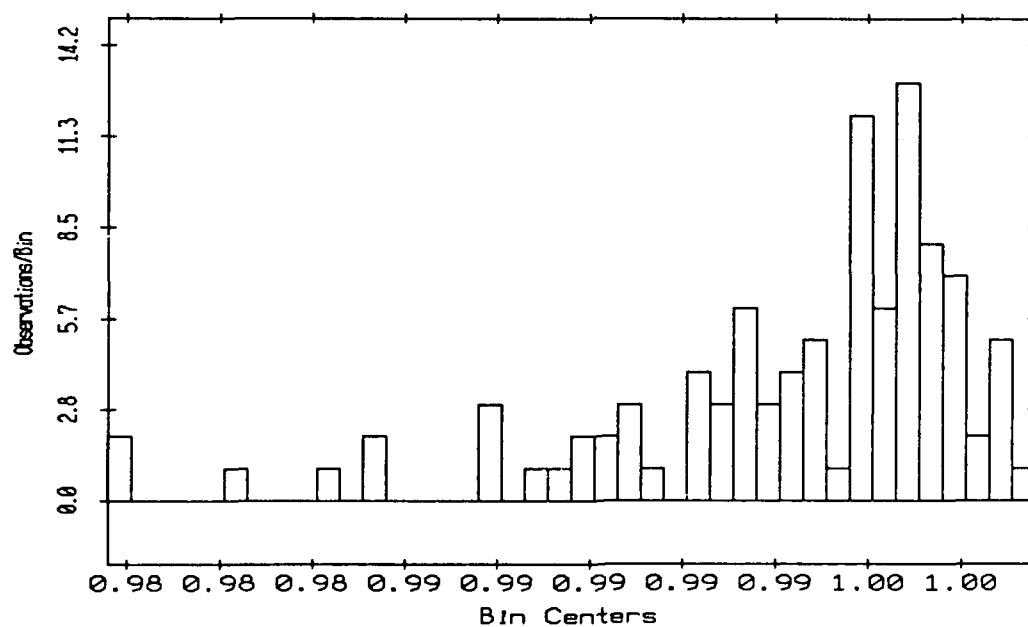


Fig. A.230 : SMC estimation, assumed model order = 1, (model 1), data length 128 points.  
Histogram of estimated SMC at normalized frequency = 4/32

## Histogram of Coherency at normalized frequency = 5/32

Mean=0.95600, Var=0.84054e-04, assumed model order = 1

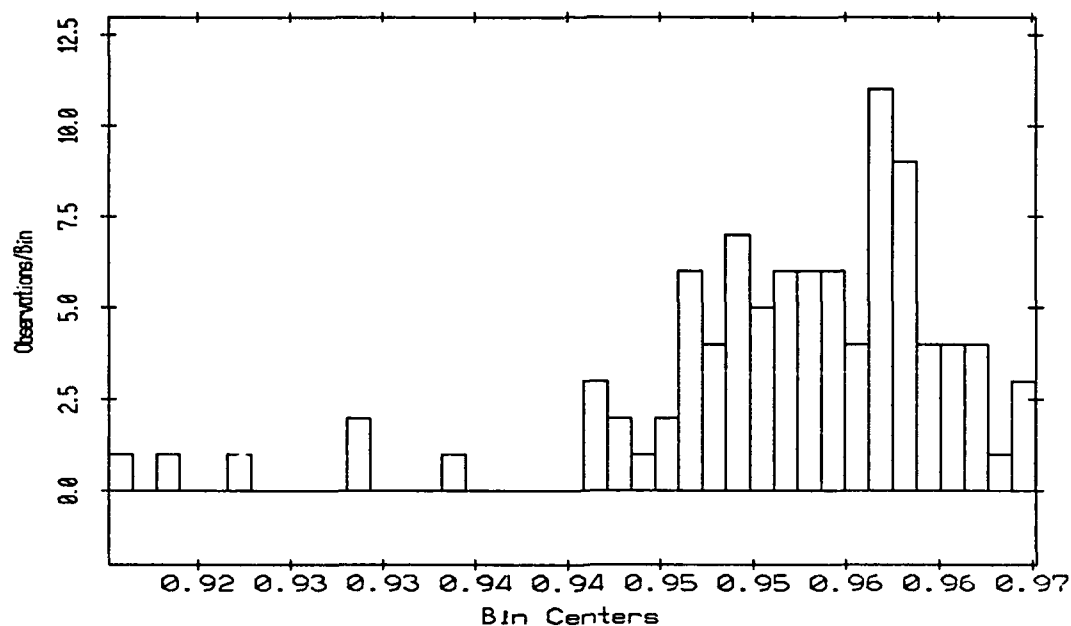


Fig. A.231 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency = 5/32

## Histogram of Coherency at normalized frequency = 6/32

Mean=0.83543, Var=0.22152e-03, assumed model order = 1

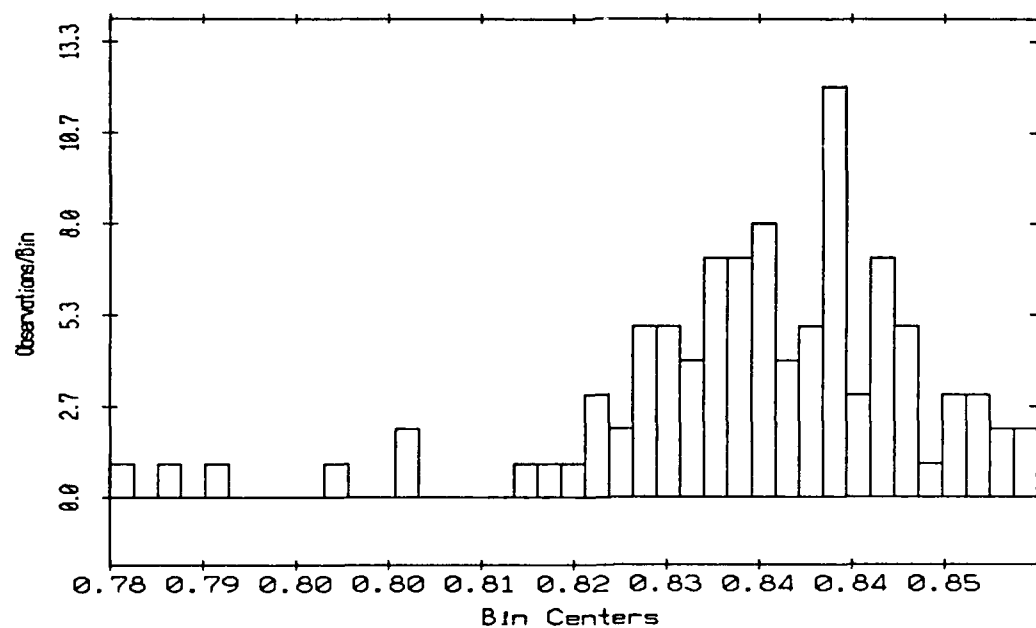


Fig. A.232 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency = 6/32

## Histogram of Coherency at normalized frequency = 7/32

Mean=0.68725, Var=0.28014e-03, assumed model order = 1

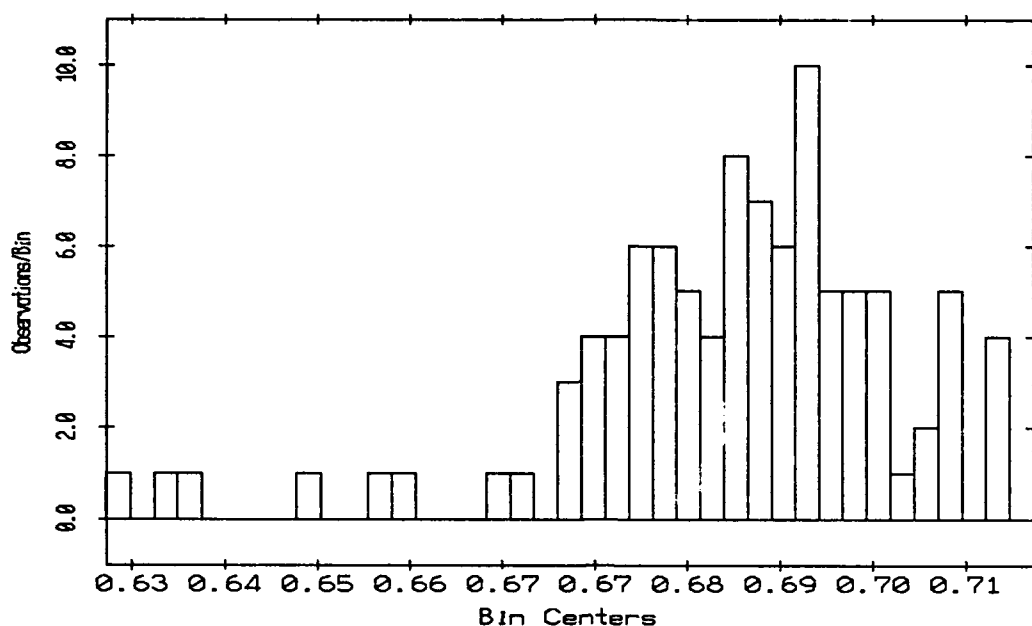


Fig. A.233 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency = 7/32

## Histogram of Coherency at normalized frequency = 8/32

Mean=0.54132, Var=0.25424e-03, assumed model order = 1

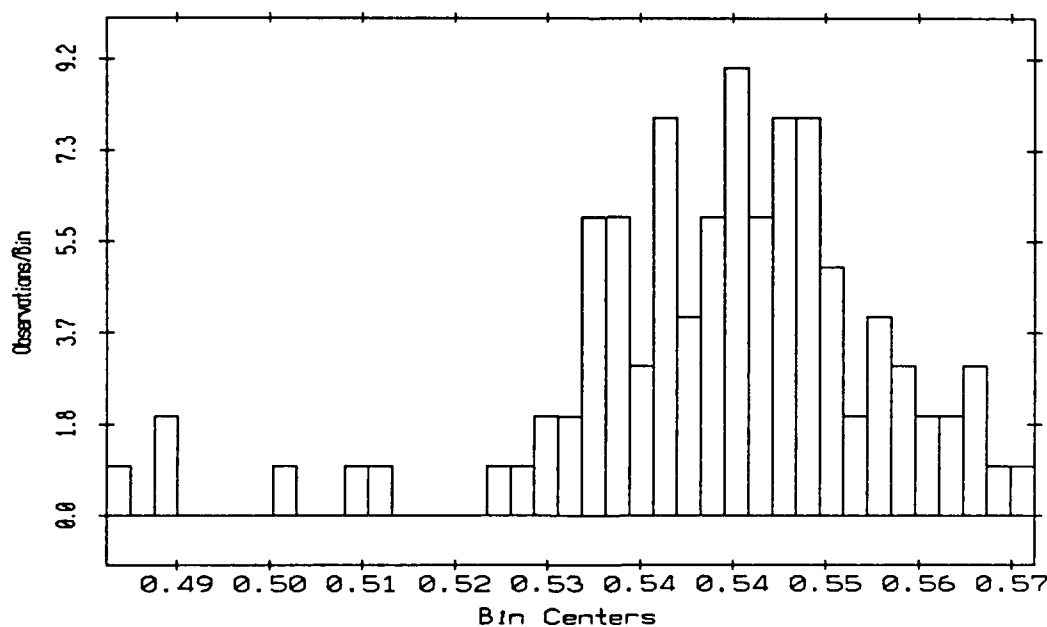


Fig. A.234 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency = 8/32

## Histogram of Coherency at normalized frequency = 9/32

Mean=0.41068, Var=0.18925e-03, assumed model order = 1

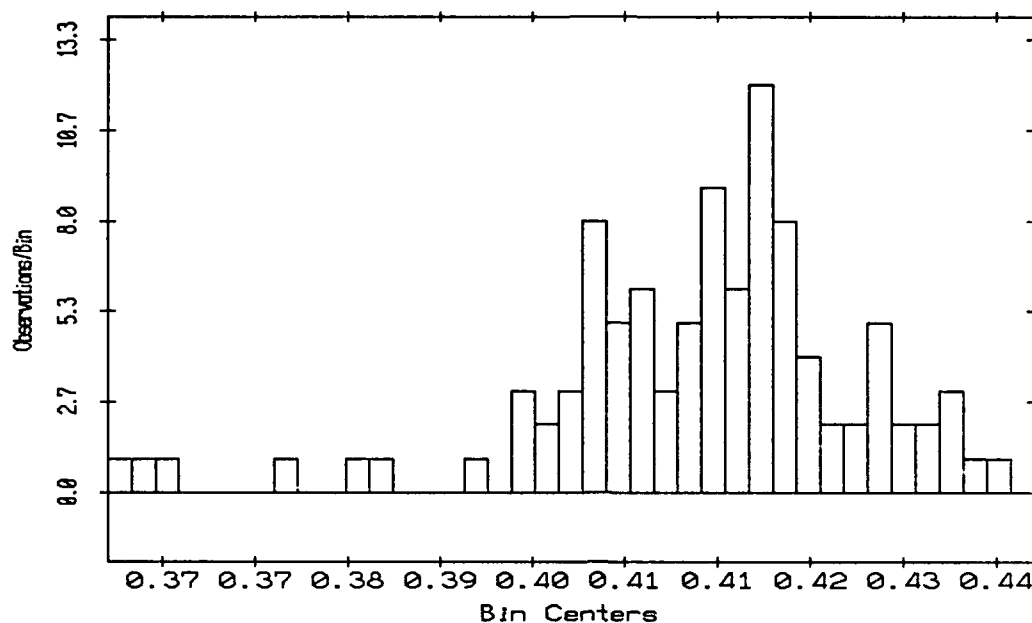


Fig. A.235 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency = 9/32

## Histogram of Coherency at normalized frequency = 10/32

Mean=0.30036, Var=0.12236e-03, assumed model order = 1

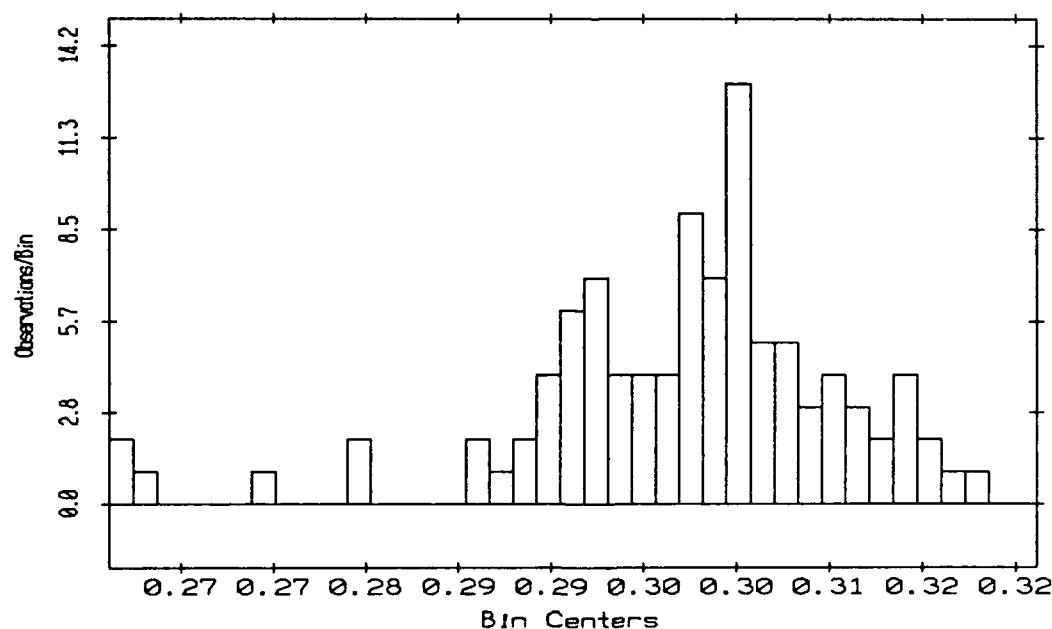


Fig. A.236 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency = 10/32

## Histogram of Coherency at normalized frequency = 11/32

Mean=0.21011, Var=0.70942e-04, assumed model order = 1

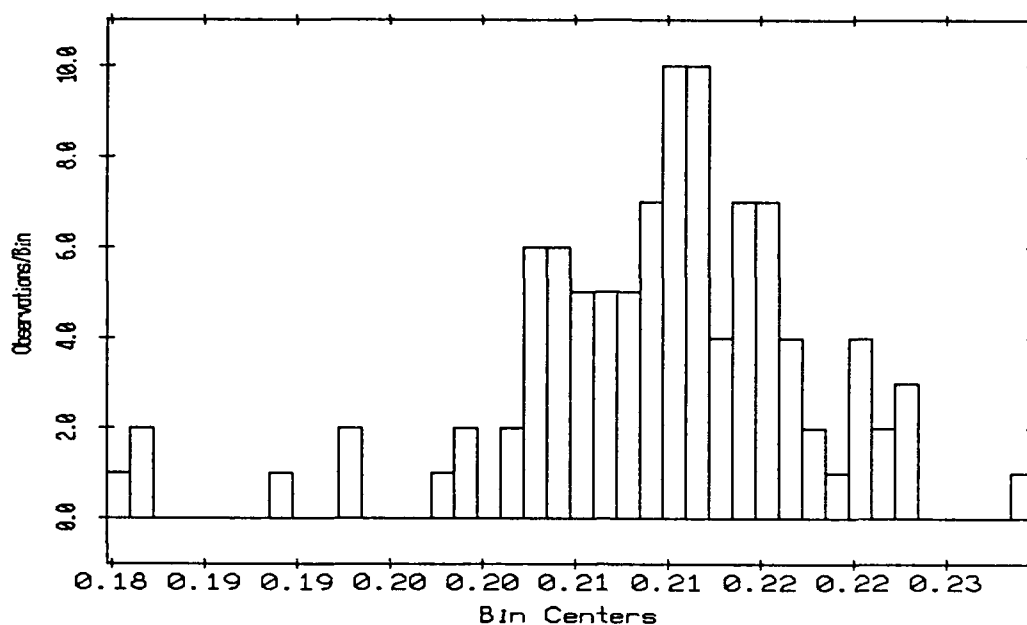


Fig. A.237 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency = 11/32

## Histogram of Coherency at normalized frequency = 12/32

Mean=0.13886, Var=0.38325e-04, assumed model order = 1

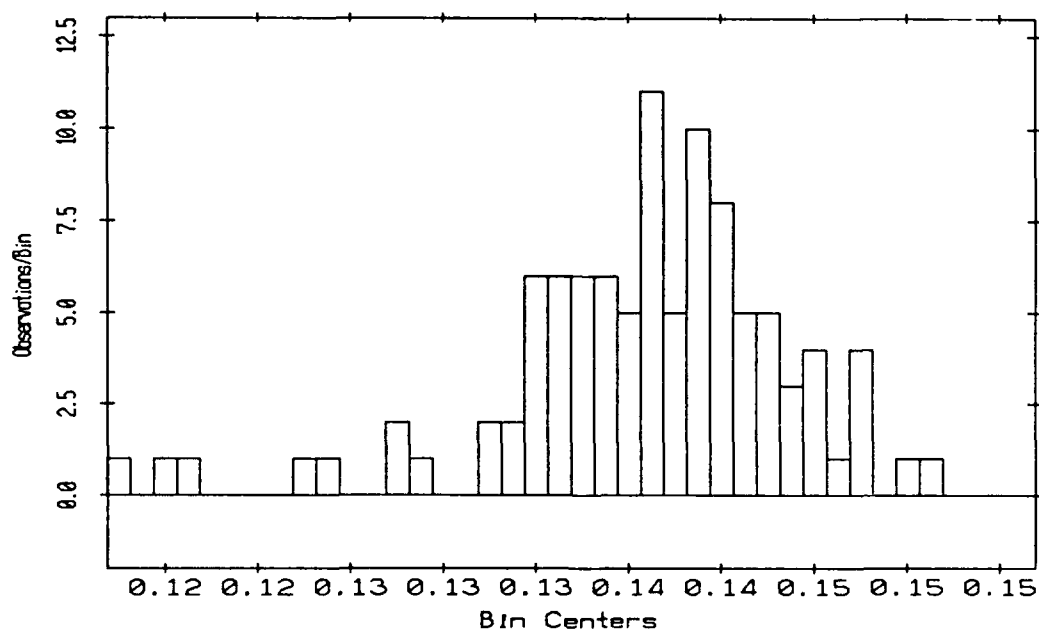


Fig. A.238 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency = 12/32

Histogram of Coherency at normalized frequency = 13/32

Mean=0.84900e-01, Var=0.21109e-04, assumed model order = 1

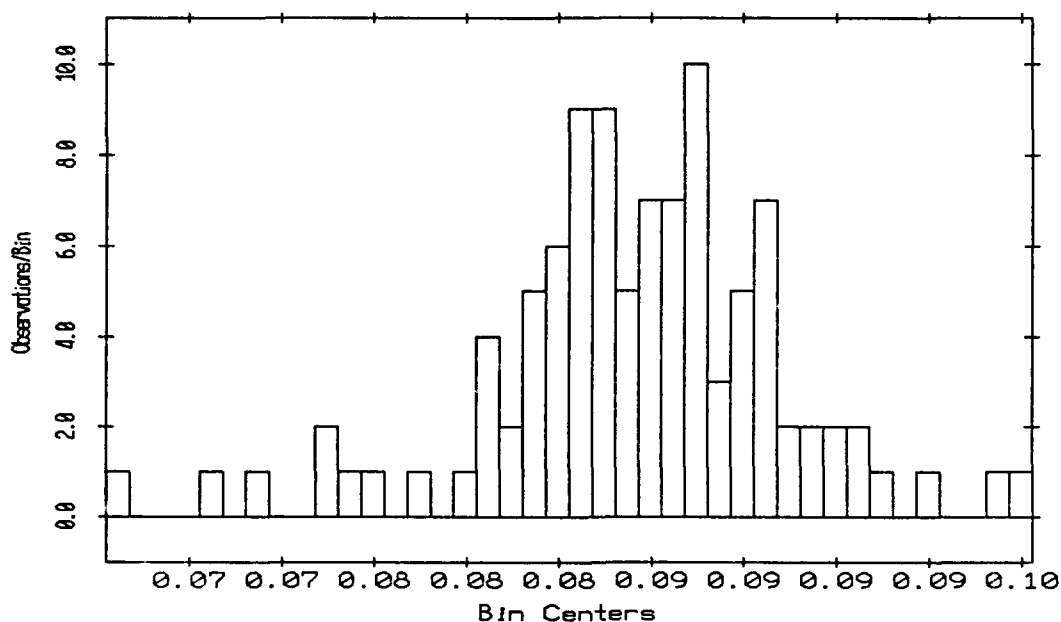


Fig. A.239 : SMC estimation, assumed model order = 1, (model 1), data length 128 points, Histogram of estimated SMC at normalized frequency = 13/32

Histogram of Coherency at normalized frequency = 14/32

Mean=0.46672e-01, Var=0.13930e-04, assumed model order = 1

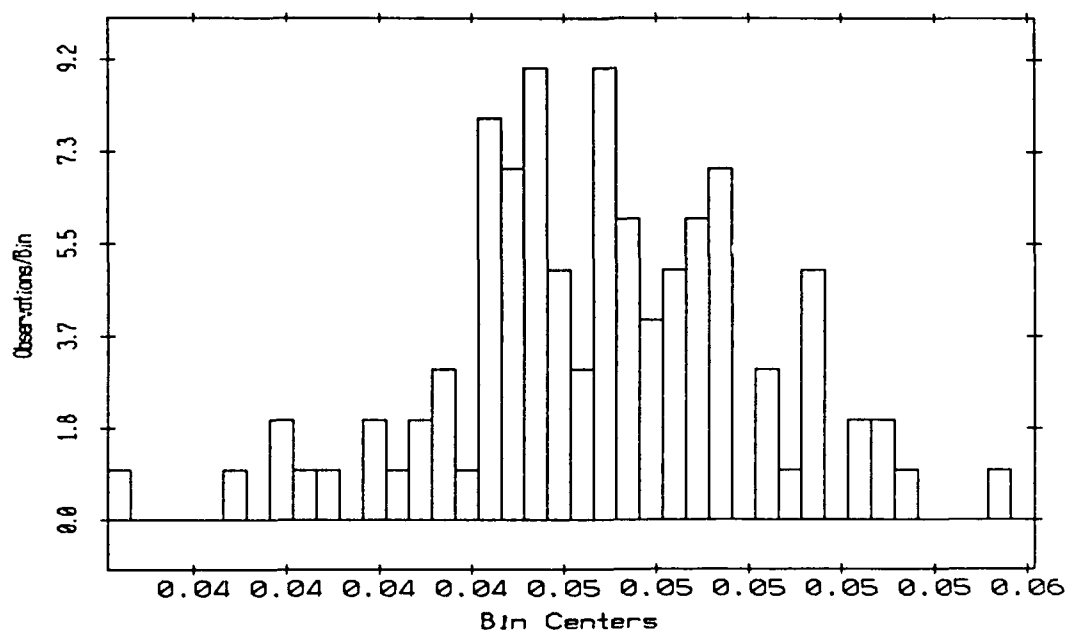


Fig. A.240 : SMC estimation, assumed model order = 1, (model 1), data length 128 points, Histogram of estimated SMC at normalized frequency = 14/32



## Histogram of Coherency at normalized frequency = 15/32

Mean=0.22945e-01, Var=0.11902e-04, assumed model order = 1

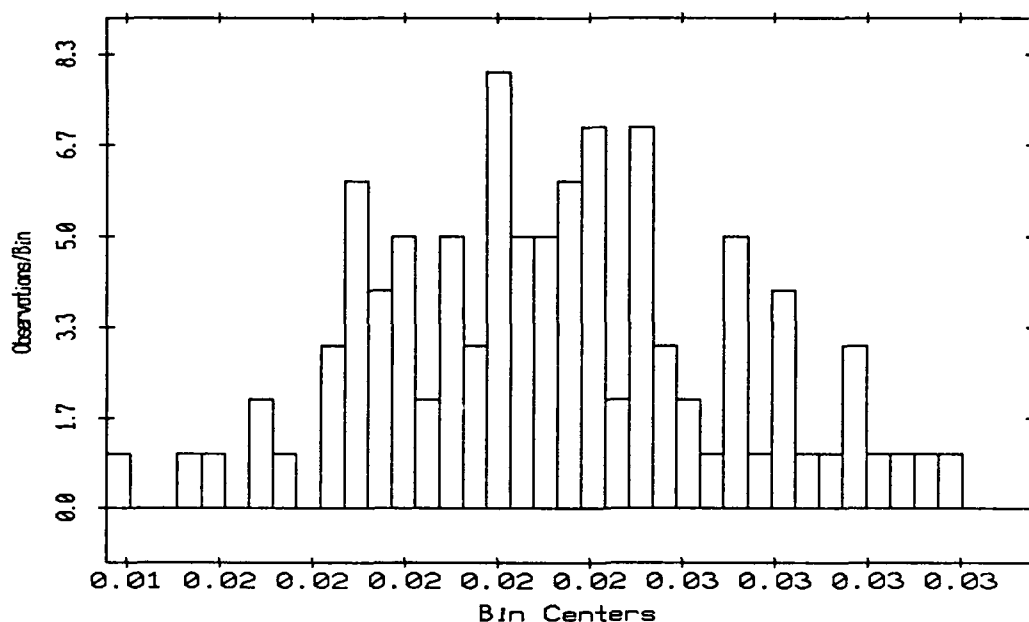


Fig. A.241 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency = 15/32

## Histogram of Coherency at normalized frequency = 16/32

Mean=0.12919e-01, Var=0.11651e-04, assumed model order = 1

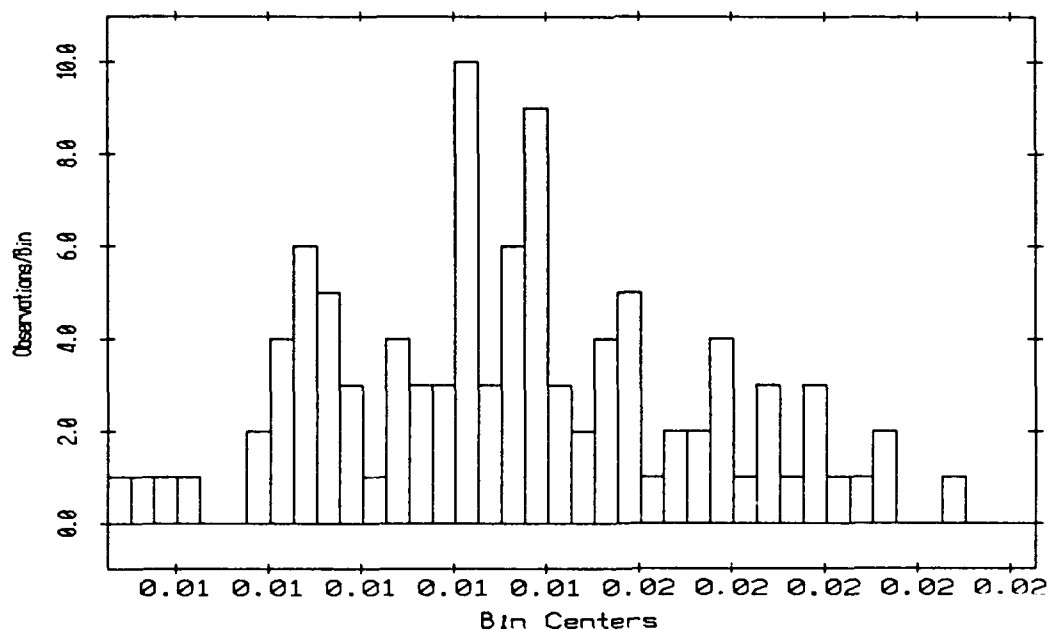


Fig. A.242 : SMC estimation, assumed model order = 1, (model 1), data length 128 points,  
Histogram of estimated SMC at normalized frequency = 16/32

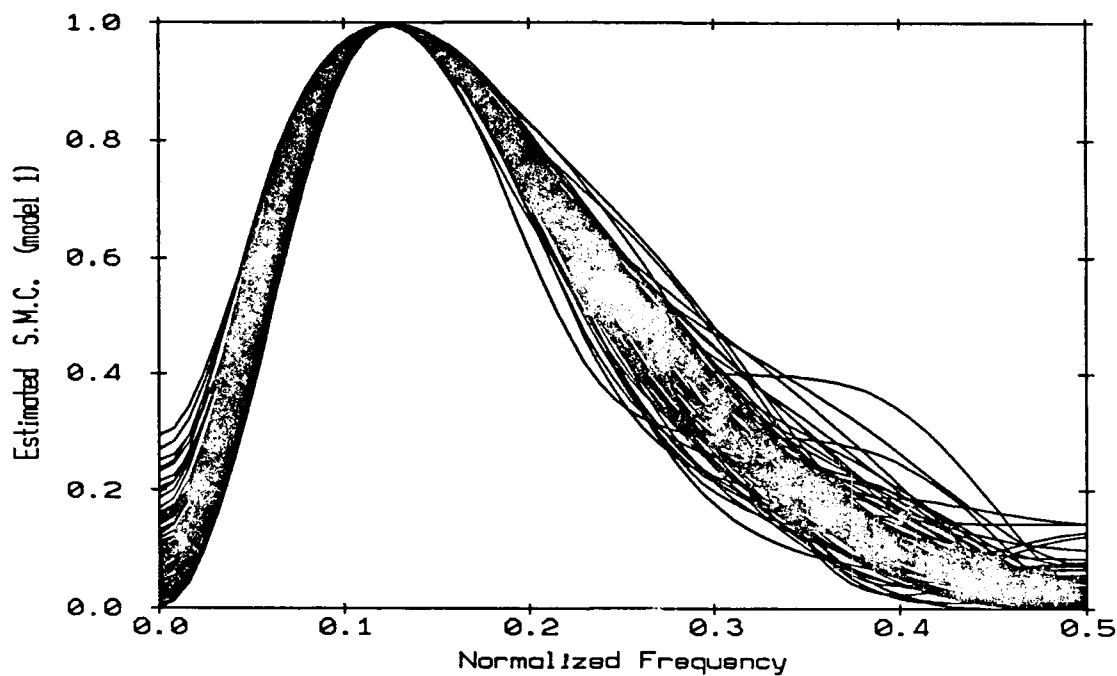


Fig. A.243 : SMC estimation, ( model 1 ), assumed model order = 4,  
data length 512 points

SMC estimation, (model 1), data length 512 points

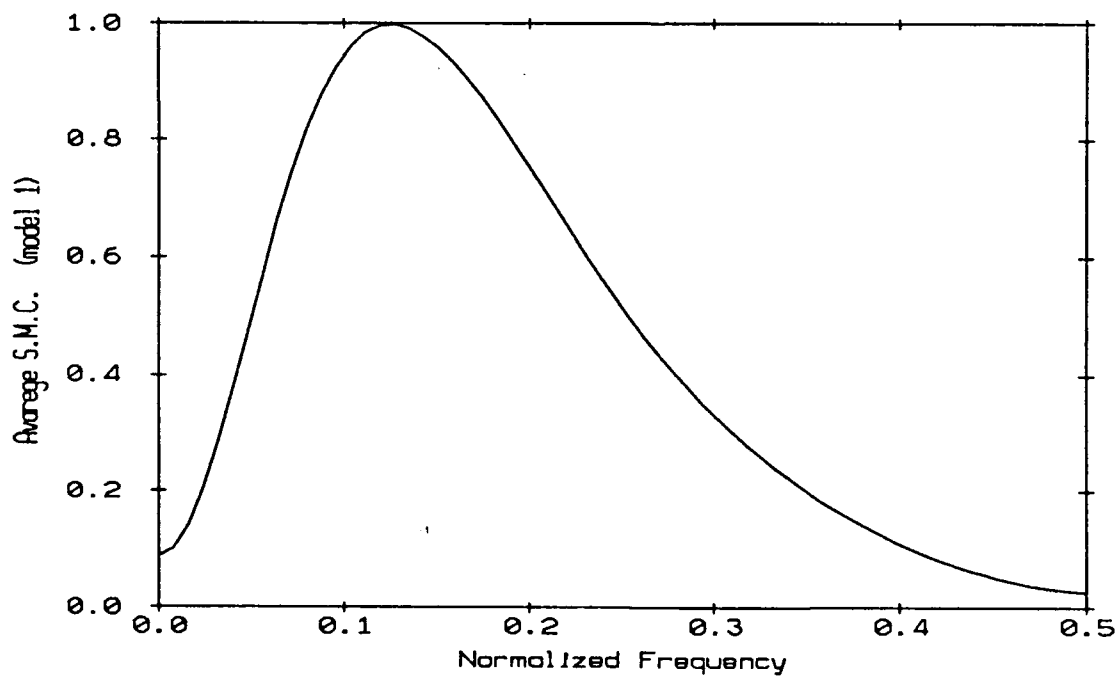


Fig. A.243.1 : SMC estimation, ( model 1 ), assumed model order = 4,  
data length 512 points, Average estimated SMC over 100 runs

SMC estimation, (model 1), data length 512 points

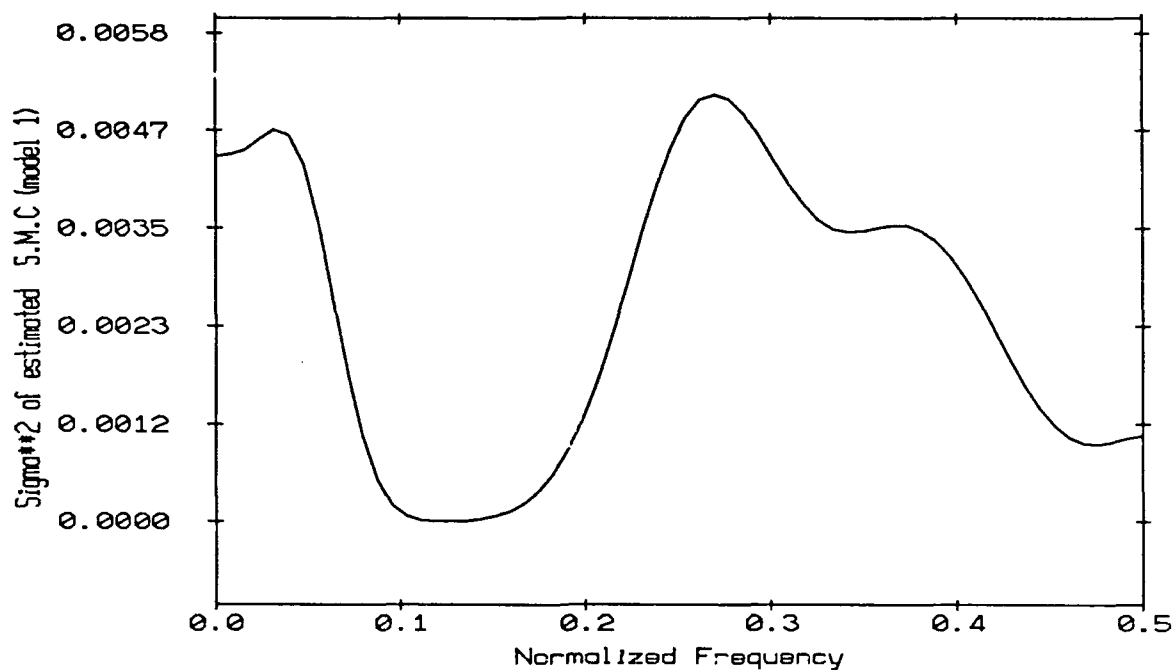


Fig. A.244 : Squared Standard Deviation, of estimated SMC,  
assumed model order= 4, 100 runs

SMC estimation, (model 1), data length 512 points

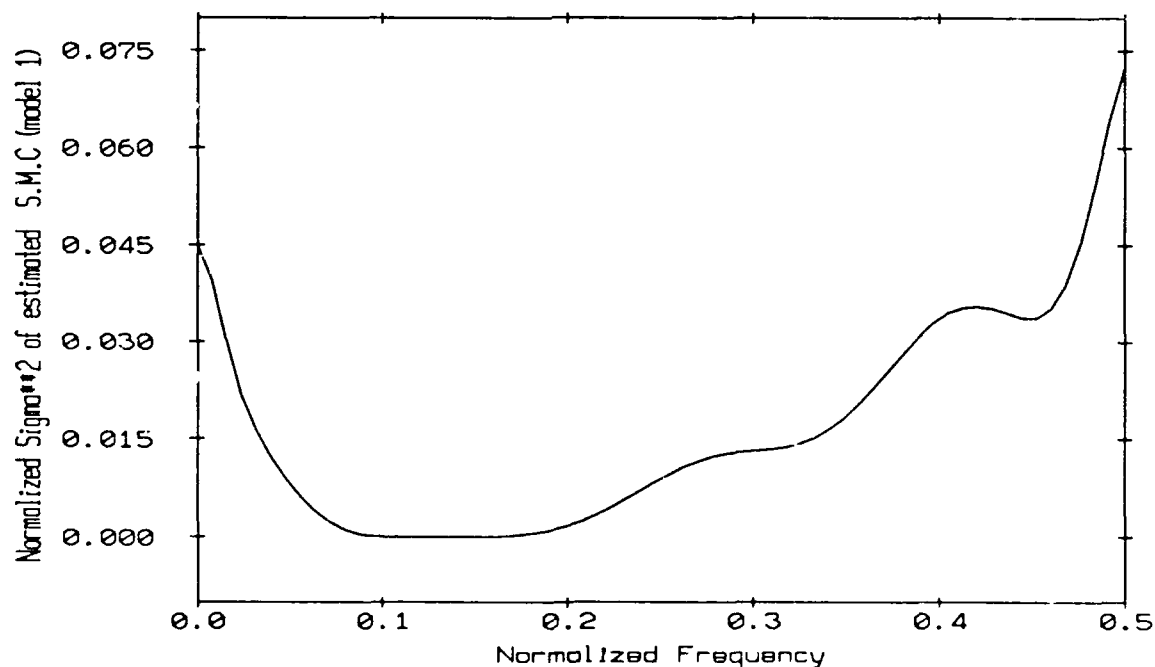


Fig. A.244.1: Normalized Squared Standard Deviation of estimated SMC,  
assumed model order= 4, 100 runs

## Histogram of Coherency at normalized frequency = 1/32

Model 1, Mean=0.20405, Var=0.45630e-02, assumed model order = 4

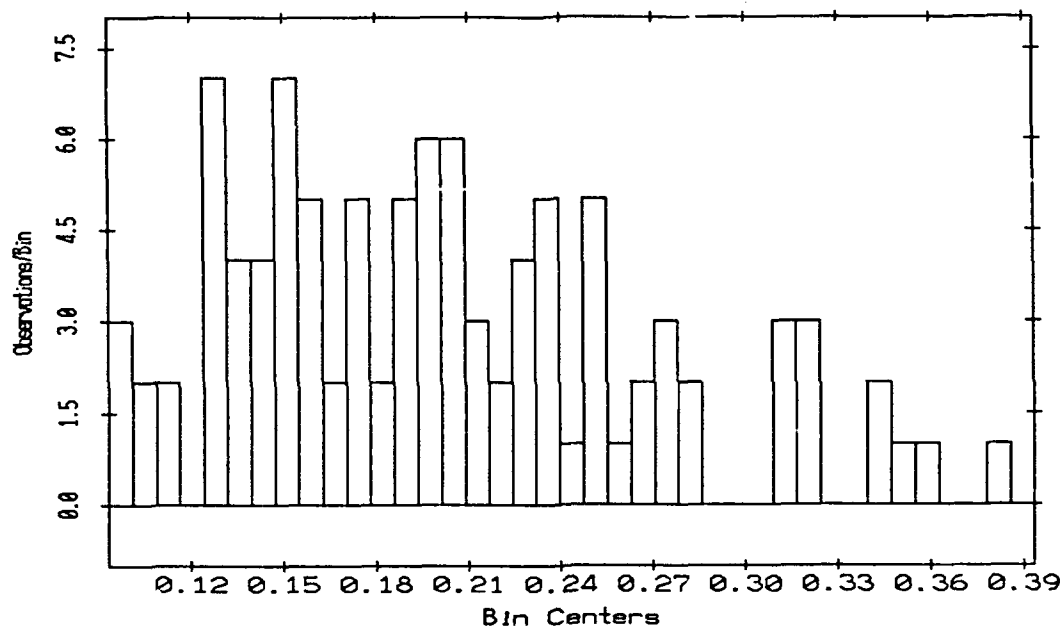


Fig. A.245 : SMC estimation, assumed model order = 4, (model 1), data length 512 points.  
Histogram of estimated SMC at normalized frequency = 1/32

## Histogram of Coherency at normalized frequency = 2/32

Model 1, Mean=0.56763, Var=0.35607e-02, assumed model order = 4

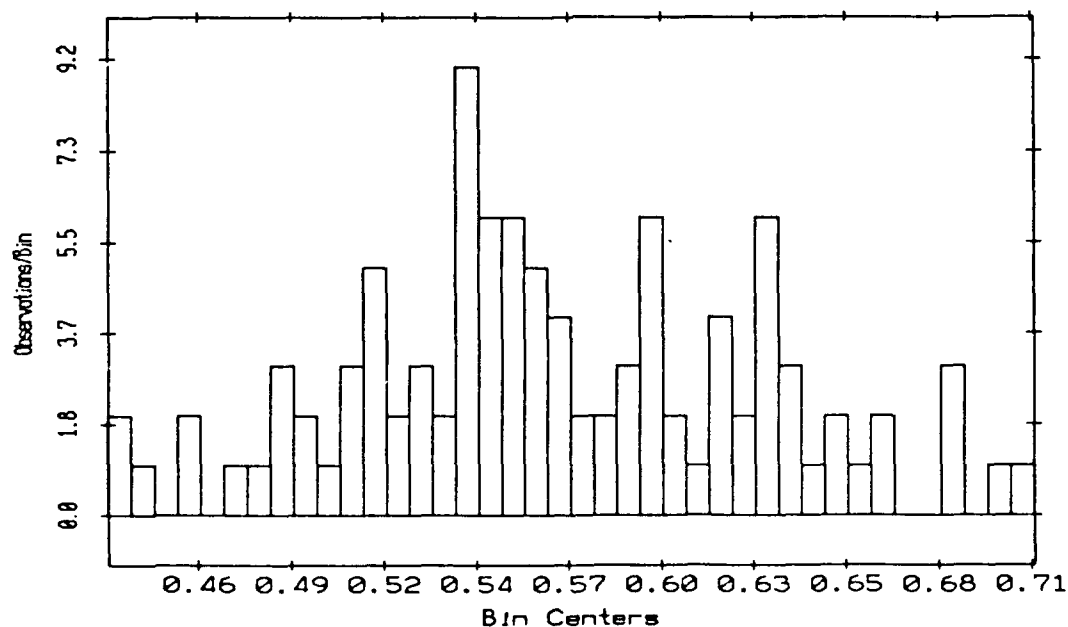


Fig. A.246 : SMC estimation, assumed model order = 4, (model 1), data length 512 points.  
Histogram of estimated SMC at normalized frequency = 2/32

## Histogram of Coherency at normalized frequency = 3/32

Model 1, Mean=0.87715, Var=0.49067e-03, assumed model order = 4

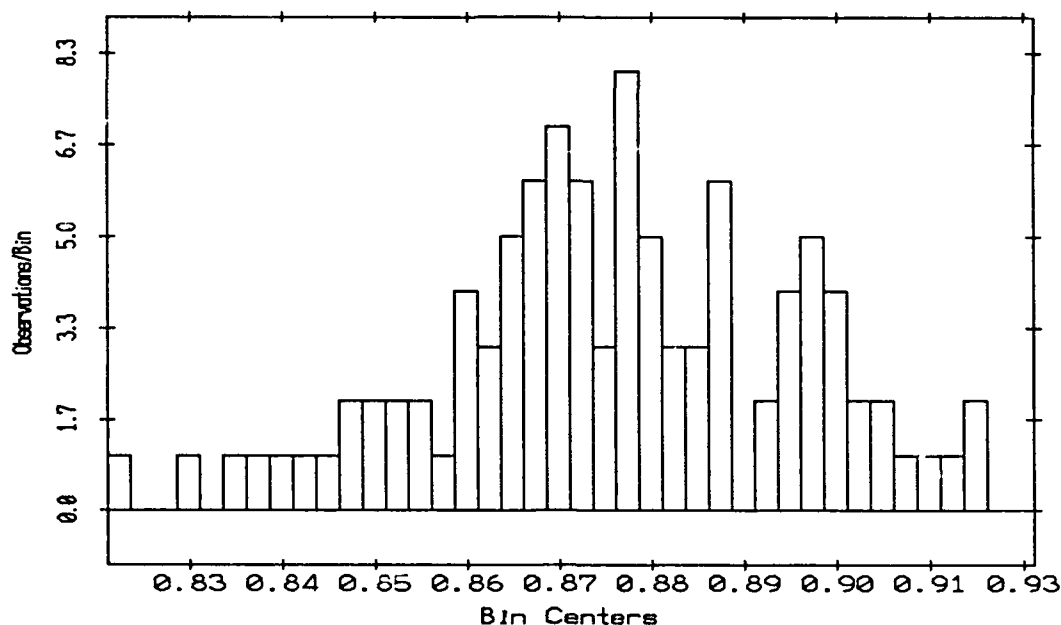


Fig. A.247 : SMC estimation, assumed model order = 4, (model 1), data length 512 points,  
Histogram of estimated SMC at normalized frequency = 3/32

## Histogram of Coherency at normalized frequency = 4/32

Model 1, Mean=0.99612, Var=0.16403e-05, assumed model order = 4

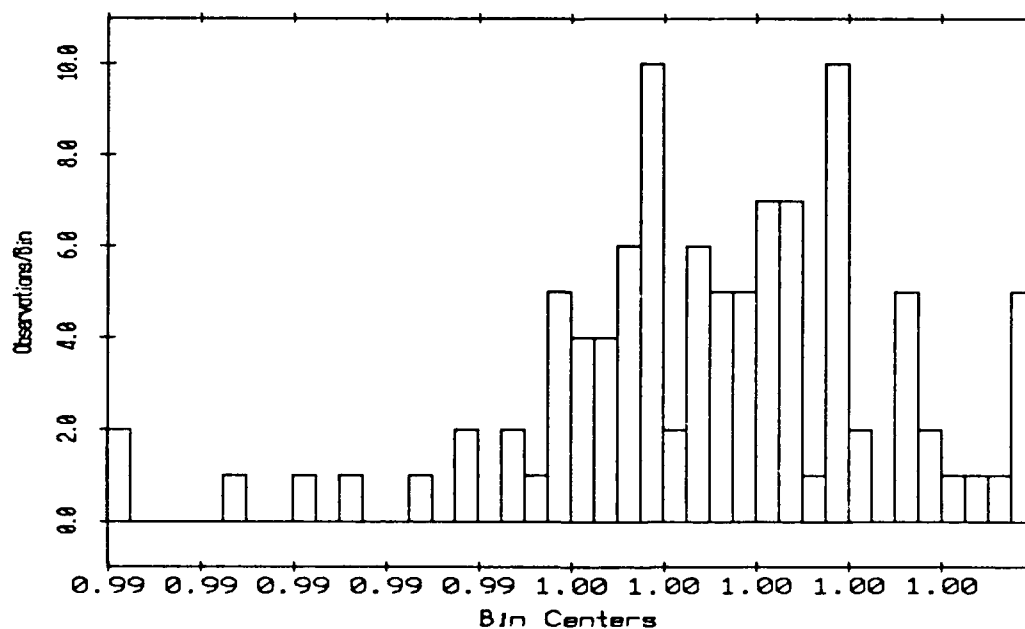


Fig. A.248 : SMC estimation, assumed model order = 4, (model 1), data length 512 points,  
Histogram of estimated SMC at normalized frequency = 4/32

## Histogram of Coherency at normalized frequency = 5/32

Model 1, Mean=0.95771, Var=0.58087e-04, assumed model order = 4

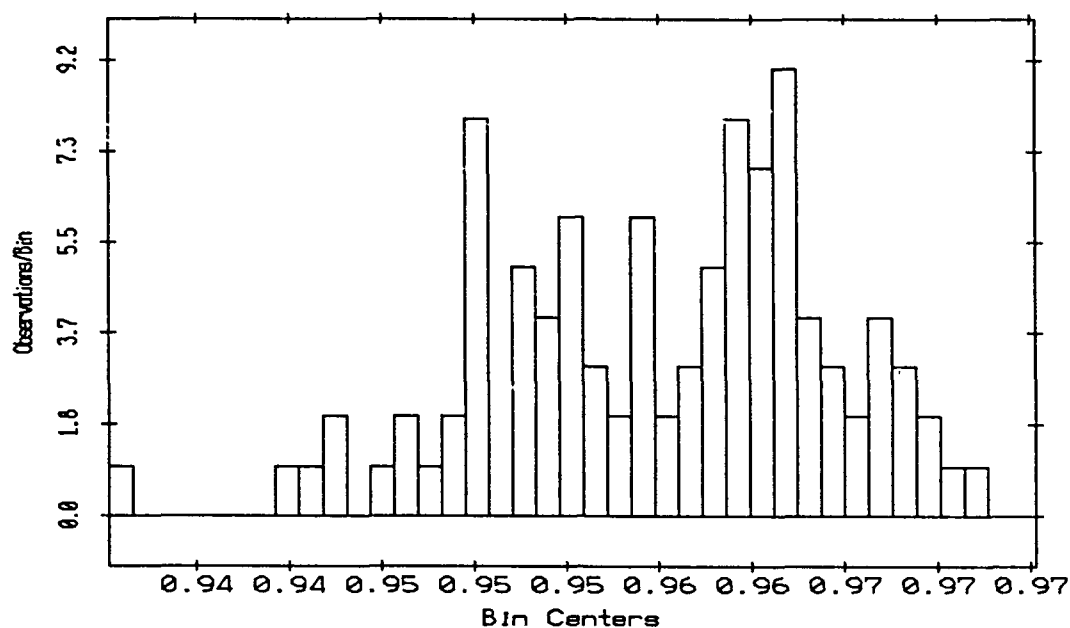


Fig. A.249 : SMC estimation, assumed model order = 4, (model 1), data length 512 points.  
Histogram of estimated SMC at normalized frequency = 5/32

## Histogram of Coherency at normalized frequency = 6/32

Model 1, Mean=0.83523, Var=0.57017e-03, assumed model order = 4

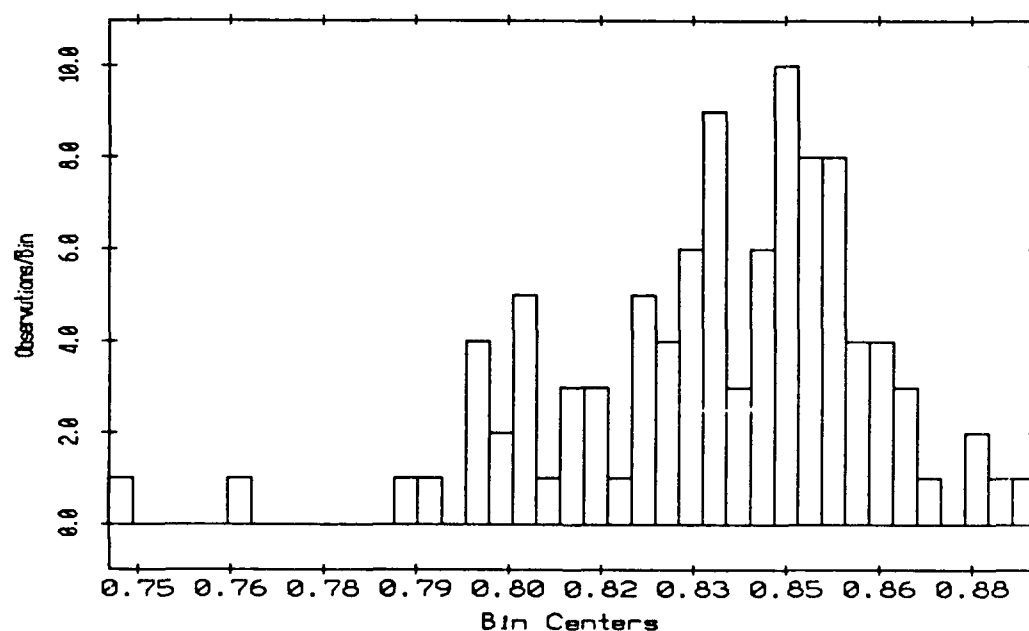


Fig. A.250 : SMC estimation, assumed model order = 4, (model 1), data length 512 points.  
Histogram of estimated SMC at normalized frequency = 6/32

## Histogram of Coherency at normalized frequency = 7/32

Model 1, Mean=0.68212, Var=0.22307e-02, assumed model order = 4

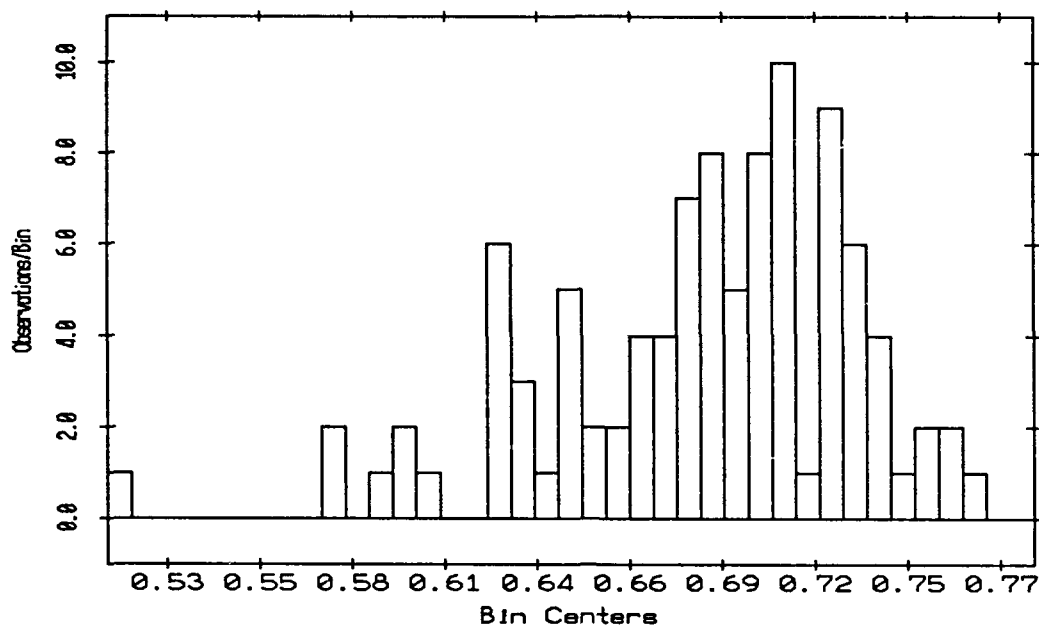


Fig. A.251 : SMC estimation, assumed model order = 4, (model 1), data length 512 points,  
Histogram of estimated SMC at normalized frequency = 7/32

## Histogram of Coherency at normalized frequency = 8/32

Model 1, Mean=0.53304, Var=0.44609e-02, assumed model order = 4

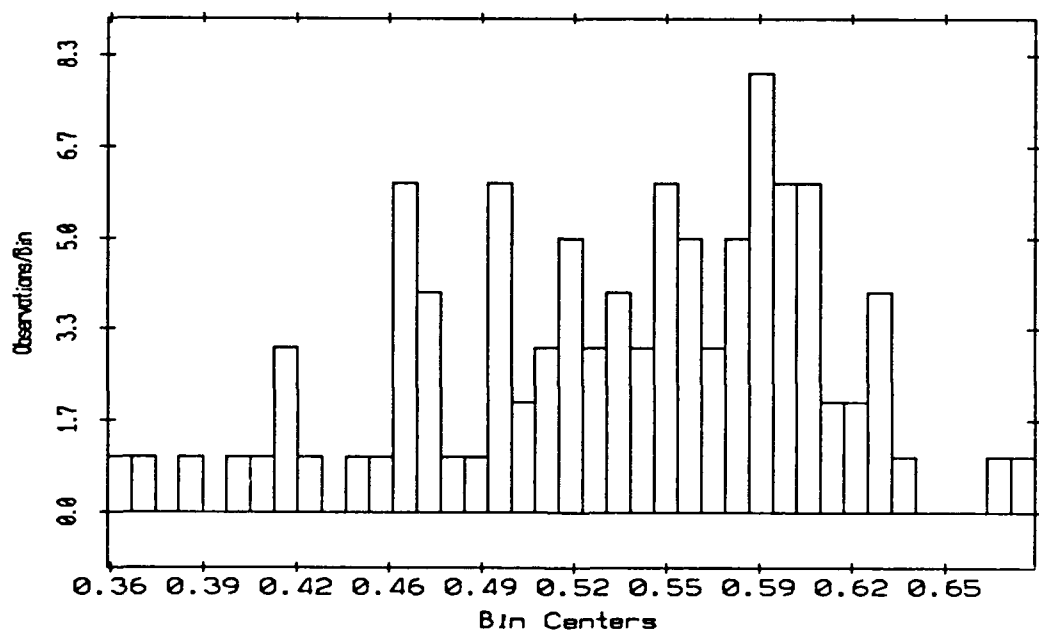


Fig. A.252 : SMC estimation, assumed model order = 4, (model 1), data length 512 points,  
Histogram of estimated SMC at normalized frequency = 8/32

## Histogram of Coherency at normalized frequency = 9/32

Model 1, Mean=0.40567, Var=0.50260e-02, assumed model order = 4

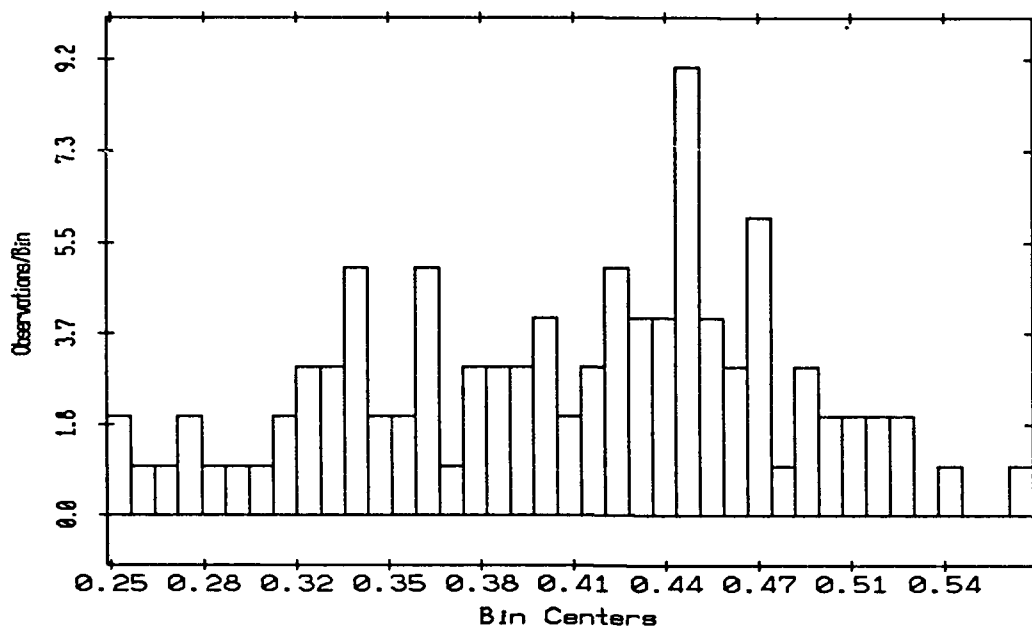


Fig. A.253 : SMC estimation, assumed model order = 4, (model 1), data length 512 points,  
Histogram of estimated SMC at normalized frequency = 9/32

## Histogram of Coherency at normalized frequency = 10/32

Model 1, Mean=0.30212, Var=0.40191e-02, assumed model order = 4

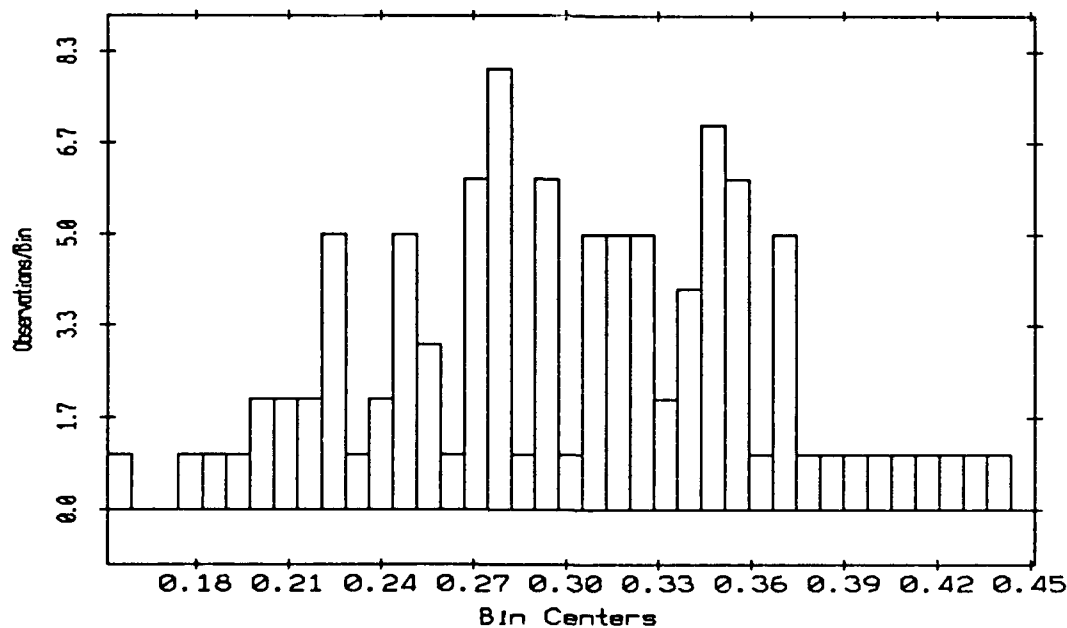


Fig. A.254 : SMC estimation, assumed model order = 4, (model 1), data length 512 points,  
Histogram of estimated SMC at normalized frequency = 10/32



## Histogram of Coherency at normalized frequency = 11/32

Model 1, Mean=0.21857, Var=0.34507e-02, assumed model order = 4

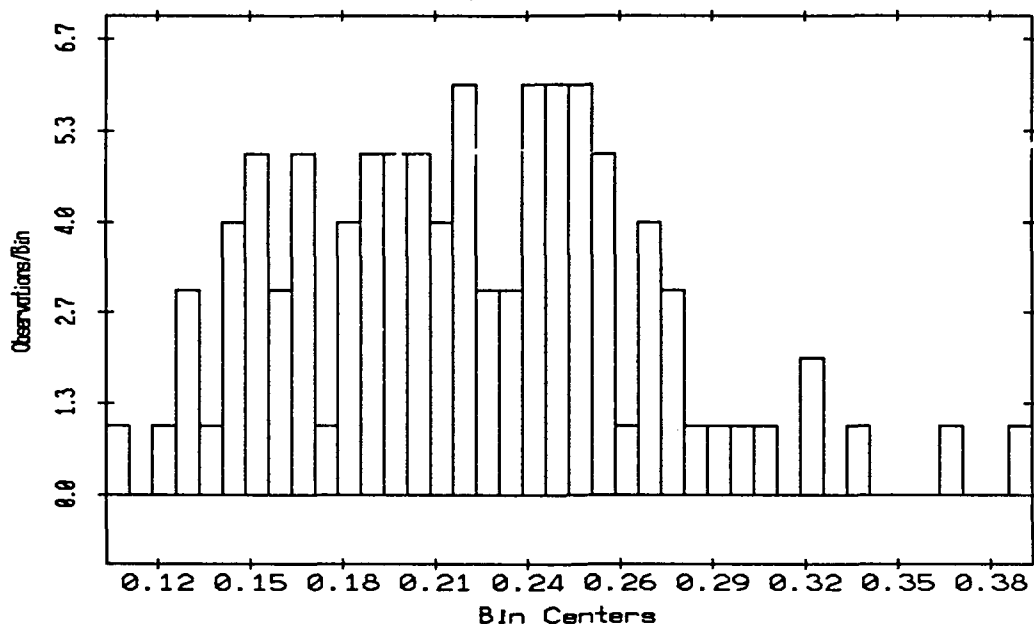


Fig. A.255 : SMC estimation, assumed model order = 4, (model 1), data length 512 points,  
Histogram of estimated SMC at normalized frequency = 11/32

## Histogram of Coherency at normalized frequency = 12/32

Model 1, Mean=0.15253, Var=0.35224e-02, assumed model order = 4

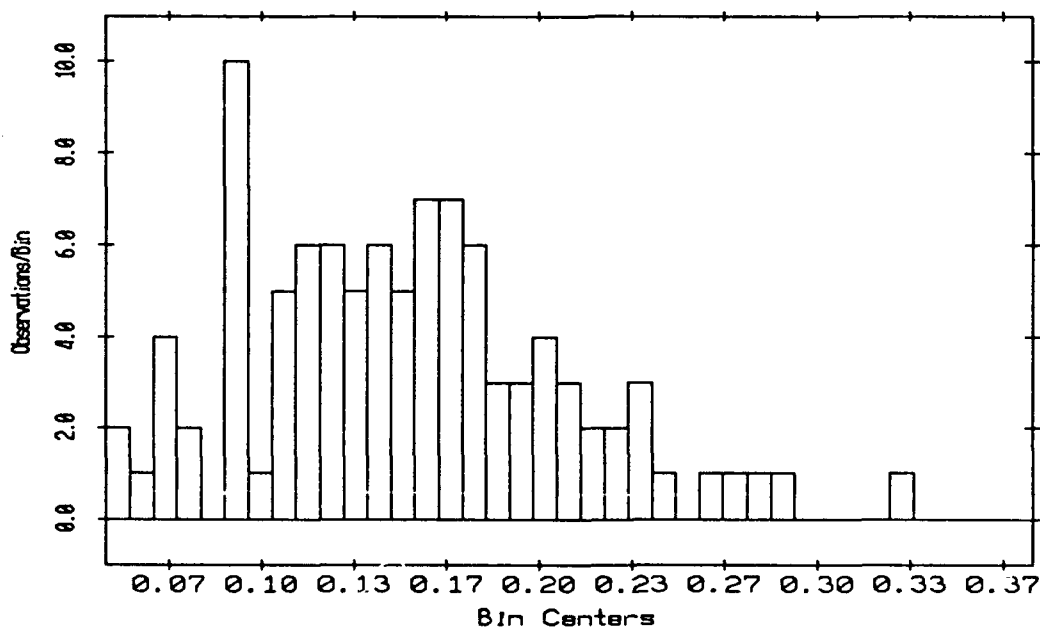


Fig. A.256 : SMC estimation, assumed model order = 4, (model 1), data length 512 points,  
Histogram of estimated SMC at normalized frequency = 12/32

## Histogram of Coherency at normalized frequency = 13/32

Model 1, Mean=0.10217, Var=0.28978e-02, assumed model order = 4

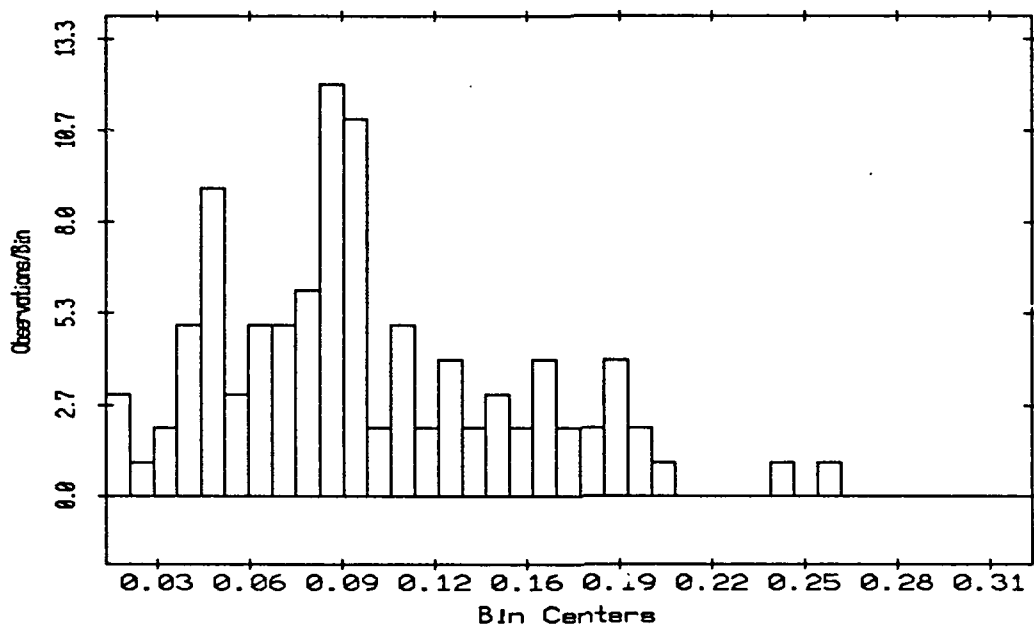


Fig. A.257 : SMC estimation, assumed model order = 4, (model 1), data length 512 points.  
Histogram of estimated SMC at normalized frequency = 13/32

## Histogram of Coherency at normalized frequency = 14/32

Model 1, Mean=0.64866e-01, Var=0.16151e-02, assumed model order = 4

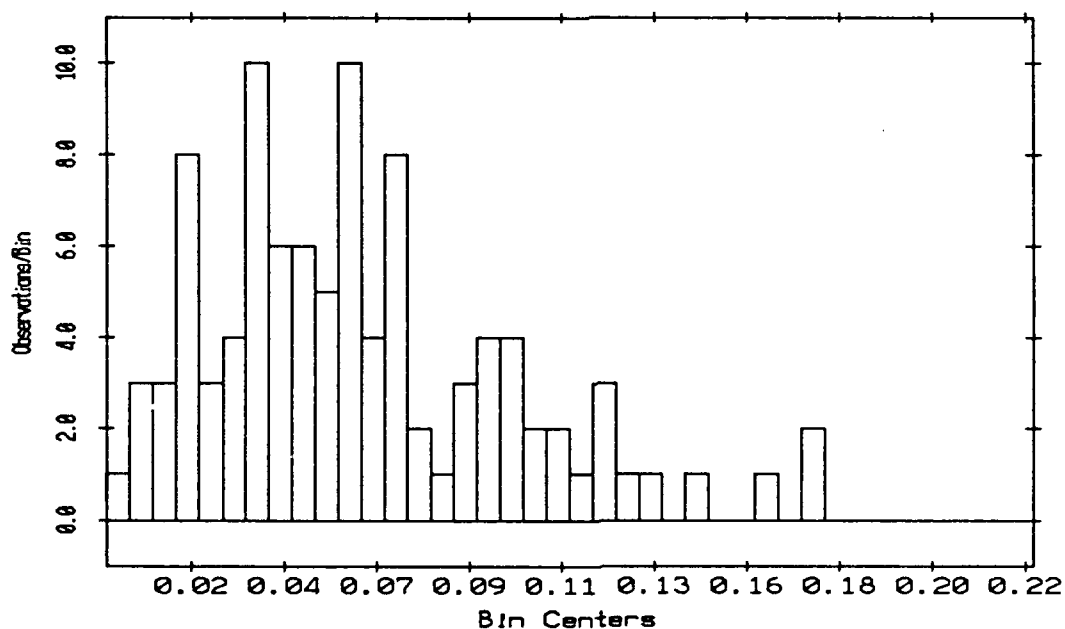


Fig. A.258 : SMC estimation, assumed model order = 4, (model 1), data length 512 points.  
Histogram of estimated SMC at normalized frequency = 14/32

## Histogram of Coherency at normalized frequency = 15/32

Model 1, Mean=0.39360e-01, Var=0.92895e-03, assumed model order = 4

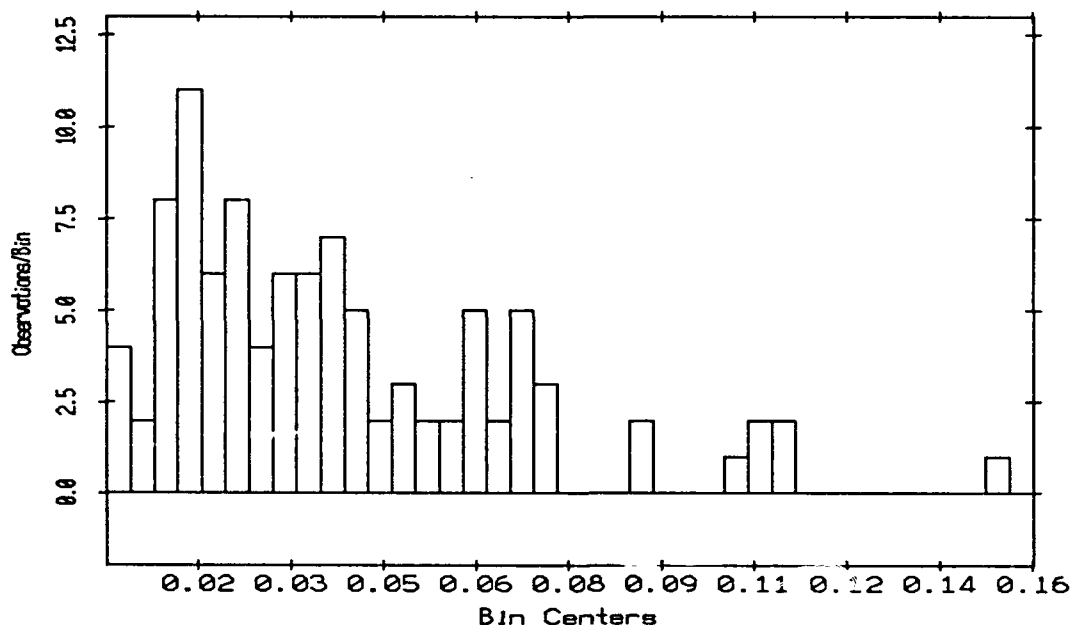


Fig. A.259 : SMC estimation, assumed model order = 4, (model 1), data length 512 points,  
Histogram of estimated SMC at normalized frequency = 15/32

## Histogram of Coherency at normalized frequency = 16/32

Model 1, Mean=0.27484e-01, Var=0.10265e-02, assumed model order = 4

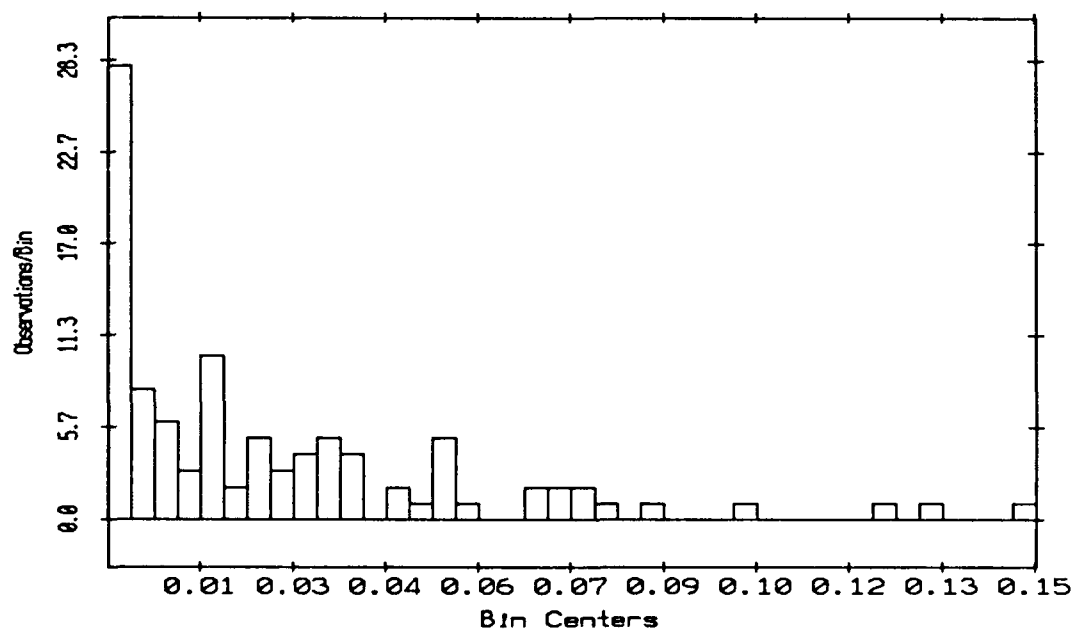


Fig. A.260 : SMC estimation, assumed model order = 4, (model 1), data length 512 points,  
Histogram of estimated SMC at normalized frequency = 16/32